Nuclear Superfluidity and Superconductivity in Neutron Stars

Nicolas Chamel

Institute of Astronomy and Astrophysics
Université Libre de Bruxelles, Belgium

IAA bruxelles ULB fnrs LA LIBERTÉ DE CHERCHER

PHAROS THE MULTI-MESSENGER PHYSICS AND ASTROPHYSICS OF NEUTRON STARS

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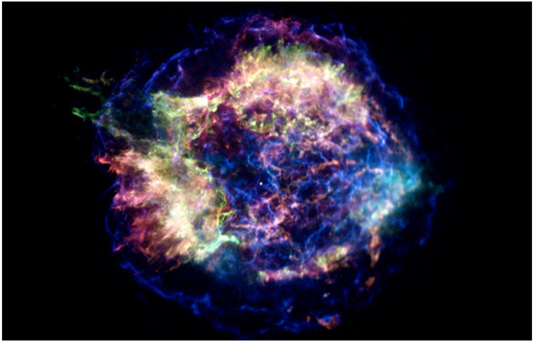
Neutron stars: laboratories for dense matter

Formed in gravitational core-collapse supernova explosions, neutron stars are the **most compact stars** in the Universe.



Nuclear physics:

$M \sim 1 - 2M_\odot$
$R \sim 10$ km
$\Rightarrow \rho \sim 10^{15}$ g cm^{-3}

Energy scale: MeV

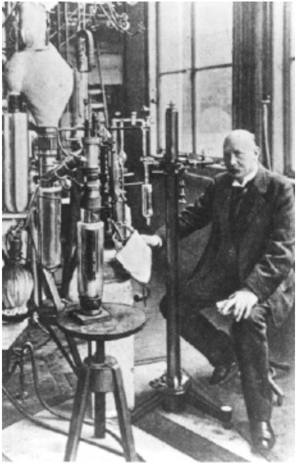
"cold" $\lesssim 10^{10}$ K $\lesssim$ "hot"

Neutron stars are initially very hot ($\sim 10^{12}$ K) but cool down to $\sim 10^9$ K within days by releasing neutrinos.

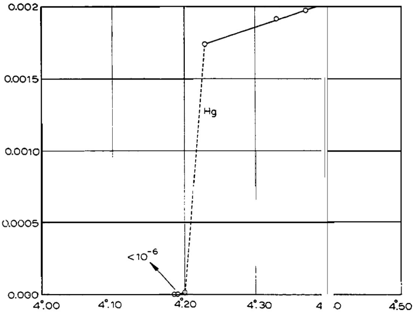
Their dense matter is thus expected to undergo various phase transitions, as observed in terrestrial materials at low-temperatures.

Discovery of "suprageleider"

Heike Kamerlingh Onnes and his collaborators were the first to liquefy helium ($T = 4.2$ K) in 1908.



On April 8th, 1911, H. K. Onnes and Gilles Holst discovered that the **electric resistance of mercury vanished** at $T_c \simeq 4.2$ K

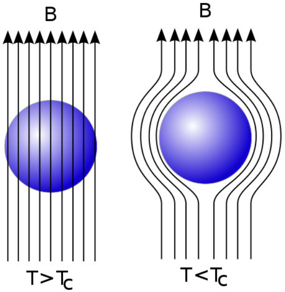


Onnes was awarded the Nobel Prize in 1913.

The year later, tin and lead were found to be also superconducting.

Meissner-Ochsenfeld effect

When placed in a weak magnetic field, a superconductor acts as a perfect diamagnet: $B = 0$.



In 1933, Walther Meissner and Robert Ochsenfeld discovered that when a material initially placed in a magnetic field is cooled below the critical temperature, the **magnetic flux is expulsed**.

Naturwissenschaften 21, 787(1933)

This phenomenon showed that a superconductor is not just a perfect conductor but correspond to a new thermodynamic state of matter.

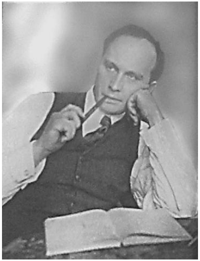
Indeed, Ohm's law $\boldsymbol{j} = \sigma \boldsymbol{E}$ implies $\boldsymbol{E} = 0$ if $\sigma \to +\infty$. From Maxwell Faraday equation, $\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E} = 0$: $\boldsymbol{B}$ should not change.

Magnetic levitation

Magnetic levitation

"Soft" vs "hard" superconductors

The Meissner effect implies the existence of a **critical magnetic field/electric current**: expulsing the flux requires an energy $\sim H^2/(8Pi)$ but energy gain by the transition is finite.



In 1935, Lev Vasilievich Shubnikov and collaborators at the Kharkov Institute of Science and Technology in Ukraine discovered that so called "hard" or type II superconductors (as opposed to "soft" or type I) exhibit **two critical fields**.

Rjabinin&Shubnikov, Phys. Z. Sow. 7, 122; Nature 135, 581 (1935)

The Meissner effect is incomplete ($B \neq 0$) in hard superconductors (metallic compounds and alloys).

Discovery of superfluidity

During the 1930s, it was found by several groups that below $T_\lambda = 2.17$ K, helium (He II) does not behave like an ordinary liquid.

"**by analogy with superconductors**, *the helium below the λ-point enters a special state which might be called* ***superfluid****.*"

Kapitza, Nature 141, 74 (1938)

Kapitza received the Nobel Prize in 1978.

"*the observed type of flow most certainly* **cannot be treated as laminar or even as ordinary turbulent flow**."

Allen & Misener, Nature 141, 75 (1938)

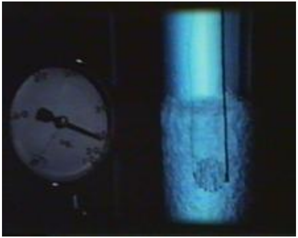
About the history of superfluidity:

Balibar in "History of Artificial Cold, Scientific, Technological and Cultural Issues", Boston Studies in the Philosophy and History of Science 299 (Springer, 2014)

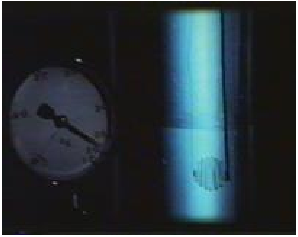
Balibar, J. Low Temp. Phys. 146, 441 (2007)

“Super heat conductivity”

- **He II does not follow Fourier’s law** for the heat current $\mathcal{J} = -\lambda \nabla T$, except in extremely fine slits or capillaries. Actually, the ratio $\mathcal{J}/|\nabla T|$ diverges as $|\nabla T| \to 0$!
  Kuerti et al., Physica 3, 266 (1936); Keesom&Keesom, Physica 3, 359; Allen et al., Nature 140, 62 (1937)
- **He II does not boil**:
  McLennan et al., Philos. Mag.14, 161 (1932)



$T > T_\lambda$

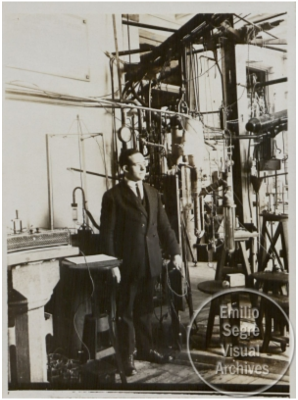


$T < T_\lambda$

http://www.lps.ens.fr/~balibar/Allen-boiling.mpg

Heat in He II is not transported according to classical laws.

Onnes and Dana observations about liquid helium



Incidentally, Kamerlingh Onnes and his collaborators discovered superfluidity without realizing it the same day they discovered superconductivity in April 1911!

Onnes noted about liquid helium:
"*Just before the lowest temperature [about 1.8 K] was reached, the boiling suddenly stops...*"

Leo Dana, a visiting student at Onnes' lab measured the lambda transition in the specific heat in 1922 but no one paid attention!

About the history of superconductivity:
van Delft&,Kes, Phys. Today, 63, 9, 38 (2010)

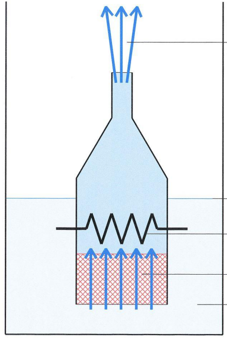
"Superfluidity"

- **"superleak"**: He II flows without resistance through very narrow slits and capillaries, almost independently of the pressure drop.
  Keesom & van der Ende. Proc. Roy. Acad. Amsterdam 33, 24 (1930)
- **"superflow"**: persistent flow of He II (note the similarity with persistent currents in superconductors)
  Reppy & Depatie, PRL 12, 187 (1964)
- "superfluidity" disappears beyond some **critical velocity** (note the similarity with critical currents in superconductors)
- on the other hand, He II exhibits similar viscosity as He I under certain circumstances

`http://www.lps.ens.fr/~balibar/Allen-superflow.mpg`

He II does not follow the classical laws of hydrodynamics.

Fountain effect

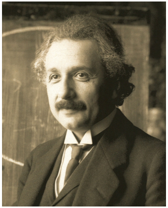


Allen & Jones, Nature 141, 243 (1938)

`http://www.lps.ens.fr/~balibar/Allen-fountain.mpg`

The superfluid flows from the cooler to the hotter region. From the second law of thermodynamics, we thus conclude that the superfluid carries no heat (no entropy).

Superfluidity and Bose-Einstein condensation



Satyendra Nath Bose and Albert Einstein predicted in 1925 that at low enough T an **ideal gas of bosons condense into a macroscopic quantum state**. But this prediction was largely ignored or considered as incorrect.

The association between **Bose-Einstein condensation** (BEC) and superfluidity was first advanced by Fritz London.

It was a key idea for developing the microscopic theory of superfluidity and superconductivity.

London, Nature 141, 643 (1938)

Two-fluid model

Following London's suggestion that superfluidity is related to BEC, Laszlo Tisza postulated that **He II contains two distinct components***:

- a superfluid that carries no entropy (condensate)
- a normal viscous fluid.

This model explained all the observed phenomena and predicted thermomechanical effects like **"temperature waves"**. *Tisza, Nature 141, 913 (1938)*

Landau further developed the two-fluid model based on **"quasiparticle" excitations in quantum fluids** without any reference to BEC. He received the Nobel prize in Physics in 1962.

Landau, Phys. Rev. 60, 356 (1941)

* This two-fluid picture was first introduced for superconductors.

Gorter & Casimir, Phys. Z. 35, 963 (1934)

Flow quantization and vortices

The circulation of the superfluid flow is quantized

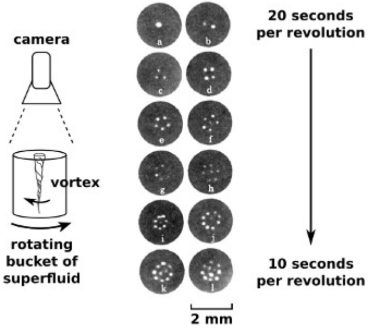
$$\oint \boldsymbol{v_s} \cdot d\ell = \frac{Nh}{m} \text{ with } N = 0, 1, \text{ etc.}$$

where $\boldsymbol{v_s}$ is the “superfluid velocity”.

Onsager, Il Nuovo Ciment 6, 279 (1949); Feynman, Prog. Low. T. Phys. 1, 17 (1955)

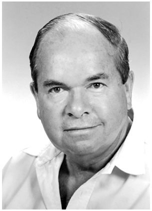
A superfluid rotating at angular frequency ω in a bucket of radius R is threaded by $N = \dfrac{2m\pi R^2\omega}{h}$ **quantized vortex lines**, each of which carries an angular momentum $\hbar$.

In between vortices, the flow is “irrotational” $\nabla \times \boldsymbol{v_s} = 0$.



Yarmchuk et al., PRL43, 214 (1979)

Abrikosov state

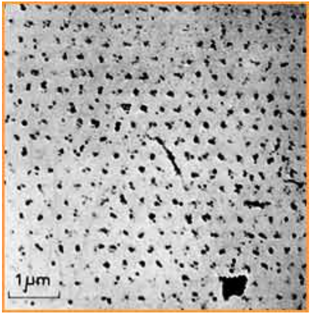


In 1957, Alekseï Alekseïevitch Abrikosov predicted that a "hard" superconductor is threaded by a **regular array of magnetic flux tubes**.

He was awarded the Nobel Prize in Physics in 2003.

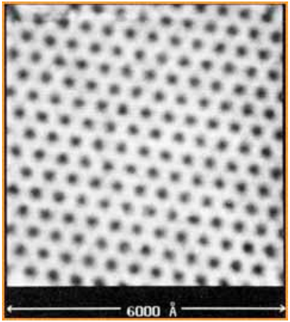
Abrikosov, Soviet Physics JETP 5, 1174 (1957)

Pb-4at%In



Essmann&Trauble, Phys. Lett. 24A, 526 (1967)

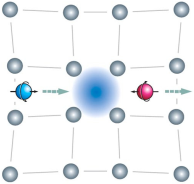
$NbSe_2$



Hess et al., PRL 62, 214 (1989)

Bardeen-Cooper-Schrieffer theory

A microscopic theory was proposed in 1957 by Bardeen, Cooper and Schrieffer (1972 Nobel Prize in Physics).



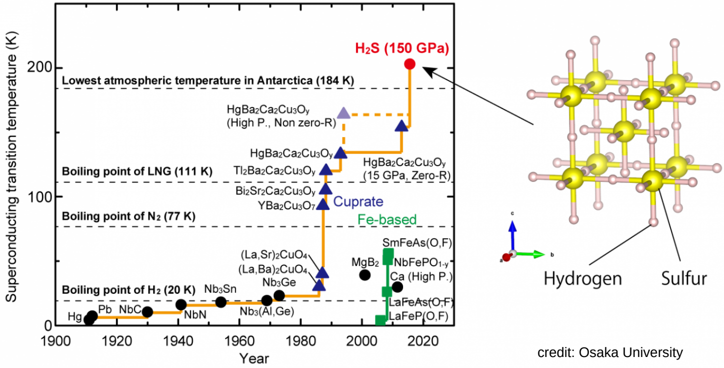
In a superconductor, the dynamical distorsions of the crystal lattice (phonons) can induce an **attractive effective interaction** between electrons of opposite spins.

- Electrons form **bosonic pairs** and can thus **condense** below T_c. A superconductor can thus be viewed as a charged superfluid.
- Pairing leads to an **energy gap** ($\sim$ binding energy of the pairs)

The BCS theory suggested that fermionic atoms could be superfluid. In 1971, Osheroff discovered ^{3}He superfluidity below 2.5 mK.

Timeline of superconductor discoveries

High-T_c cuprate superconductors were discovered in 1986 by IBM researchers G. Bednorz and K.A. Müller (Nobel Prize in 1987).

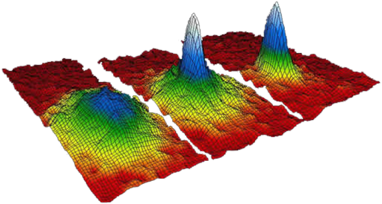


credit: Osaka University

Recently CH_8S has been found to be superconducting (under high pressures) at room temperatures

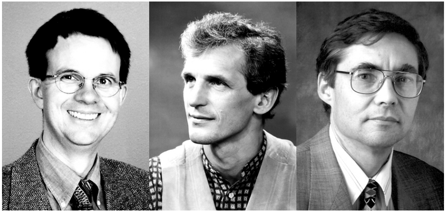
Snider et al., Nature 586, 373 (2020)

Bose-Einstein condensation in dilute atomic gases



On June 5, 1995, the first dilute BEC was produced by Eric Cornell and Carl Wieman at the University of Colorado at Boulder NIST-JILA, with $\sim$ 2000 rubidium ^{87}Ru atoms cooled to 170 nK.

Shortly thereafter, Wolfgang Ketterle's team at MIT obtained a BEC of $\sim 5 \times 10^5$ sodium ^{23}Na atoms cooled to 2 μK.

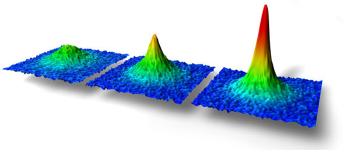


For their achievements, Cornell, Ketterle and Wieman were awarded the 2001 Nobel Prize in Physics.

Since that time, dilute BEC have been produced by other groups using various kinds of atoms.

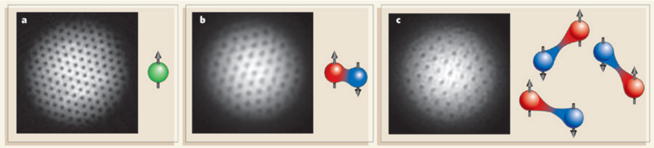
Fermionic condensates: from BEC to BCS

On December 16, 2003, the first dilute fermionic condensate was produced by Deborah Jin at JILA with 500 000 potassium ^{40}K atoms cooled to 50 nK.



By varying the pairing interaction with a magnetic field, it is possible to study the **crossover from a BEC to a BCS state**.

Leggett&Zhang in "The BCS-BEC Crossover and the Unitary Fermi Gas", Lecture Notes in Physics 836 (Springer, 2012), pp. 33-47



Quantized vortices in: (a) a BEC of bosonic sodium atoms, a fermionic condensate of ^{6}Li atoms in the BEC (b) and BCS (c) states.

Zwierlein et al, Nature 435, 1047 (2005)

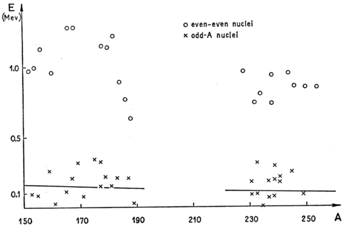
Nuclear superfluidity and superconductivity

The implications of the BCS theory (published in January 1957) for atomic nuclei were first discussed by A. Bohr, B. R. Mottelson, and D. Pines during the Summer of 1957.

D. Pines in "BCS: 50 Years" (World Scientific, 2011), pp.85-105

Bohr, Mottelson, and Pines speculated that nuclear pairing might explain the **energy gap** in the excitation spectra of nuclei.

Phys. Rev. 110, 936 (1958)

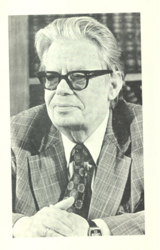


They also anticipated that nuclear pairing could explain **odd-even mass staggering, and the reduced moments of inertia** of nuclei.

"**The present data are insufficient to indicate the limiting value for the gap in a hypothetical infinitely large nucleus.**"
Bohr, Mottelson, Pines.

Superfluidity and superconductivity in neutron stars

In the 1960's, several superconductors were known but only one superfluid, ^{4}He (superfluidity of ^{3}He was discovered at the beginning of the 1970's).



Bogoliubov, who developed a microscopic theory of superfluidity and superconductivity, was the first to explore its application to nuclear matter.

Dokl. Ak. nauk SSSR 119, 52 (1958)

Neutron-star superfluidity was predicted by Arkady Migdal in 1959, and first studied by Ginzburg & Kirzhnits in 1964 **before the discovery of pulsars** in 1967.

Migdal, Nucl. Phys. 13, 655 (1959)

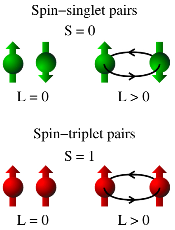
Ginzburg & Kirzhnits, Zh. Eksp. Teor. Fiz. 47, 2006 (1964)

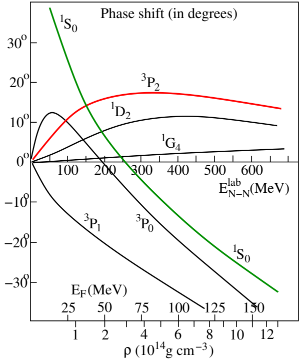
Neutron pairing channels

Below $\sim 10^{10}$ K, neutrons form pairs and condense into a superfluid phase

Pairing ($^{2S+1}L_J$) if $\delta > 0$

Gezerlis, Pethick, Schwenk in Novel Superfluids, Vol.2, Bennemann&Ketterson (Oxford University Press, 2014), Chap. 22





Microscopic calculations:

- diagrammatic methods
- variational methods
- quantum Monte Carlo methods.

For a recent review: *Sedrakian & Clark, Eur. Phys. J. A55, 167 (2019)*

Superfluid and superconducting phases

The interior of a neutron star is not only made of neutrons, but consists of protons, leptons, hyperons, and possibly mesons, and even deconfined quarks!

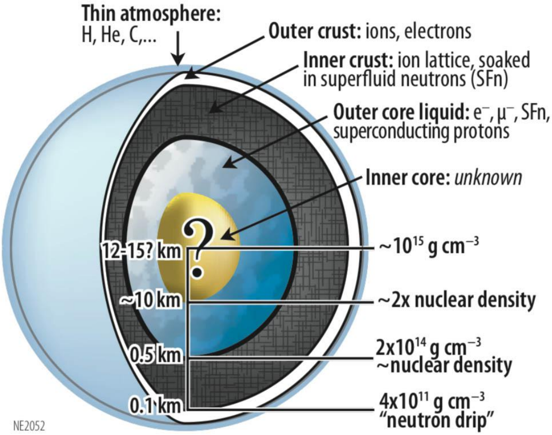
Possible phases:

- $^1S_0$ and $^3PF_2$ proton pairing
- neutron-proton pairing
- hyperon-hyperon pairing ($^1S_0$ $\Lambda\Lambda$)
- hyperon-nucleon pairing ($^1S_0$ $n\Lambda$, $^1S_0$ $n\Sigma^-$, $^3SD_1$ $n\Sigma^-$)
- quark pairing

Although $^1S_0$ proton superconductivity is well established, the other superfluid/superconducting phases are more uncertain.

Anatomy of a neutron star

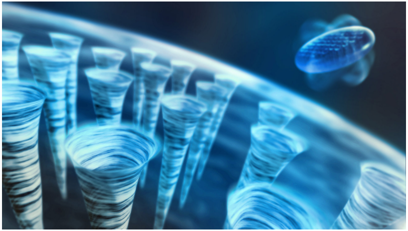
The huge gravity of neutron stars produces the highest-T_c ($\sim 10^{10}$ K) superfluids and superconductors known in the Universe:



Quantized vortices and fluxoids in neutron stars

A rotating superfluid neutron star is threaded by **neutron vortices**, each carrying an angular momentum $\hbar$.

Surface density of neutron vortices:
$n_v(\mathrm{cm}^{-2}) \sim 6 \times 10^5/P(10\ \mathrm{ms})$.



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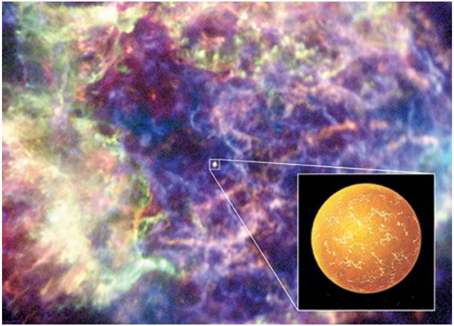
A neutron star is also threaded by **proton fluxoids**, each carrying a flux $\Phi_0 = hc/(2e)$ (assuming type II superconductivity).

Surface density of proton fluxoids: $n_\Phi(\mathrm{cm}^{-2}) \sim 5 \times 10^{18}/B(10^{12}\ \mathrm{G})$.

In a neutron star, $n_\Phi \sim 10^{13} - 10^{14} n_v$ but type I superconductivity is not excluded (superconductivity may be neither of type I nor II !).
Sedrakian & Clark, in "Pairing in Fermionic Systems", (World Scientific, 2006)

Superstars

The huge gravity of neutron stars produces the highest-T_c ($\sim 10^{10}$ K) and largest superfluids and superconductors known in the Universe !



Neutron stars	**$\sim 10^{10}$ K**
$\vdots$	$\vdots$
CH_8S	288 K
Cuprates	1 – 130 K
Electrons in ordinary metals	1 – 25 K
Helium-4	2.17 K
Helium-3	2.491×10^{-3} K
Bosonic condensates	$\sim 10^{-6}$ K
Fermionic condensates	$\sim 10^{-8}$ K

Predicted long ago, these quantum phases may be probed through astrophysical observations.