Superfluid neutron star dynamics

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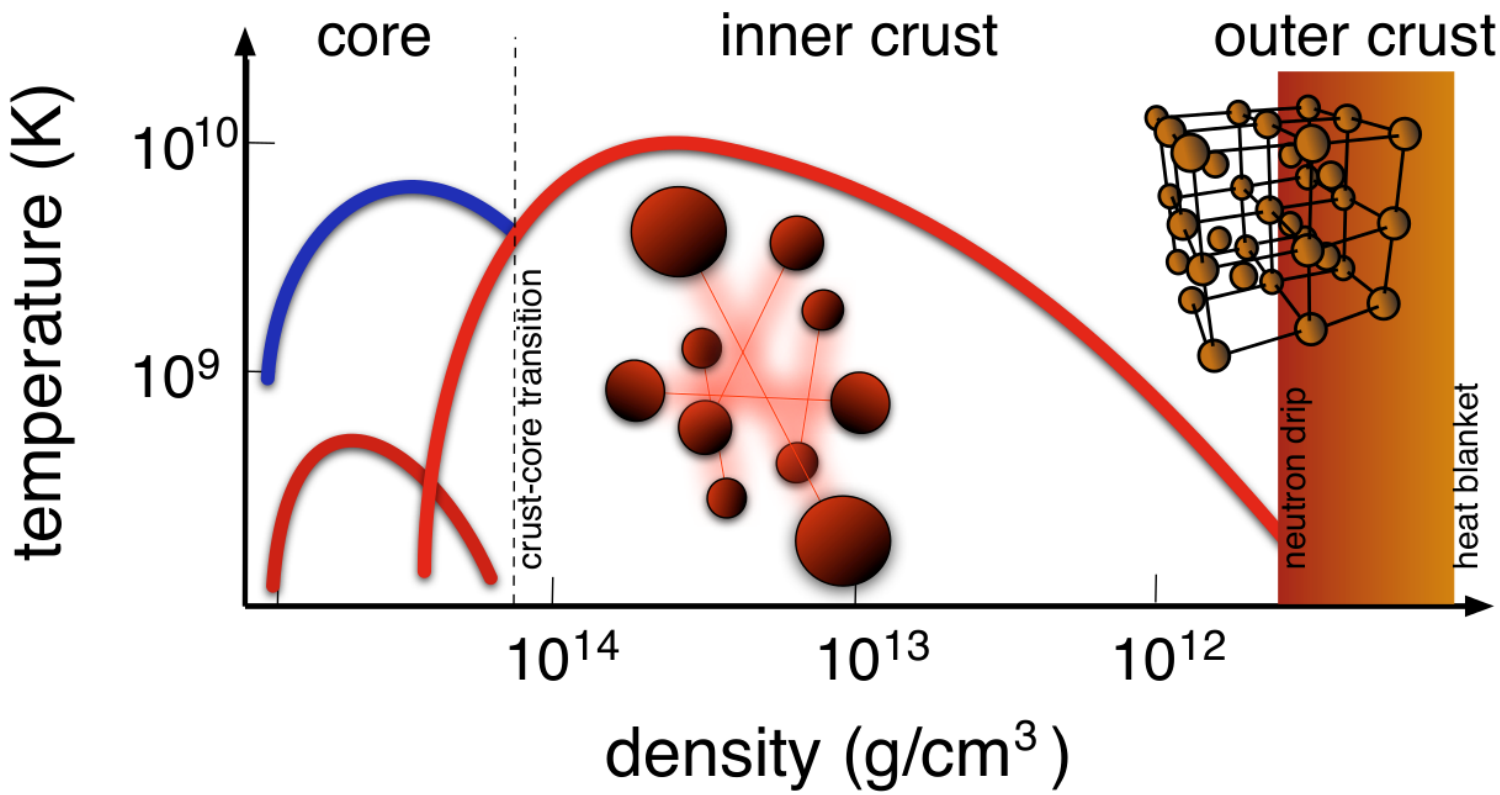
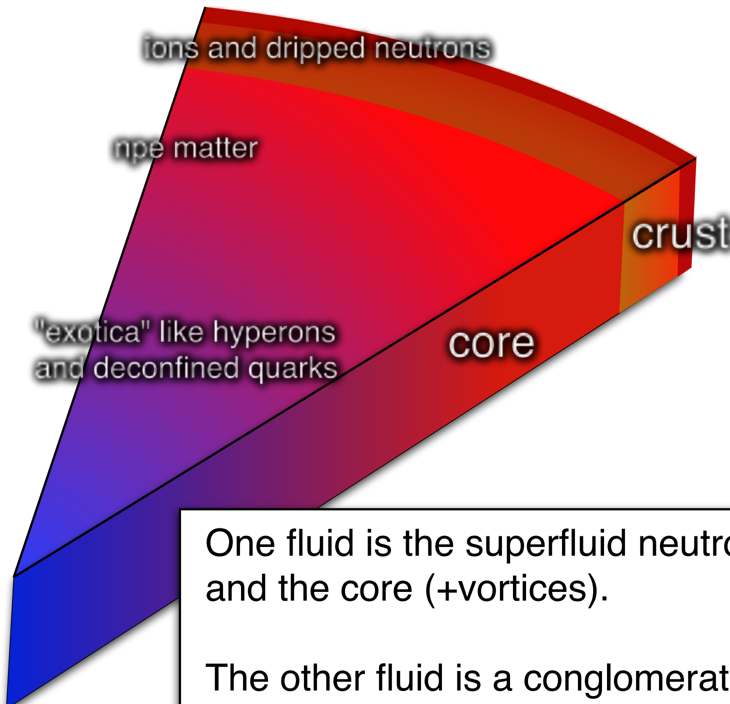
Mature neutron stars are "cold" (10^8K<< T_{Fermi}=10^{12}K) so they **should** be either solid or superfluid.



Since late 1950s, nuclear physics calculations indicate "BCS-like" pairing gaps for neutrons and protons.

Many pulsars are perfect "clocks", but in some cases the spin is not so regular. "Glitches" (=sudden spin-up events, observed in more than 100 systems) provide "evidence" for superfluidity.

Phenomenology inspired by **classic two-fluid model** for superfluid Helium.



One fluid is the superfluid neutrons in the crust and the core (+vortices).

The other fluid is a conglomerate of all charged components (ions, protons, electrons...)

In laboratory systems like Helium, the two fluids represent atoms and excitations (e.g. phonons).

In a neutron star:

– electrons/muons in the core are coupled (electromagnetically) to the protons on very short timescales

– vortices and fluxtubes are sufficiently dense that smooth averaging can be performed

Need to model the dynamics of two interacting fluids.

As a starting point, assume that superfluid vortices are sufficiently dense that smooth-averaging makes sense (note: no longer irrotational!).

We have two continuity equations and two momentum equations:

$$\partial_t n_{\mathrm{x}} + \nabla_i (n_{\mathrm{x}} v_{\mathrm{x}}^i) = 0$$

$$(\partial_t + v_{\mathrm{x}}^j \nabla_j) p_i^{\mathrm{x}} + \nabla_i (\Phi + \tilde{\mu}_{\mathrm{x}}) + \varepsilon_{\mathrm{x}} w_j^{\mathrm{yx}} \nabla_i v_{\mathrm{x}}^j = f_i^{\mathrm{x}}$$

where the relative velocity is $w_i^{\mathrm{yx}} = v_i^{\mathrm{y}} - v_i^{\mathrm{x}}$ and the momenta are given by

$$p_i^{\mathrm{x}} = v_i^{\mathrm{x}} + \varepsilon_{\mathrm{x}} w_i^{\mathrm{yx}}$$

This encodes the (non-dissipative) **entrainment effect**, due to which the velocity of each fluid does not have to be parallel to its momentum.

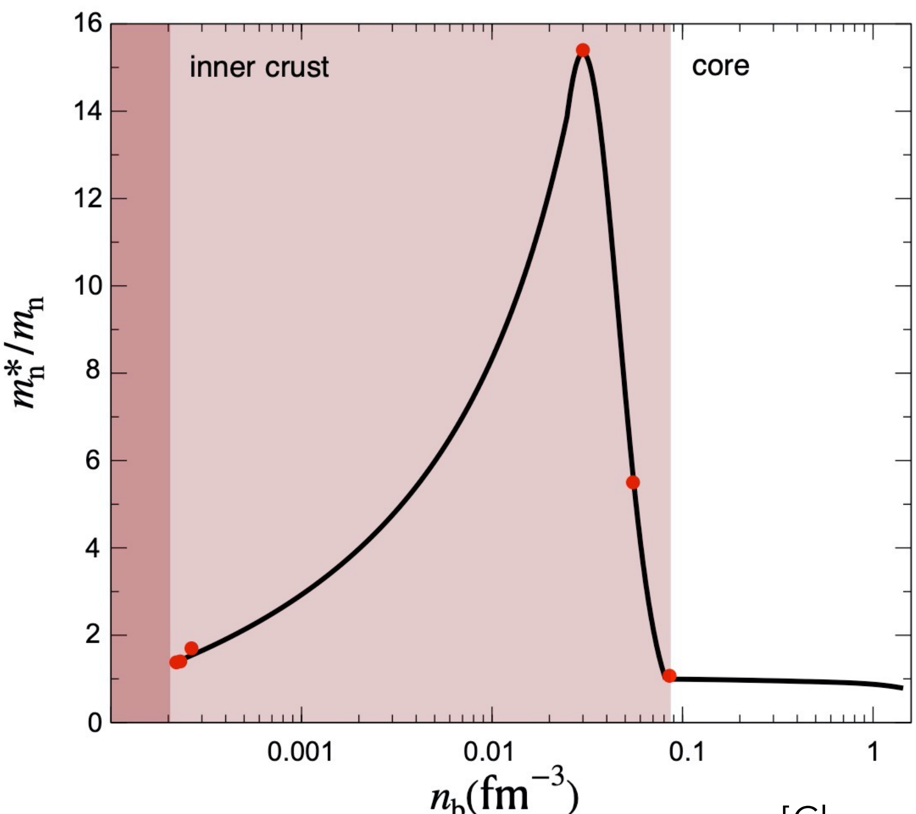
Comment: In the Helium problem, the entrainment encodes the “normal fluid” density (the thermal excitations). For neutron stars we (at least initially) ignore thermal excitations (although, note transitions!).

Entrainment can be thought of in terms of an "effective mass";

$$\rho_p \varepsilon_p = n_p (m_p - m_p^*)$$

In the star's core, it is a fairly weak effect, due to the strong interaction.

In the inner crust, the effective neutron mass may be large (due to Bragg scattering off of the nuclear lattice).



[Chamel+ 2005]

Also need to consider new dissipation channels. Ignore particle scattering, so no normal "friction".

In superfluid Helium, the presence of vortices leads to "mutual friction" due to scattering off of "normal atoms" in the vortex core.

Standard form (Hall+Vinen 1950s, for a straight vortex array);

$$f_i^{\mathrm{mf}} = \frac{R}{1+R^2}\varepsilon_{ijk}\hat{\omega}_{\mathrm{n}}^j \varepsilon^{klm}\omega_l^{\mathrm{n}} w_m^{\mathrm{np}} + \frac{R^2}{1+R^2}\varepsilon_{ijk}\omega_{\mathrm{n}}^j w_{\mathrm{np}}^k$$

Where the vorticity is given by

$$\omega_{\mathrm{n}}^i = \varepsilon^{ijk}\nabla_j p_k^{\mathrm{n}}$$

– electron scattering off vortices generally leads to

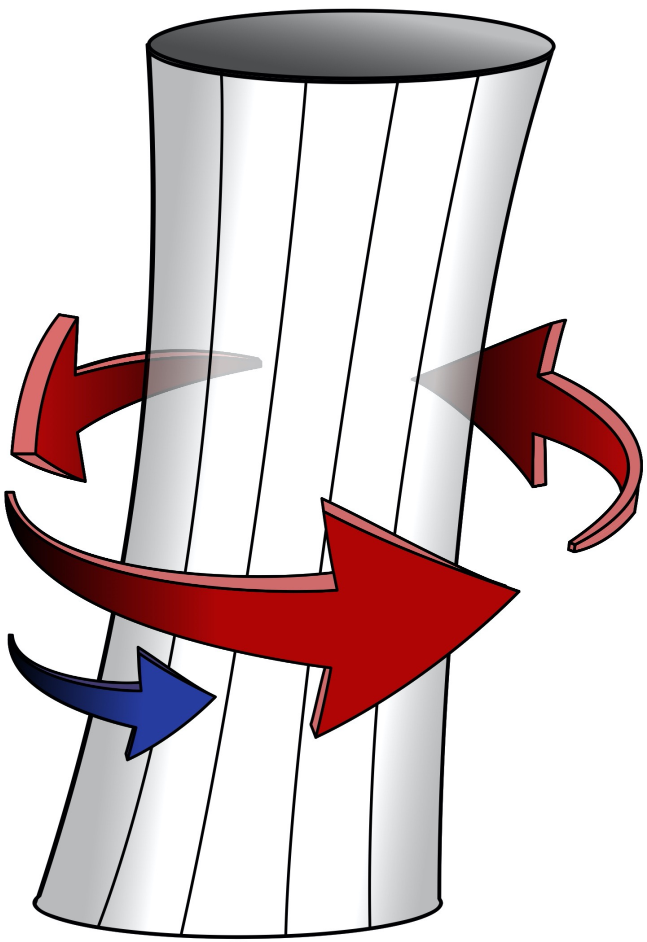
$$R << 1$$

– entrainment leads to magnetised vortices and stronger friction

– Kelvin waves may be excited due to interactions with the crust nuclei

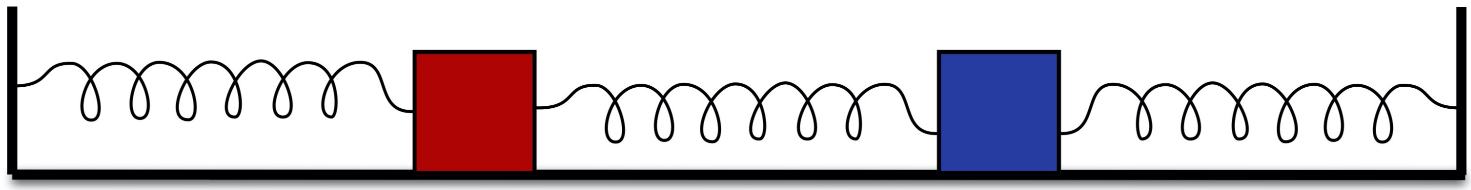
– vortex/fluxtube interactions may lead to a strong effect (velocity dependent)

– the form of the friction is different for vortex tangles (=turbulence)

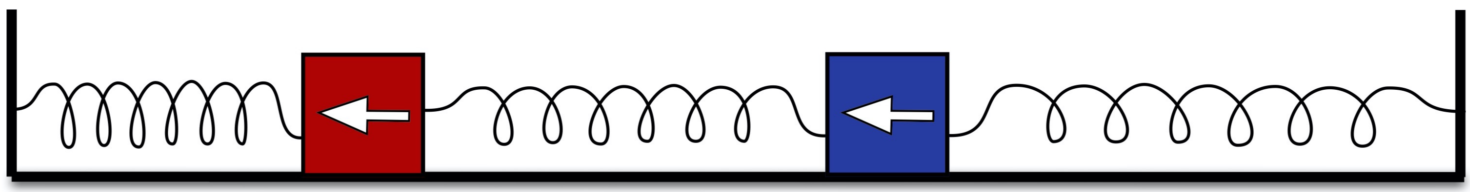


1. the neutron circulation (red) is quantised
2. entrainment leads to a proton current (blue) around the vortex core (electrons cannot react on the small scale as the vortex core << the mean free path)
3. the proton current generates a (strong) magnetic field on the vortex
4. electrons scatter off of the magnetic field, leading to dissipation

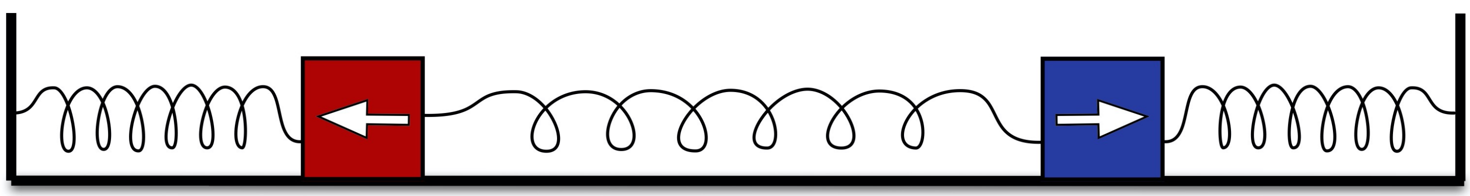
Simple toy model for **oscillations**: Masses coupled by springs



Relative to equilibrium, the system has two degrees of freedoms.



1. The masses move together.



2. The masses move out of sync.

Rewrite the perturbation equations for a non-rotating star, using displacement

$$\delta v_{\mathrm{x}}^{i} = \partial_t \xi_{\mathrm{x}}^{i}$$

working with two components n and p (including the electrons), we introduce:

- Total density perturbation:

$$\delta\rho = \delta\rho_{\mathrm{n}} + \delta\rho_{\mathrm{p}}$$

- Total mass flux, with associated weighted displacement:

$$\rho\xi^{i} = \rho_{\mathrm{n}}\xi_{\mathrm{n}}^{i} + \rho_{\mathrm{p}}\xi_{\mathrm{p}}^{i}$$

- Velocity difference, leading to:

$$\psi^{i} = \xi_{\mathrm{n}}^{i} - \xi_{\mathrm{p}}^{i}$$

- Perturbed pressure:

$$\delta p = \rho_{\mathrm{n}}\delta\tilde{\mu}_{\mathrm{n}} + \rho_{\mathrm{p}}\delta\tilde{\mu}_{\mathrm{p}}$$

- Deviation from beta equilibrium:

$$\delta\beta = \delta\tilde{\mu}_{\mathrm{n}} - \delta\tilde{\mu}_{\mathrm{p}}$$

The two Euler equations then take the form:

$$\partial_t^2 \xi_i + \frac{1}{\rho}\left(\rho_{\rm n} \nabla_i \delta\tilde{\mu}_{\rm n} + \rho_{\rm p} \nabla_i \delta\tilde{\mu}_{\rm p}\right) + \nabla_i \delta\Phi = 0$$

where

$$\frac{1}{\rho}\left(\rho_{\rm n} \nabla_i \delta\tilde{\mu}_{\rm n} + \rho_{\rm p} \nabla_i \delta\tilde{\mu}_{\rm p}\right) = \nabla_i\left(\frac{\delta p}{\rho}\right) - \nabla_i\left(\frac{\rho_{\rm p}}{\rho}\right)\delta\beta$$

and

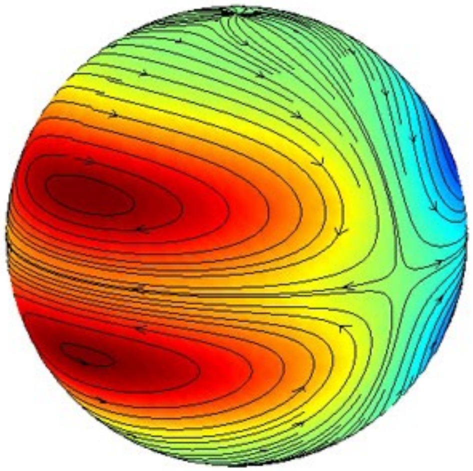
$$\left(1 - \varepsilon_{\rm p} - \varepsilon_{\rm n}\right)\partial_t^2 \psi_i + \nabla_i \delta\beta = 0$$

When the equations are written in this form, it is easy to identify the two degrees of freedom. In general, they are coupled by:

1. The equation of state
2. Gravity
3. Entrainment

Also… mutual friction enters through the difference, so requires $\delta\beta \neq 0$.

Neutron stars have a zoo of oscillation modes, more or less directly associated with the various "restoring forces" in the system.



In principle, observations can be used to probe the star's interior.

Superfluids have additional degrees of freedom:

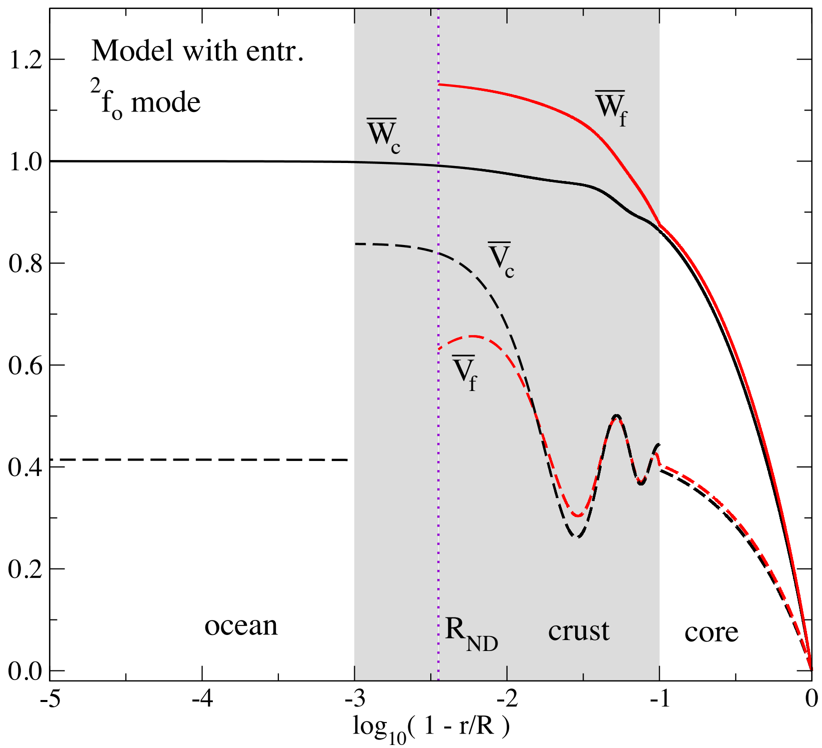
- Acoustic modes restored by pressure.
- Superfluid modes restored by deviation from chemical equilibrium.

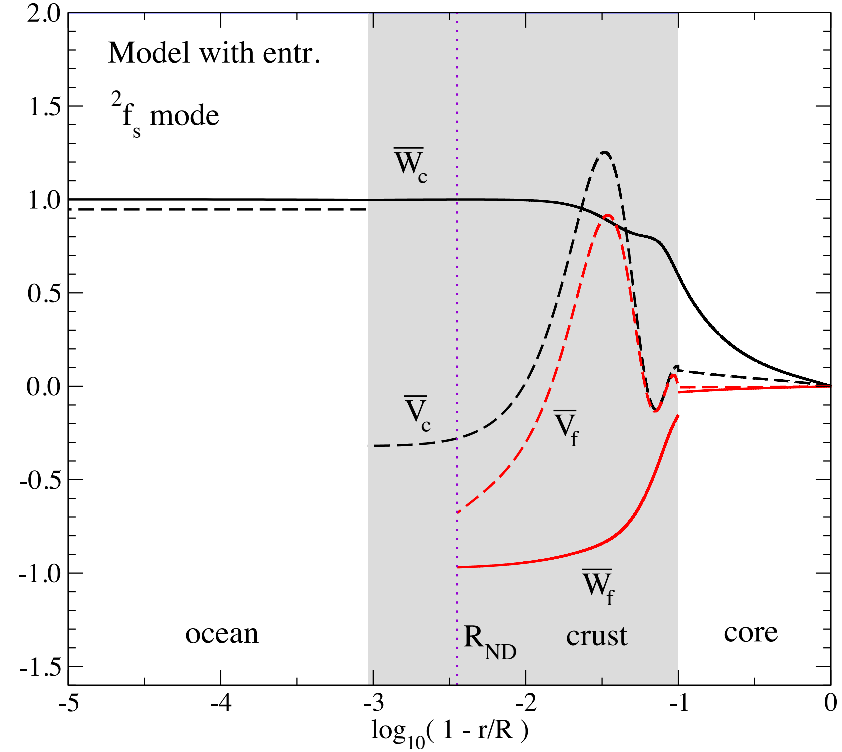
$$\omega^2 \approx \frac{l(l+1)c_s^2}{r^2} \qquad\qquad \omega^2 \approx \frac{m_p}{m_p^*}\frac{l(l+1)c_p^2}{r^2}$$

The problem is complicated by the different phase transitions (crust/core/ocean) and the need to use "realistic" parameters (=relativity).

Note: In the crust we distinguish neutrons+protons locked in nuclei (c) from the "free" neutrons (f) – not the same identification as in the core!

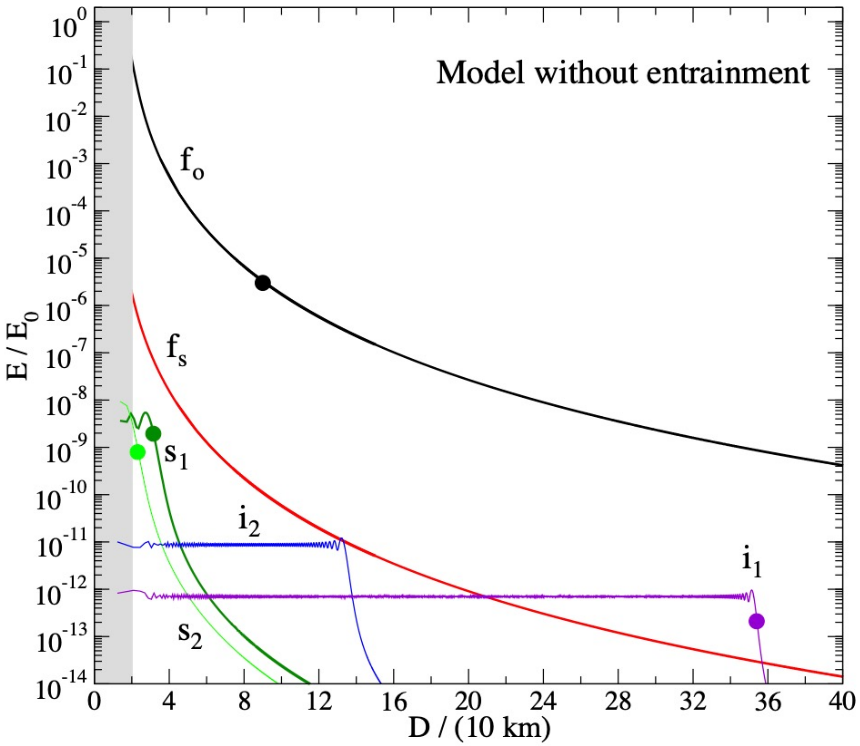
Example: the two fundamental modes for a model with “realistic” entrainment profile and elastic crust.

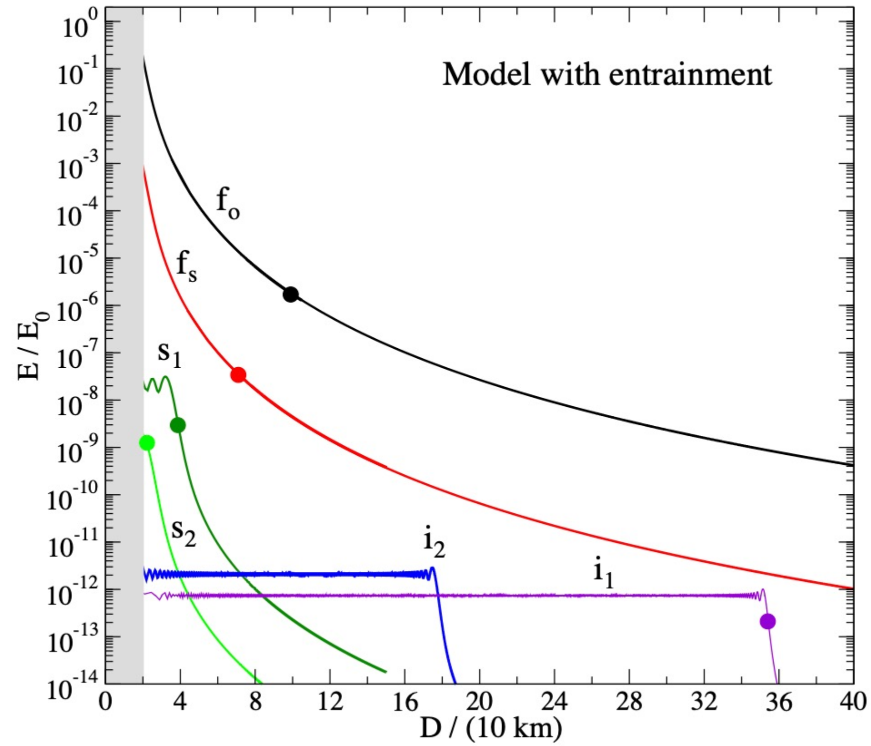




[Passamonti, NA & Pnigouras 2022]

Recent application: Dynamical tides in neutron star binaries. Does the presence of superfluidity make a difference?





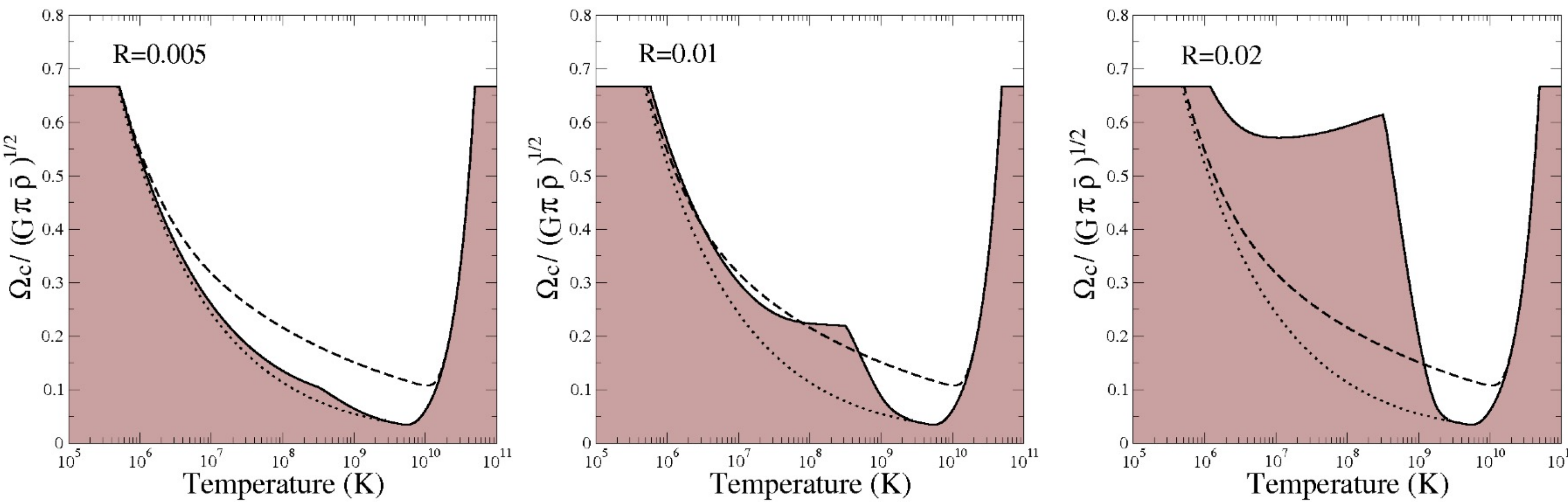
[Passamonti, NA & Pnigouras 2022]

The oscillations of a rotating star may be driven unstable by the emission of gravitational waves. The most important unstable modes are thought to be the fundamental f-mode and the inertial r-mode.

In each case, the instability "window" depends on dissipation (bulk/shear viscosity, boundary layers…).

Mutual friction may **completely suppress** the gravitational-wave driven instability of the star's fundamental mode.

The situation is less "dramatic" for the unstable r-modes.



[Haskell, NA & Passamonti 2009]

reflections

Superfluidity is relevant for pulsar glitches, neutron star cooling, free precession, seismology (gravitational waves/magnetar flares), magnetic field evolution…

We do not (yet) have clear answers to the what/why/how of many aspects of the problem, but there have been "interesting" results;

— relevance of entrainment and the mutual friction damping;

— two-stream instability (perhaps triggering turbulence);

— pinning and role of core superconductivity;

— various aspects of glitches

The main challenge relates to the vast range of scales involved - from the 100 fm coherence length of a vortex to the 10-100 m that can be resolved in numerical (relativity) simulations…

Laboratory systems provide insight, but… we need to improve our "fluid" models (relativity!). Step by (potentially painful) step.