IOCSYM
2026

Conference

The 1st International Online Conference on Symmetry

20–22 October 2026, online



Program and Abstract Book

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The 1st International Online Conference on Symmetry

20–22 October 2026 | Online

MDPI

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Table of Contents

Welcome from the Chair 1
Keynote Speakers 3
Invited Speakers 5
Program at a Glance 7
IOCSYM 2026 Program 8
Session 1. Mathematical Physics 18
Session 2. High Energy Physics, Particles and Fields 55
Session 3. Applied Physics 91
Session 4. Astronomy, Astrophysics and Cosmology 106
Session 5. Algebra and Geometry 146
Session 6. Mathematical Analysis and Statistics 165
Session 7. Crystalline Metals and Alloys 181

Welcome from the Chair

We are pleased to announce the 1st International Online Conference on Symmetry (IOCSYM2026). This event is organized by the MDPI open access journal *Symmetry* (ISSN: 2073-8994; Impact Factor: 2.2) and will take place online from 20 to 22 October 2026. We look forward to bringing together researchers to explore the world of symmetry in this brand-new, interactive online format.

This conference represents a unique opportunity for symmetry scientists around the world to share their latest research, communicate with each other, solve problems, and suggest alternative options for creating a better future.

As well as benefiting from the energetic and dynamic nature of virtual international conferences, another advantage is that you do not need to travel to attend. You can take advantage of this opportunity from wherever you live.

This conference provides an excellent platform for researchers to present their latest findings, engage in direct discussions, and participate in online Q&A sessions.

The main topics and sessions of the conference are as follows:
S1. Mathematical Physics;
S2. High Energy Physics, Particles and Fields;
S3. Applied Physics;
S4. Astronomy, Astrophysics and Cosmology;
S5. Algebra and Geometry;
S6. Mathematical Analysis and Statistics;
S7. Applied Mathematics.

All submitted abstracts will be reviewed by the conference committee (before being published) and displayed on the website of the conference. The conference will be an online-only event, with **no registration fee**, and no requirement for travel or related expenses.

Outstanding contributions from each session will be eligible for conference awards. Moreover, instead of limiting presentations to a live poster session, all participants have the opportunity to upload and display their research in the form of a poster via the poster gallery on the conference website. Therefore, all participants with poster submissions for the poster gallery are now eligible for awards!

We invite you to join us online for what promises to be an enriching exchange of ideas and knowledge. Together, let us continue to unravel the mysteries of symmetry and its profound impact on our understanding of the natural world.



Prof. Dr. Sergei Odinstov
Conference Chair
Institute of Space Sciences (IEEC-CSIC), Barcelona, Spain



Dr. Calogero Vetro
Conference Chair
Department of Mathematics and Computer Science, University of Palermo, Palermo, Italy



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Symmetry (ISSN 2073-8994), an international and interdisciplinary scientific journal, publishes reviews and regular research papers. Our aim is to encourage scientists to publish their experimental and theoretical research in as much detail as possible. There is no restriction on the maximum length of the papers. Full experimental and/or methodical details must be provided, so that results can be reproduced. There are in addition three unique features:

- Manuscripts regarding research proposals and research ideas will be particularly welcomed.
- Comments on any related papers published in this journal and other journals can be published as short letters.
- Electronic files regarding the full details of the calculation and experimental procedure, if unable to be published in a normal way, will be deposited as supplementary material.

Impact Factor: 2.2 (2025); 5-Year Impact Factor: 2.1 (2025)

Keynote Speakers



Prof. Dr. Boris Malomed

Department of Physical Electronics, Faculty of Engineering, Tel Aviv University, Tel Aviv, Israel

Prof. Dr. Boris Malomed was born in Minsk (Belarus). He received his MS in physics from Belarusian State University in 1977, his PhD in theoretical physics from Moscow Institute of Physics & Technology in 1981, and his Doctor of Sciences degree (habilitation) from the Institute for Theoretical Physics, Ukrainian Academy of Sciences (Kiev) in 1989. Until 1991, he was a senior researcher at the Institute for Oceanology, USSR Academy of Sciences (Moscow). Since 1991, he has been an Associate Professor, and since 1998 a Full Professor at Tel Aviv University. He has published 1,300+ papers and four books. The total number of his citations exceeds 63,000. His h-index is 96 (Web of Science) / 100 (Scopus) / 114 (Google Scholar). He is an editor of the journals "Physics Letters A" and "Chaos, Solitons & Fractals", an editor-in-chief of the new journal "Photonic and Quantum Waves", and a board member of several others. He has been an advisor to 31 MS and 14 PhD students and 11 postdocs. He is a Senior Member of Optica and a Foreign Member of the Serbian Academy of Nonlinear Sciences. In 2025, he was a recipient of the NSC (Nonlinear Science & Complexity) Lagrange award for lifetime achievement in Nonlinear Physical Science.



Prof. Dr. Salvatore Capozziello

Southern High School, Naples, Italy; National Institute for Nuclear Physics—Naples Section (INFN-NA),Naples, Italy; Department of Physics "E. Pancini", University of Naples "Federico II", Monte S. Angelo University Complex, Naples, Italy



Prof. Dr. Kazuharu Bamba

Faculty of Symbiotic Systems Science, Fukushima University, Fukushima, Japan

Kazuharu Bamba is a full Professor at Fukushima University in Japan. He works on inflationary cosmology, the issue of dark energy, alternative theories of gravity alongside general relativity, and the physics of black holes. He has been an Editor of various international peer-reviewed journals such as Entropy, Universe, and Symmetry (MDPI).



Prof. Dr. Victor Flambaum

School of Physics, UNSW Sydney, Sydney, Australia

Leading Scientist at Institute of Nuclear Physics and Professor at Novosibirsk State University. In 1991 joined University of New South Wales, Sydney as Professor of Theoretical Physics. Scientia Professor since 2003. Visiting Fellowships include Harvard University, Harvard–Smithsonian observatory (led topical group), Princeton University, Princeton Institute for Advanced Study, JILA and Argonne Laboratory (USA), Humboldt and Gutenberg (Germany)



Prof. Shu Li

State Key Laboratory of Dark Matter Physics, Tsung-Dao Lee Institute & School of Physics and Astronomy, Shanghai Jiao Tong University

Prof. Shu Li is presently a tenured fellow at Tsung-Dao Lee Institute (TDLI), Shanghai Jiao Tong University (SJTU). He obtained his bachelor and PhD degree from University of Science and Technology of China and pursued postdoctoral research at Duke University and CERN before joining TDLI/SJTU in 2017 as the first T.D. Lee Fellow. His research focuses on high-energy experimental particle physics, including Higgs&Electroweak physics and also new physics beyond the Standard Model utilizing the ATLAS high energy proton-proton collision data from Large Hadron Collider (LHC) at CERN and led several international research groups of LHC-ATLAS collaboration. He is also the Principal Investigator of future DarkSHINE experiment R&D, being the first accelerator-based fixed-target experiment initiative in China dedicated to dark photon search with high repetition rate electron beam from SHINE (Shanghai HIgh repetitioN rate XFEL and Extreme light facility). He has been awarded several national and Shanghai municipal talent programs and been the leading Principal Investigators of NSFC international key research project and original exploration program, MOST national key research and development program sub-taskforce, and Shanghai pilot program for basic research.

Invited Speakers



Prof. Dr. Herbert Weigel

Physics Department, Stellenbosch University, South Africa



Prof. Charalampos Moustakidis

Aristotle University of Thessaloniki, Thessaloniki, Greece



Prof. Dr. Joan Solà Peracaula

University of Barcelona Theoretical Physics, Barcelona, Spain



Prof. Qing-Wen Wang

Department of Mathematics, Shanghai University, Shanghai, China



Prof. Dr. Egon Schulte

Department of Mathematics, Northeastern University, Boston, USA



Prof. Dr. Muhammad Mubashir Bhatti

College of Mathematics and Systems Science, Shandong University of Science and Technology, China



Dr. Alexander Kemper

Department of Physics, North Carolina State University, Raleigh, USA



Dr. Sebastian Deffner

Department of Physics, University of Maryland, Baltimore County, Baltimore, MD 21250, USA



Dr. Renhai Wang

School of Mathematical Sciences, Guizhou Normal University, Guiyang, China



Prof. Dr. Carla Manuela Pinto

School of Engineering, Polytechnic of Porto, Rua Dr António Bernardino de Almeida, Porto, Portugal



Prof. Dr. Jorge Segovia

Department of Physical, Chemical and Natural Systems, Pablo de Olavide University, Sevilla, Spain



Dr. Konstantin Zloshchastiev

Institute of Systems Science, Durban University of Technology, Durban, South Africa



Dr. Andrew Fowlie

Xi'an Jiaotong-Liverpool University



Prof. Aroonkumar Beesham

Department of Mathematical Sciences, University of Zululand, South Africa; Faculty of Applied and Health Sciences, Mangosuthu University of Technology, South Africa



Prof. Dr. Wiesław Leoński

Institute of Physics, University of Zielona Góra, Poland



Prof. Dr. Lorentz Jäntschi

Department of Physics and Chemistry, Technical University of Cluj-Napoca, Romania



Prof. Graham Hall

Institute of Mathematics, University of Aberdeen, Scotland



Prof. Dr. Roberto Passante

Dipartimento di Fisica e Chimica, Università degli Studi di Palermo, Via Archirafi 36, I-90123 Palermo, Italy

Program at a Glance

	Day 1	Day 2	Day 3
Morning	**Chair Welcome Speech** **Session 4.** Astronomy, Astrophysics and Cosmology **Flash Poster (Session 4)**	**Session 5.** Algebra and Geometry **Flash Poster (Session 5)**	**Session 7.** Applied Mathematics **Flash Poster (Session 7)**
Afternoon	**Session 1.** Mathematical Physics **Flash Poster (Session 1)**	**Session 2.** High Energy Physics, Particles and Fields **Flash Poster (Session 2)**	**Session 3.** Applied Physics **Flash Poster (Session 3)** **Session 6.** Mathematical Analysis and Statistics **Flash Poster (Session 6)**

IOCSYM 2026 Program

20 October 2026 (Tuesday)

Session 4. Astronomy, Astrophysics, and Cosmology

Time: 9:00 (CEST, Basel) | 03:00 (EDT, New York) | 15:00 (CST Asia, Beijing)

CEST Time	Speaker	Title
09:00–09:10	*Opening Speech* *Prof. Dr. Sergei Odintsov* *Dr. Calogero Vetro* *Event Chairs*	
09:10–09:20	*Welcome from the Session Chair* *Dr. Sándor Frey* *Session Chairs of Session 4*	
09:20–09:40	**Prof. Dr. Charalampos Moustakidis** Invited Speaker	The Nuclear Physics of Exotic Compact Objects
09:40–10:00	**Prof. Dr. Joan Solà Peracaula** Invited Speaker	Running Vacuum Framework: A Unification Paradigm for the Expansion History of the Universe
10:00–10:20	**Prof. Dr. Aroonkumar Beesham** Invited Speaker	Reconstruction of Cosmic Expansion History Through a Logarithmic Deceleration Parametrization
10:20–10:35	**Alexandr E. Volvach** Oral Speaker	Nanohertz Gravitational-Wave Signatures from the Supermassive Black Hole Binary Candidate in 3C 454.3
10:35–10:50	**Judit Muraközy** Oral Speaker	Does the 4+4 Alternation Persist in Recent Solar Cycles?
10:50–11:05	**Vesselin G. Gueorguiev** Oral Speaker	Resolving the Hubble Tension via Time Reparametrization Invariant Scaling Symmetry
11:05–11:20	**Alkiviadis Kanakis-Pegios** Oral Speaker	Bulk Viscosity Effects on Dissipative Tidal Deformability of Neutron Stars
11:20–11:35	**Duvier Suarez Fontanella** Oral Speaker	Axion Propagation in a Gravitational Wave Background
11:35–12:00	**Flash Poster Presentations**	**Lina Yildiz** A Curvature–Elasticity Criterion for Viable Unified F(R) Cosmology **Ke Chen** Feasibility Analysis of Holographic Dark Energy based on Generalized Entropy **Wentao Liu** Lorentz Violation Without Gravitational Ringdown Modification in Einstein-Bumblebee Black Holes **Wen-Di Guo** Scalar Quasinormal Modes of Rotating Black Holes in Low-Energy Horava Gravity **Angel Rincon** QNMs and Shadows of Charged Scale-Dependent Black Holes
BREAK		

Session 1. Mathematical Physics

Time: 14:00 (CEST, Basel) | 8:00 (EDT, New York) | 20:00 (CST Asia, Beijing)

CEST Time	Speaker	Title
14:00–14:10	*Welcome from the Session Chair* *Prof. Dr. David Laroze* *Prof. Dr. Kazuharu Bamba* *Session Chairs of Session 1*	
14:10–14:40	**Prof. Dr. Boris Malomed** Keynote Speaker	Spontaneous Symmetry Breaking in Nonlinear Dual-Core Systems
14:40–15:10	**Prof. Dr. Salvatore Capozziello** Keynote Speaker	To be announced
15:10–15:40	**Prof. Dr. Kazuharu Bamba** Keynote Speaker	Inflationary Cosmology and the Trans-Planckian Censorship Conjecture
15:40–16:00	**Dr. Konstantin Zloshchastiev** Invited Speaker	Superfluid Vacuum Interpretation of Quantum Mechanics: Fluid Waves, Guiding Spacetime and Emergence of Classicality
16:00–16:15	**Babalic Elena Mirela** Oral Speaker	Equations of Motion on the Extended Siegel-Jacobi Space Generated by a Linear Hamiltonian
16:15–16:30	**Emanuel Indrei** Oral Speaker	The Equilibrium Shape of a Crystal
16:30–16:45	**Concetta Caglioti** Oral Speaker	Complexity and Geometry of Potential Energy Surfaces: A Structural NP-Hardness Framework
16:45–17:00	**Flash Poster Presentations**	**Pavel Makarov** Elements of Symmetry in the FDTD Method for Selected Problems of Computational Electrodynamics **Rodrigo Francisco dos Santos** Optical Geometry from Critical Superfluid Dynamics: Einstein-Euler Parametrization and Non-Integrable Causal Structure **Felix Lev** Problems of Foundation of Mathematics and Quantum Theory
END OF DAY 1		

21 October 2026 (Wednesday)

Session 5: Algebra and Geometry

Time: 9:00 (CEST, Basel) | 03:00 (EDT, New York) | 15:00 (CST Asia, Beijing)

CEST Time	Speaker	Title
09:00–09:10	*Welcome from the Session Chair* *Prof. Dr. Manuel de León* *Session Chair of Session 5*	
09:10–09:30	**Prof. Dr. Qing-Wen Wang** Invited Speaker	Matrix Equations and Their Applications Over the Dual Quaternion Algebra
09:30–09:50	**Prof. Dr. Egon Schulte** Invited Speaker	Skeletal Polyhedra, Geometric Graphs, and Crystallographic Groups
09:50–10:05	**Álvaro Antón-Sancho** Oral Speaker	Involutions and Lagrangian Submanifolds in Higgs Bundle Moduli Spaces Over Curves
10:05–10:20	**Evan Angelone** Oral Speaker	Chiral polyhedra from AGL(1, q)
10:20–10:35	**Beyza Tekeoğlu Akkaya** Oral Speaker	On Gaussian Balancing-Like Sequences and Their Asymptotic Geometry
10:35–10:50	**Jieying Jin** Oral Speaker	Symmetric Circle Configurations from Regular Skeletal Polyhedra
10:50–11:05	**Johnny Muhindo Bahavira** Oral Speaker	Automorphism Groups and Multiscale Symmetry Breaking in a Rooted Watershed Tree: An Algebraic–Geometric Study of the Funa Drainage Network
11:05–11:20	**Aslınur Karabek** Oral Speaker	Fibonacci Tower Vector Fields: Algebraic Structure, Stability and Finitely Generated Abelian Groups
11:20–11:30	**Flash Poster Presentations**	**Macarena Boix** Diagonalizability Conditions for a (2,1)-Band Matrix of Order 4 with Prescribed Spectrum **Nasir Ali** Characterizing Rings via Domination in Compressed Zero-Divisor Graphs and Applications to Network Optimization

BREAK

Session 2. High Energy Physics, Particles and Fields

Time: 14:00 (CEST, Basel) | 8:00 (EDT, New York) | 20:00 (CST Asia, Beijing)

CEST Time	Speaker	Title
14:00–14:10	*Welcome from the Session Host* *Prof. Dr. Ruilin Zhu* *Session Host*	
14:10–14:40	**Prof. Dr. Victor V. Flambaum** Keynote Speaker	Violation of Symmetries in Atoms and Nuclei and Search for Dark Matter
14:40–15:10	**Prof. Dr. Shu Li** Keynote Speaker	Recent Multi-Boson and Multi-Higgs Results from LHC-ATLAS
15:10–15:30	**Prof Dr. Herbert Weigel** Invited Speaker	Classical Solitons with BPS Symmetry May Be Unstable in the Quantum World
15:30–15:50	**Prof. Dr. Jorge Segovia** Invited Speaker	Towards Complete Pion GPDs: From QCD Constraints to EIC Signatures
15:50–16:05	**Denis Tolkachev** Oral Speaker	Summation of Logarithms in an Arbitrary Local Quantum Field Theory
16:05–16:20	**Yukun Zhang** Oral Speaker	Symmetries and Kinematics of Spinning Test Particles Orbiting Kerr-Bertotti-Robinson Black Holes
16:20–16:35	**Francesco Giovanni Celiberto** Oral Speaker	Exotic and Rare Hadrons Across Flavor Sectors: Fragmentation, Symmetries, and Collider Phenomenology
16:35–16:50	**Jia-Zhou Liu** Oral Speaker	Black Hole Entropy Beyond the Wald Term in Nonminimally Coupled Gravity: A Covariant Phase Space Decomposition

16:50–17:00	**Flash Poster Presentations**	**Daniel Piasecki** Origin of the Covariant Wigner Operator as a Quantum Amplitude in QCD **Mingyu Zhao** Zero-Equation: Unified Criterion for Real-Virtual Conjugate Symmetry and Necessary and Sufficient Condition for Symmetry Breaking
END OF DAY 2		

22 October 2026 (Thursday)

Session 7. Applied Mathematics

Time: 9:00 (CEST, Basel) | 03:00 (EDT, New York) | 15:00 (CST Asia, Beijing)

CEST Time	Speaker	Title
9:00–9:10	*Welcome from the Session Chairs* *Prof. Dr. Yufeng Zhang* *Prof. Dr. Quanxin Zhu* *Session Chairs of Session 7*	
09:10–09:30	**Dr. Muhammad Mubashir Bhatti** Invited Speaker	Lie Symmetry and Spectral Relaxation Approach to Study Non-Newtonian Mixed Convection Flows
09:30–09:50	**Prof. Dr. Renhai Wang** Invited Speaker	Multivalued and Finite-Dimensional Random Dynamics of Critically Nonlinear BBM Equations Driven by Colored Noise
09:50–10:10	**Prof. Dr. Carla Manuela Pinto** Invited Speaker	Blue "Notes" and Red Flags in Fractional Epidemiology
10:10–10:25	**Fatma Zohra Daoud** Oral Speaker	Fractional-Order Memory Effects and Chaotic Dynamics in Nonlinear Delay Systems: Computational Analysis of Bifurcation and Multi-Scroll Attractors
10:25–10:40	**Walid Remili** Oral Speaker	A Numerical Approach for Solving Load-Leveling Energy System Models Governed by First-Kind Volterra Integral Equations
10:40–10:55	**Mrinmoy Maity** Oral Speaker	Lie Symmetry Analysis of the Duffing Oscillator for Nonlinear Vibration Control and Stability in Mechanical Systems
10:55–11:10	**Zermani Manel** Oral Speaker	Symmetry Properties and Chaotic Dynamics of Trisenne–Lucas Numbers
11:10–11:25	**Ambreen Khan** Oral Speaker	Numerical Investigation of Chromatographic Models with a Tri-Langmuir Isotherm
11:25–11:40	**Vanessa Botero-Gómez** Oral Speaker	Enhancing the Multiverse Optimizer Through Symmetry-Guided Search Operators
11:40–11:55	**Flash Poster Presentations**	**Usman Paul Ismail** Oscillation of the Ar2 Molecule Using the LAMMPS Molecular Dynamics Simulation Tool **Kaishuo Lin** Bayesian Fisher Information-Based Photoacoustic Optimal Waveform Design via Fredholm Integral Equation **Soukaina Hadiri** A New Centered Gaussian Process
BREAK		

Session 3. Applied Physics (Parallel Session)

Time: 14:00 (CEST, Basel) | 08:00 (EDT, New York) | 20:00 (CST Asia, Beijing)

CEST Time/CST Asia Time	Speaker	Title
14:00–14:10	*Welcome from the Session Chairs* *Prof. Dr. Joao Pacheco B.C. de Melo* *Prof. Dr. Jim Freericks* *Session Chairs of Session 3*	
14:10–14:30	**Dr. Alexander Kemper** Invited Speaker	Lie Algebras in Quantum Computing
14:30–14:50	**Prof. Dr. Humberto Belich Junior** Invited Speaker	Lorentz Symmetry Violation in Dirac Materials
14:50–15:10	**Prof. Dr. Graham S Hall** Invited Speaker	Four-Dimensional Manifolds Admitting Homothetic Vector Fields with a Zero
15:10–15:30	**Prof Dr. Roberto Passante** Invited Speaker	Casimir and Casimir-Polder Effects: Theory, Experiments and Nanotechnology Applications

15:30–15:50	**Dr. Sebastian Deffner** Invited Speaker	Miscellaneous Contemplations on the Dynamics of Nonlinear Quantum Systems
15:50–16:05	**Tanzeela Tasneem** Oral Speaker	Symmetry-Reduced Optimal Control of Quadrotor Dynamics via Noether's Theorem
16:05–16:20	**Nikolai Sergeevich Akintsov** Oral Speaker	Symplectic Symmetry Preservation in Relativistic Particle Dynamics via Physics-Informed Neural Network Integrators
16:20–16:35	**Alexandra Farcas** Oral Speaker	Enhanced Targeted Therapeutics: Investigating DNA Coverage Density Effects on the Structural Dynamics of DNA-Functionalized Gold Nanoparticles for Precision Nanomedicine
16:35–16:50	**Saida Mammadova** Oral Speaker	Theoretical Study of the Electronic and Magnetic Behavior of Ca-Doped Ni–Zn Ferrites
16:50-17:00	**Flash Poster Presentations**	**Aswin Karakadakattil** A Fused Approach of Lie Group SE2(3) and Hill-Clohessy-Wiltshire Optimization for Relative Navigation of a Group of Small Objects

Session 6. Mathematical Analysis and Statistics (Parallel Session)

Time: 14:00 (CEST, Basel) | 08:00 (EDT, New York) | 20:00 (CST Asia, Beijing)

CEST Time/CST Asia Time	Speaker	Title
14:00–14:10	*Welcome from the Session Chairs* *Dr. Nguyen Lam* *Prof. Dr. Salim Bouzebda* *Session Chairs of Session 6*	
14:10–14:30	**Dr. Andrew Fowlie** Invited Speaker	Symmetries and Paradoxes in Bayesian Inference
14:30–14:50	**Prof. Dr. Lorentz Jäntschi** Invited Speaker	Symmetry in Molecular Descriptor Families and Its Structural Significance
14:50–15:10	**Prof. Dr. Wiesław Leoński** Invited Speaker	Double Bose-Hubbard Chain Model and a Transfer of Quantum Steering
15:10–15:25	**Pietro Sileci** Oral Speaker	Coupled Critical Growth for Anisotropic Systems
15:25–15:40	**Azizur Rahman** Oral Speaker	Efficient and Accurate Variational Inference for Multilevel Threshold Autoregressive Models in Intensive Longitudinal Data
15:40–15:55	**Pierros Ntelis** Oral Speaker	A Foundational Analysis of Local Functional Calculus
15:55–16:10	**Selin Saraç Güleryüz** Oral Speaker	Finite Mixture Models as a Framework for Symmetry Breaking in Statistical Distributions: Theory, Estimation, and Applications to Asymmetric Data
16:10–16:25	**Aileen Aviles Diansuy** Oral Speaker	A Jacobian-Based Generalization of Volume of a Solid of Revolution Along an Oblique Axis
16:25–16:40	**Mehtap Dursun** Oral Speaker	An Experimental and Statistical Approach to Optimize Filtration Rate in a Chemical Process
16:40–17:00	**Flash Poster Presentations**	**Md. Ismail Hossain** Approximate Bayesian Inference for Functional Mixed Effects Regression with Partially Observed Functional Covariates **Md. Nurul Raihen** A Symmetry-Regularized Fuzzy Graph Learning Framework for Spatio-Temporal Water Quality Prediction **Nitika Chandel** Fixed Point Results via a Triplet of Auxiliary Functions in Relational Partial Metric Spaces **Istiak Ahmed** Functional Data Analysis of Sentinel-2 Derived Vegetative Phenology for Potato Yield Estimation Under Normal and Cyclone Conditions
END OF IOCSYM 2026		

Poster List

Submission ID	Authors	Title
Session 1: Mathematical Physics		
Sciforum-190867	Atika Mehedi	Dynamical Correlation in Binary Mixtures of Bose–Einstein Condensates (BECs)
Sciforum-190721	Claudio Armas	An Individual-Based Socio-Epidemiological Model for Crowd Evacuations and Pedestrian Crossings
Sciforum-194863	Pavel Makarov	Elements of Symmetry in the FDTD Method for Selected Problems of Computational Electrodynamics
Sciforum-195347	Cleber Souza Correa	Holonomic Resonances as Emergent Signatures of a Trigonometric Topological Mapping in the Extended Alpha Group Matrix
Sciforum-189503	Kibrom kidane Kahsay	The Octavian Coherence Gate: The Four Irreducible Integers of the E8(1) ⊃G2(1) ×F4(1) Conformal Embedding
Sciforum-200105	Aayush Sinha	Quantized Curvature Reconstruction via SU(2) Generators: Extending Absolute Unified Mathematics of Quantum Gravitation
Sciforum-182640	Joanna K. Kalaga	Analysis of Multimode Squeezing in Three-Qubit Systems
Sciforum-189541	Genki Sumiyama	Analytic Direct Calculation of the Topological Charge Structure for Calorons on S1×R3
Sciforum-170230	Mingyu Zhao	Wave-Particle Symmetry: Mass Dependence of Quantum Tunneling in Discrete Spacetime
Sciforum-189916	Vadim Khoruzhenko	A Geometric Model of Electromagnetic Interaction Based on the Volumetric Curvature of Space
Sciforum-190858	Tom Lawrence	Symmetries of Field Configurations on Compact Spaces, Spinors, and No-Go Theorems
Sciforum-158190	Felix Lev	Problems with the Foundations of Mathematics and Quantum Theory
Sciforum-188496	Rodrigo Francisco dos Santos	Optical Geometry from Critical Superfluid Dynamics: Einstein–Euler Parametrization and Non-Integrable Causal Structure
S2. High Energy Physics, Particles and Fields		
Sciforum-189099	Xiang-Cheng Meng	Precession of Spherical Orbits and Observational Constraints on Black Hole Parameters from the Jet of M87*
Sciforum-190644	Akash Das	Extreme Geometrical Effects on Relativistic Isobaric Collisions
Sciforum-192265	Chengyi Li	String-Inspired Foam and Lorentz Violation: Reconciling Light-Speed Variation with Air Shower Observations
Sciforum-184183	Yan Zhou	Vacuums at the Planck scale
Sciforum-188290	Xinhua Ma	Consideration of Flavor-Related Additive Quantum Numbers
Sciforum-193676	Muhammad Younas	Symmetry-Driven Optical Response of GaAs: A First-Principles Density Functional Theory Investigation
Sciforum-190742	Brahim Ait Ouazghour	Phenomenology of Charged Higgs Bosons in 2HDM at Future Lepton Colliders
Sciforum-195625	Francesco Giovanni Celiberto	Exotic and Rare Hadrons Across Flavor Sectors: Fragmentation, Symmetries, and Collider Phenomenology

Sciforum-190740	Daniel Piasecki	Origin of the Covariant Wigner Operator as a Quantum Amplitude in QCD
Sciforum-190785	Abdelkader Cheddad	Effective Partonic Entropy Suppression Near the QCD Crossover
Sciforum-194144	Mingyu Zhao	Zero-Equation: Unified Criterion for Real-Virtual Conjugate Symmetry and Necessary and Sufficient Condition for Symmetry Breaking
Sciforum-190805	Rokaya Djida	Study of The Thermal Deconfining Phase Transition at Zero Baryochemical Potential for Different Hadronic Phase Configurations
Sciforum-190859	Mohamed Amine Lakehal	The Impact of the Polyakov Effective Potential on the Deconfinement Transition of QCD Matter
Sciforum-190860	Faiza Ramdani	A Switching-Function Approach to Parametrizing the Temperature-Driven Deconfinement Phase Transition, at Zero Chemical Potential, in a Finite Volume
S3. Applied Physics		
Sciforum-199140	Ilya Galaktionov	Symmetry Breaking in Fiber Optic Connector Endface Geometry: Correction via Controlled Manual Polishing
Sciforum-199132	Ilya Galaktionov	Symmetric vs. Asymmetric PON Architectures: A Comparative Analysis of Power Budgets and Reach for Next-Generation Access Networks
Sciforum-189729	Paraskevi K. Askouni	Comparative Seismic Performance of Symmetrical Mixed RC–Steel and Reinforced Concrete Buildings
Sciforum-198807	Ilya Galaktionov	Adaptive Restoration of Wavefront Symmetry in Turbulent Atmospheric Optical Channels: An Applied Physics Perspective on Control Algorithms
Sciforum-190766	Amide Alasgar Sadigova	DFT Investigations on the Thermoelectric and Electronic Properties of Ni0.40Zn0.60Fe2O4
S4. Astronomy, Astrophysics and Cosmology		
Sciforum-200173	Preeti Joshi	Validity of Generalized Second Law of Thermodynamics in Rastall Cosmology: Generalized Chaplygin Gas with Tsallis Entropy.
Sciforum-189150	Alexander Lelyakov	Asymptotics of the Gravitational Field for a Homogeneous Phase of a Gas of Thin Tubes of a Massless Scalar Field
Sciforum-188342	Samprity Das	Study of Anisotropic Compact Stars in Modified Gravity with a Bose–Einstein Dark Matter Density Profile
Sciforum-200295	Wen-Di Guo	Scalar Quasinormal Modes of Rotating Black Holes in Low-Energy Hořava Gravity
Sciforum-188548	Papadopoulos Stylianos	Gravitational Instantons and Chiral Reduction in Spin(4) Gauge theory of spacetime.
Sciforum-194147	Mingyu Zhao	Spacetime–Mass Equivalence Principle: A New Gravitational Field Equation and Its Cosmological Applications
Sciforum-190830	Ioannis Stergakis	Machine and Deep Learning Regression Techniques for the Inverse Stellar Structure Problem
Sciforum-189148	Alexander Lelyakov	The Evolution of Dark Energy in a Gas of Thin, Closed Tubes of a Massless Scalar Field with Radially Varying Sizes
Sciforum-179354	Rashmita Jena	Studying Hyperon Puzzle in a Unified SU(3)–SU(6) Baryon–Meson Coupling Scheme

Sciforum-198043	Sonia Sharma	Anisotropic Transition Cosmology Driven by Bulk Viscosity and Gaussian Expansion Dynamics
Sciforum-201001	Angel Rincon	QNMs and Shadows of Charged Scale-Dependent Black Holes
Sciforum-190846	Ioannis Mavroudis	Ground-State Properties of Astrophysical Waiting-Point Nuclei under Extreme Magnetic Fields
Sciforum-196974	Muhammad Azzam Alwan	Neutron Star Cooling in Covariant f(Q) Gravity
Sciforum-164417	Shreesham Pandey	**Induced Curvature Singularities in Modified Theories of Gravity**
Sciforum-200042	Ke Chen	Feasibility Analysis of Holographic Dark Energy Based on Generalized Entropy
S5. Algebra and Geometry		
Sciforum-188528	Siritharkumar Thilushan	From Smith Theory to the Dold–Thom Theorem: A Unified Study of Symmetry in Algebraic Topology
Sciforum-195160	Rayan Das	A Pedagogical Note on the Algebraic Instability of the Pythagorean Relation for Exponents Greater Than Two
Sciforum-181562	Siritharkumar Thilushan	Symmetrically Fuzzy β-Connected and Disconnected Sets in Fuzzy Topological Spaces
Sciforum-184547	Cleber Souza Correa	Topological Structures of the Alpha Group via Monte Carlo Simulations
Sciforum-200624	Siritharkumar Thilushan	Group-Equivariant Fuzzy Topological Vector Spaces: A New Framework via Symmetric Group Actions
Sciforum-190165	Macarena Boix García	Diagonalizability Conditions for a (2,1)-Band Matrix of order 4 with a Prescribed Spectrum
Sciforum-190535	Nikita Chandel	Establishment of Fixed-Point Results via a Triplet of Auxiliary Functions in Relational Partial Metric Spaces
Sciforum-200431	Md Nurul Raihen	A Symmetry-Regularized Fuzzy Graph Learning Framework for Spatio-Temporal Water Quality Prediction
Sciforum-190730	Abdul Al Shorif	Grouped Robust Functional Time Series Forecasting Model of Mortality Rates: A case study of Matlab Health and Demographic Surveillance System, Bangladesh
Sciforum-190839	Istiak Ahmed	Functional Data Analysis of Sentinel-2 Derived Vegetative Phenology for Potato Yield Estimation Under Normal and Cyclone Conditions
Sciforum-175070	Md. Ismail Hossain	Approximate Bayesian Inference for Functional Mixed Effects Regression with Partially Observed Functional Covariates
Sciforum-199304	Andrei Yermolenko	Contact Problem for a Circular Simply Supported Plate and a Rigid Base
Sciforum-190157	Daniel López-Rodríguez	Structure-Preserving Numerical Schemes for Nonlinear Adsorption Models with Mass Conservation
Sciforum-198894	Diego Restrepo-Leal	Advanced Differentiable Hybrid Frameworks for Time Series Reconstruction and Forecasting in Data-Scarce Regimes
Sciforum-190675	Zuwen Sun	Bayesian Fisher Information Based Photoacoustic Optimal Waveform Design via Fredholm Integral Equation

Sciforum-190725	Feige Zeng	Constructive Symmetry: Hidden Structure in Prime Arithmetic Progressions Revealed by the Base Primorial
Sciforum-198905	Usman Paul Ismail	Oscillation of the Ar2 Molecule using the LAMMPS Molecular Dynamics Simulation Tool
Sciforum-195545	Amudhamalar V	Spherical Fuzzy Multisets: Symmetric Algebraic Structures and Applications in Multi-Criteria Decision-Making
Sciforum-198686	Mohamed Abdelsabour Fahmy	Symmetry-Guided Boundary Element Modeling of Microneedle-Delivered Nanoparticle Photothermal Hyperthermia in Multilayer Skin Tumors

Session 1. Mathematical Physics

Sciforum-158190: Problems with foundation of mathematics and quantum theory

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As shown in Gödel's incompleteness theorems, standard mathematics has foundational problems because it invokes actual infinity from the very beginning. We describe a theorem which states that the infinite ring of integers Z is a limit of finite rings $R_p=(0,1,2,...p-1)$ of residues modulo p when $p\to\infty$, and the proof invokes only potential infinity. Then it can be shown that a finite quantum theory (FQT) in which the state space is a linear space over R_p+iR_p, and operators of physical quantities act in this space, is more general than standard quantum theory (SQT). This means that any result of SQT can be recovered in FQT if p is chosen to be sufficiently large, and at the same time, in FQT there are results which cannot be recovered in SQT, which contains particles and antiparticles (e.g., electron–positron, proton–antiproton) and the laws of conservation of electric charge, baryon number, etc. These results arise because the algebras in SQT are such that in their irreducible representations (IRs), the energies of particles can only be either positive or negative, and there are no IRs in which particles have states with different energy signs. On the contrary, in finite mathematics, IRs necessarily contain both positive and negative energies. The case when there is one IR uniting positive and negative energies has a higher symmetry than the case when this IR splits into two independent IRs with positive and negative energies. Then the concept of particle–antiparticle can only be approximate when p is very large. Therefore, the challenge of constructing fundamental quantum theory lies in constructing a theory that does not assume that p is anomalously large, and such a theory will be based on new physical concepts.



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Sciforum-158645: Spontaneous symmetry breaking in nonlinear dual-core systems

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Models of various physical media based on configurations with two parallelidentical nonlinear waveguides (cores) amount to systems of linearly coupled nonlinearSchrödinger equations (NLSEs). Well-known examples are models of nonlinear dual-core optical fibers or planar optical waveguides. In the realm of Bose–Einstein condensates (BECs), similar settings represent dual-core traps for matter waves. In the linear limit, such symmetric systems support modes which are symmetric with respect to the underlying double-core system. If the intra-core self-focusing (self-attractive) nonlinearity is strong enough, its competition with the linear inter-core coupling gives rise to spontaneous symmetry breaking (SSB). Via the corresponding symmetry-breaking bifurcations, the SSB destabilizes the states that are symmetric with respect to the underlying dual-core structure, and replaces them by nontrivial asymmetric states. In particular, the SSB phenomenology takes a well-defined form for solitons in the dual-core systems. Obvious symmetric solitons are destabilized and replaced by nontrivial asymmetric ones through the respective supercritical or subcritical symmetry-breaking bifurcations, which take place when the soliton's norm exceeds a specific critical value. In particular, the subcritical bifurcation gives rise to a narrow region of bistability between coexisting symmetric and asymmetric solitons, while no bistability occurs in the case of the supercritical symmetry-breaking bifurcation.

The talk aims to present an overview of basic models, results, and physical realizations of the SSB phenomenology in nonlinear photonic and matter-wave (BEC) media. In particular, two basic models are considered, which makes it possible to explore the SSB in an explicit semi-analytical form: an infinitely deep potential box split by a delta-functional barrier,and a ring symmetrically split by two repulsive barriers [4]. The talk also includes a summary of relatively recent experimental results, which for the first time have demonstrated SSB of optical temporal solitons in symmetric dual-core optical fibers (which was theoretically predicted 30 years earlier).



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Sciforum-164979: Symmetry-Aware Artificial Intelligence and Machine Learning for Cybersecurity: An Integrated Mathematical Physics, Algebraic Geometry, and Statistical Learning Framework for Robust Threat Detection and Secure Systems

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The rapid evolution of cyber threats presents major challenges to digital infrastructure, requiring mathematically rigorous and adaptive cybersecurity frameworks capable of detecting complex and evolving attack patterns. This research proposes a Symmetry-Aware Artificial Intelligence (AI) and Machine Learning (ML) framework that integrates group theory, algebraic geometry, differential geometry, mathematical physics, and statistical learning to improve intrusion detection, anomaly recognition, predictive threat intelligence, and secure communication systems. The framework models cyber environments as high-dimensional geometric manifolds in which network states, packet flows, and user behaviors are represented through algebraic and topological structures. Symmetry transformations, invariant operators, and Lie group actions are employed to identify the conserved behavioral patterns and hidden structural regularities associated with both legitimate and malicious cyber activities.

The proposed methodology implements symmetry-preserving learning architectures using Graph Neural Networks (GNNs), convolutional neural networks with invariant feature mappings, variational autoencoders, and Bayesian probabilistic inference models to analyze dynamic cybersecurity data streams. Algebraic geometry techniques, including polynomial ideals, algebraic varieties, manifold embeddings, and tensor-based feature representations, are applied in multidimensional attack-surface modeling, latent anomaly localization, and nonlinear feature-space optimization. Symmetry breaking principles derived from mathematical physics are utilized to detect deviations from stable system states, enabling early identification of zero-day attacks, ransomware propagation, distributed denial-of-service (DDoS) attacks, and coordinated intrusion campaigns. Statistical learning components incorporate entropy-based risk estimation, stochastic optimization, and probabilistic decision boundaries to improve prediction reliability and reduce uncertainty in cyber threat classification.

Experimental evaluation was conducted using benchmark cybersecurity datasets, including NSL-KDD, CICIDS2017, and UNSW-NB15, with performance assessed using accuracy, precision, recall, F1-score, ROC-AUC, and false-positive rate metrics. The results demonstrate that the proposed symmetry-aware framework significantly improves detection accuracy, adversarial robustness, interpretability, and false-positive reduction compared to conventional deep learning and statistical intrusion detection models, providing a scalable and mathematically grounded foundation for next-generation cybersecurity systems.



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Sciforum-170230: Wave-Particle Symmetry: Mass Dependence of Quantum Tunneling in Discrete Spacetime

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Conventional quantum tunneling theory, based on the continuous spacetime assumption, relies on the "borrowed energy" interpretation from vacuum fluctuations, suffers from the Hartman superluminal paradox, and fails to reveal the essence of wave-particle duality at the ontological level of spacetime. Based on the fundamental axioms of discrete spacetime, this paper starts from the iron law of nearest-neighbor transition of particles and derives an exact single-particle equation of motion and a stationary Schrödinger difference equation without any approximations. On this basis, we obtain the rigorous exact solution for quantum tunneling probability $T=|r_1|^{2N}$ and systematically investigate the dependence of tunneling probability on rest mass. Our results show that in the weak-field low-energy regime, the tunneling probability decays exponentially with $m_0^{1/2}$, which is similar in form to the continuous spacetime WKB approximation but differs by approximately 40% in coefficient (the WKB formula is only an approximate formula with an inherent error of 10%-50%). In the high-barrier high-mass limit, the tunneling probability decays as a power law with mass $T\approx m_0^{-2a/l_P}$, which is a revolutionary prediction unique to discrete spacetime. This paper further proposes the wave-particle separation condition without external energy input and derives the critical mass threshold for wave-particle transition. This theory completely eliminates the vacuum fluctuation assumption and the superluminal paradox in conventional tunneling theories, provides a solid discrete spacetime ontological foundation for wave-particle symmetry, and presents three quantitative predictions that can be directly verified by experiments.



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Sciforum-182367: Memristive-Modified Hybrid System with a Line of Equilibria

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This paper proposes and investigates a novel four-dimensional (4D) fractional-order chaotic system obtained by coupling a flux-controlled memristor with a classical Jerk oscillator. The resulting model is governed by a set of nonlinear fractional differential equations involving the Caputo derivative of order $0<\alpha\leq 1$, which introduces memory effects into the system dynamics. The memristive nonlinearity is incorporated through a quadratic memductance function $W(w)=a+bw^2$, while μ acts as a control parameter and β represents the damping coefficient.

The inclusion of both fractional-order operators and memristive coupling leads to richer dynamical behavior compared to the classical integer-order Jerk system. A rigorous qualitative analysis is performed to establish the existence and uniqueness of solutions. The equilibrium analysis shows that the system possesses a continuous line of equilibrium points $E=(0,0,0,w*)$, where $w*\in R$, which is a typical feature of memristive systems.

Local stability is studied using eigenvalue analysis combined with Matignon's fractional-order stability criterion. The results indicate that the interaction between the fractional order α and memristive parameters significantly affects stability regions and the onset of oscillatory behavior.

Extensive numerical simulations reveal a rich variety of nonlinear dynamics, including periodic oscillations, chaotic attractors, coexistence of attractors, and hidden attractors. Notably, hidden chaotic attractors are observed even when the equilibrium manifold is locally stable, highlighting the complexity induced by dual memory effects.

Overall, the proposed system combines fractional-order and memristive memory mechanisms, generating complex dynamics with potential applications in nonlinear circuits, secure communications, and neuromorphic engineering.



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Sciforum-182640: Analysis of Multimode Squeezing in Three-Qubit Systems

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Introduction:The ability to generate and control quantum states is fundamental for the development of quantum information theory and its applications. Key quantum features, such as superposition, entanglement, and squeezing, are essential resources in quantum communication, metrology, and information processing. In particular, understanding the relationship between entanglement and squeezing is crucial for a better characterisation of nonclassical states. This work focuses on multimode squeezing in three-qubit systems exhibiting genuine tripartite entanglement.

Methods:We consider a theoretical model of three-qubit states, including classes of states characterised by non-zero tripartite entanglement. Following the established classification of three-qubit entangled states, we analyse different categories of these states. Entanglement is quantified using negativity [1], while squeezing properties are described by quadrature operators and principal squeezing variances [2, 3]. We derive and evaluate the conditions for two-mode and three-mode squeezing, both analytically and numerically, paying particular attention to the structure of the quantum states.

Results:Our analysis shows that two- and three-mode squeezing can be observed for certain configurations. Furthermore, relationships between squeezing parameters and entanglement measures are established, demonstrating how the presence of multipartite entanglement influences multimode squeezing properties.

Conclusions:Our results confirm that multimode squeezing can be realised in three-qubit systems and is closely related to the structure of entanglement. The identified connections between squeezing and entanglement measures provide additional insight into nonclassical correlations in multipartite systems and may be useful for future developments in quantum information and quantum optics.

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Sciforum-182668: Advanced maths in cosmology

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In this talk, I will present recent advancements in mathematical and cosmological frameworks, drawing from my key studies. I will introduce functorial methods in Functors of Actions Theories (FATs), bridging mathematical rigor with cosmological phenomena, by predicting actions. In response to the need for advanced math, I will introduce the exploration of novel tensor-based approaches from Advancing Tensor Theories (ATTs), and Advanced Manifold–Metric Pairs (AMMPs), offering new tools for modeling. ATTs merge novel advanced indices with corresponding tensors which are defined using standard calculus operators and fractional calculus operators. AMMPs introduce more abstract Manifold–Metric Pairs in which the line elements are constructured through a functional tensor of (U,L)-rank, rather than a (0,2)-rank, which opens a new road to introduce more sophisticated mathematical structure to physics problems. Finally, I will discuss the dynamics of the quintessence in the ϕCDM, simple $\phi\Lambda$CDM, and polyΛCDM models, providing insights into scalar field-driven cosmology and its implications for dark energy and modified gravity. The analytical polyΛCDM model incorporates scalar–vector–tensor modified gravity theories into a lagrangian, and under appropriate assumptions, it results in a polynomial depenence on energy density ratios that can be measured by observational data, and distinguishes which modified gravity model is more apparent from the background cosmology data. If time permits, I will talk about a novel test of fundamental principles of gravity in extra dimensions given large-scale structure surveys with contaminated samples [8]. These works collectively highlight innovative mathematical methods to advance our understanding of the universe's evolution and structure.



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Sciforum-184197: Geometry of Irreversible Neural Computation in Non-Reciprocal Attractor Networks

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Neural computation is often described through attractor landscapes, where stable activity states correspond to minima of an effective potential and rare transitions are controlled by barrier heights. This landscape picture is powerful, but it becomes incomplete when neural dynamics is non-reciprocal and violates detailed balance. In such systems, two networks may share the same stationary density and the same leading quasipotential barrier while exhibiting different transition routes, irreversible probability currents, and thermodynamic precision costs.

This talk presents a geometric and stochastic-thermodynamic framework for analyzing irreversible attractor computation. Using Freidlin–Wentzell large-deviation theory, I describe rare neural transitions as minimum-action paths rather than as local fluctuations around stable states. In a solvable bistable model, the non-reciprocal component of the drift leaves the quasipotential unchanged but bends escape and relaxation trajectories into directed loops. This produces a path-asymmetry index that grows linearly with non-reciprocity, while the entropy production rate grows quadratically. The result is a quantitative path–current–dissipation relation: geometric irreversibility becomes an observable signature of hidden nonequilibrium currents beneath an apparently equilibrium-like landscape.

The framework is then extended to high-dimensional non-reciprocal attractor networks, where full-space geometric minimum-action optimization confirms the persistence of the same structure. Monte Carlo simulations validate the Freidlin–Wentzell barrier, and time-integrated current observables satisfy thermodynamic uncertainty bounds. These results suggest a falsifiable principle for neural and neuromorphic systems: reliable directed computation through attractor space requires not only stable states and transition barriers, but also a measurable thermodynamic cost associated with irreversible probability flow.



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Sciforum-185657: Thermodynamic Signatures of a Dunkl–Pauli Oscillator under Aharonov–Bohm Flux

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We investigate a two-dimensional Dunkl–Pauli oscillator in the presence of Aharonov–Bohm (AB) flux, focusing on the interplay between reflection symmetry and topological gauge effects. The model is formulated by introducing Dunkl operators into the Pauli Hamiltonian, which incorporates both spin interactions and a singular magnetic flux confined to a flux tube. By solving the stationary eigenvalue problem with appropriate boundary and regularity conditions, we show that the combined action of Dunkl deformation and AB flux imposes a nontrivial constraint on the admissible quantum states.

This constraint leads to the emergence of a lowest allowed angular quantum number, which depends explicitly on the flux and reflection parameters and effectively restructures the Hilbert space rather than producing a simple shift in the energy spectrum. Using the exact energy eigenvalues, we construct the canonical partition function and derive the corresponding thermodynamic quantities, including internal energy, entropy, and heat capacity.

The thermodynamic behavior reveals clear signatures of the spectral constraint: at low temperatures, the system is dominated by the modified ground state, while the heat capacity exhibits a flux-dependent Schottky-type anomaly. At high temperatures, the system approaches the classical oscillator limit. These findings highlight how the interplay between Dunkl symmetry and topological flux can generate observable thermodynamic effects, with potential applications in mesoscopic quantum systems and engineered gauge-field environments.



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Sciforum-185659: Exactly Solvable Many-Body Calogero–Dunkl Systems with Dihedral Angular Crystal Fields

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We introduce a class of exactly solvable many-body quantum systems combining Calogero–Moser interactions with dihedral angular crystal fields in two dimensions. The model describes NNN identical particles confined in a harmonic potential and interacting through inverse-square pairwise potentials along Cartesian directions, together with angular barriers arranged according to the dihedral group symmetry. By constructing Dunkl operators associated with a reducible reflection group structure, we demonstrate that the Hamiltonian can be mapped, on an invariant subspace, to a system of independent harmonic oscillators up to a coupling-dependent constant shift.

This algebraic reduction yields an explicit closed-form energy spectrum and ground-state wavefunction, characterized by a Jastrow-type factor encoding both pairwise and angular singularities, multiplied by a Gaussian envelope. The resulting ground state exhibits strong angular localization and nontrivial correlation properties reminiscent of fractional statistics in one-dimensional systems. We further analyze physical observables, including the one-body density matrix, dynamic structure factor, and thermodynamic functions, highlighting how the interplay between many-body correlations and discrete rotational symmetry manifests in measurable quantities.

The model provides a unified and exactly solvable framework that combines two distinct integrable structures—Calogero interactions and dihedral Dunkl symmetry—within a single Hamiltonian. Possible experimental realizations in ultracold atoms, optical lattices, and quantum Hall-type systems are discussed, suggesting that such systems may offer a platform to probe symmetry-driven many-body effects and engineered angular potentials.



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Sciforum-188496: Optical Geometry from Critical Superfluid Dynamics: Einstein–Euler Parametrization and Non-Integrable Causal Structure

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We construct an effective optical geometry associated with a hypothetical geometric fluid from a small fluctuation of Minkowski geodesics. The fluctuation is encoded through a discrepancy term constrained by a refractive-index perturbation (RIP) structure, leading to a velocity-dependent optical deformation of spacetime. This construction naturally introduces a bounded kinematic domain\[V＜v＜c,\]where the lower and upper velocity limits are connected through a reciprocal causal structure relating infrared and ultraviolet sectors.

The lower bound VVV is interpreted as a Landau-type critical velocity associated with the transition between laminar and turbulent regimes of the effective fluid. From the resulting optical structure, we derive an optical time function and construct an Einstein–Euler basis leading to an Arnowitt–Deser–Misner (ADM) decomposition of the effective spacetime. In this framework, the infrared critical sector dynamically selects a distinguished foliation associated with the critical regime of the optical geometry.

The deviation vector parametrizing the foliations encodes geometric and hydrodynamical properties of the system, including vorticity and rotational effects analogous to the Sagnac effect. The optical metric is further explored as the basis for a kinematic K-essence model whose dynamics exhibit a richer structure than standard K-essence fluids due to the presence of reciprocal infrared and ultraviolet sectors.

Finally, we investigate effective flow properties derived from the Hawking–Ellis energy conditions and discuss the ability of the resulting optical geometry to reproduce compact gravitational configurations while avoiding singular behavior through small geometric deformations. These results suggest a connection between analogue gravity, relativistic hydrodynamics, superfluid vacuum models, and disformal optical geometries. This work constitutes the first part of a broader program devoted to the study of the geometric, causal, and hydrodynamical properties of the proposed optical metric.



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Sciforum-189317: Analysis of observables in FLRW spacetime

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We consider a positively curved Friedmann–Lemaitre–Robertson–Walker spacetime as a background in which a nonrelativistic quantum particle propagates according to the Schroedinger equation. The probability fluid for the corresponding quantum states is taken as a model for the cosmological fluid filling this Friedman–Lemaitre–Robertson–Walker Universe. The Hamiltonian operator governing this fictitious particle is proportional to the Laplacian operator derived from the Friedman–Lemaitre–Robertson–Walker metric, while the mass of the particle equals the overall matter of the Universe (baryonic and dark). A complete, orthonormal set of quantum eigenstates of the Hamiltonian is obtained. Restricting to radially symmetric states, the latter are then used to compute matrix elements and expectation values of two observables for which quantum operators are identified, namely, the cosmological constant and the gravitational Boltzmann entropy. This value of the entropy is regarded as corresponding to a positively curved Friedman–Lemaitre–Robertson–Walker geometry when the cosmological fluid filling the Universe occupies a given quantum state. Our results comply closely with the upper bound on the entropy set through the holographic principle stated independently by 't Hooft and Susskind in the 1990's.



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Sciforum-189503: The Octavian Coherence Gate: The Four Irreducible Integers of the E8(1) ⊃G2(1) ×F4(1) Conformal Embedding

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We prove a constraint theorem for the octonionic unification program: any construction that fixes its dimensionless data (conformal weights, quantum dimensions, or coupling ratios) with zero continuous parameters is forced to use a single algebraic engine, and we identify it completely. Three requirements define the class of theories under study: (R1) the underlying lattice satisfies five closure axioms (integrality, evenness, unimodularity, positive-definiteness, and minimality of rank); (R2) both factors of its conformal sub-algebra are automorphism groups of octonionic structures; (R3) the emergent Z3 of the broken SU(3) closes through a dimension-one current extension. Under R1–R3, the lattice is E8, the sub-algebra is G2(1) ×F4(1), and the derivation chain uniquely determines four irreducible integers (n7,n26,d10,d11) = (7,26,2,3). Three faces of the conformal-embedding identity c(G2,1) + c(F4,1) = c(E8,1) = 8 lock them together, and every formal single-integer perturbation violates at least one constraint. An exhaustion theorem shows that all five alternative conformal embeddings of E8(1) fail independently. The result is a constraint theorem on an existing research program: any octonionic unification that fixes its data with zero continuous parameters must use the engine E8(1) ⊃G2(1) ×F4(1) and inherit the dictionary of rational constants assembled here. A single quarantined identification hypothesis is tested against the charged-lepton spectrum: the exported Koide ratio Q0 = 2/3 and Brannen phase θ= 2/9 each require mτ = 1776.97 MeV, consistent with the PDG 2024 value at the 1σ level.



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Sciforum-189541: Analytic Direct Calculation of the Topological Charge Structure for Calorons on $S^1 \times R^3$

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Introduction: Topological solitons are classical solutions to field equations whose stability is guaranteed by topological invariants known as topological charges. Among these, calorons are defined as topological solitons in SU(N) gauge theories on the compactified space $S^1 \times R^3$, where one spatial direction is periodic, representing the inverse temperature. Due to the direct-product structure of this background space, the total topological charge Q of a caloron exhibits a nontrivial internal structure, conventionally split into Q = q1 + q2 [1]. However, a rigorous mathematical procedure to directly derive this exact formula from explicit gauge configurations of caloron solutions has not been widely established.

Methods: In this study, we perform a rigorous analytic evaluation of the topological charge and its internal structure directly utilizing explicit caloron configurations. By meticulously evaluating the boundary integrals and the behavior of the fields under general gauge configurations, we clarify the specific conditions under which the internal components of the charge vary, thereby providing a systematic mathematical classification of the topological sectors.

Results: While the conventional Nye–Singer formula [1] holds under restricted gauge conditions, its mathematical interpretation and correspondence with explicit integration in general gauge configurations have been ambiguous. Our analysis explicitly investigates the singularities of the integrand, the regularizing effects, the contributions in the monopole limit, and the precise behavioral changes under varying holonomy parameters [2]. Through these steps, we successfully uncover the detailed mathematical mechanism governing the direct derivation of the caloron's topological charge.

Conclusions: Clarifying the mathematical framework of calorons under general gauge choices provides foundational insights into the nonperturbative vacuum structure of finite-temperature gauge theories, which is deeply related to the quark confinement mechanism.

[1] T. M. W. Nye and M. A. Singer, “An L^2-Index Theorem for Dirac Operators on $S^1 \times R^3$,” J. Funct. Anal. 177 (2000) 203–218 [arXiv:math/0009144 Search...].[2] A. Nakamula and G. Sumiyama, “Topology of Calorons Re-examined,” [arXiv:hep-th/2601.08543].



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Sciforum-189612: Symmetry with respect to positive and negative energies of the electromagnetic field

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It is well known that the Dirac equation (DE) is symmetric with respect to positive and negative energy solutions. Symmetry means that positive and negative eigenvalues have a one–one correspondence and have contrary numbers to each other. The positive and negative eigenvalues correspond to positive kinetic energy (PKE) and negative kinetic energy (NKE) states, respectively. This symmetry is retained when the non-relativistic approximations of the DE are made, which are the Schrödinger equation (SE) and the NKE SE. The relativistic quantum equations for spin-0 particles also have this symmetry. In all three cases, it is seen that the non-relativistic motion of particles, the relativistic motion of particles with spin-0 and spin-1/2, and quantum mechanics equations are symmetric with respect to the PKE and NKE.

The energy spectrum of the PKE (NKE) branch has a lower (an upper) limit and has no upper (lower) limit, so the system has a positive (negative) temperature. A PKE system absorbs or emits photons with positive energies when it transits between energy levels, while an NKE system has to absorb or emit photons with negative energies.

The energy–momentum relation of photons is . This means that a photon can carry either a positive energy $E=cp$ or a negative energy . The electromagnetic field with negative energy is called negative radiation (NR). We take the second quantization of the NR. The spectrum of the radiation (NR) has a lower (upper) limit and no upper (lower) limit, so it has a positive (negative) temperature, which is called the symmetry between the radiation and NR.

In our universe, the PKE (NKE) systems absorb and emit the radiation (NR). We say that PKE (NKE) systems match with radiation (NR) and call this pair of matches a symmetry of the universe.



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Sciforum-189916: A Geometric Model of Electromagnetic Interaction Based on the Volumetric Curvature of Space

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This work presents a hypothetical geometric model in which electric charge and electromagnetic interaction are interpreted as consequences of localized volumetric deformation of space. Instead of introducing electric charge as a primitive entity, the model starts from a local linear deformation coefficient of space, denoted by alpha, and defines the volumetric curvature intensity as Delta K_v = 1 - alpha^3. A charged source is modeled as a localized spherical region with constant internal volumetric curvature intensity and an external compensating deformation.

The external distribution is derived from a variational argument. A scalar volume-change function Delta V(r) is introduced and, in the external region, satisfies Laplace's equation. This leads to an external curvature-intensity profile proportional to r^(-4). The geometric charge is then defined as an integral characteristic of the external curvature flux, through the divergence of a radial vector field associated with this deformation.

The interaction between two non-overlapping spherical sources is constructed using a rationalized bilinear interaction functional. The leading term of the resulting force has the Coulomb form. Comparison with the standard Coulomb law relates the geometric stiffness, or energy density, of space to the electric constant. A magnetism-like interaction is interpreted as a consequence of moving geometric sources: the motion of the associated deformation produces an energy flux, and the corresponding momentum flux is obtained by division by c^2.

The model also interprets rest mass as the creation energy required to form the full deformation profile of a charged source. The proposed approach is not intended as a completed theory, but as an exploratory mathematical–physical framework for discussing charge, mass, and electromagnetic interaction.



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Sciforum-190128: Superfluid Vacuum Interpretation of Quantum Mechanics: Fluid Waves, Guiding Spacetime, and Emergence of Classicality

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Known interpretations of quantum mechanics are somewhat disconnectedfrom a theory of relativity, which is predominantly used to describethe dynamics of elementary particles and forms the foundation ofquantum field theory and the contemporary Standard Model of particlephysics. Conventional quantum mechanics relies on Galilean-covariantevolution equations of the Schrödinger type, whereas relativistic waveequations are Lorentz-covariant, of the Klein–Gordon type; therefore,their solutions do not have a straightforward probabilisticinterpretation. The superfluid vacuum conjecture reconciles these twofundamental symmetries, integrating both within a unified formalism.It uses two formal mappings, which endow wavefunctions with afluid–mechanical interpretation, relate the Schrödinger equation tofluid flow, describe its small fluctuations, and ultimately give riseto a four-dimensional Lorentzian spacetime. The latter naturallyincorporates gravity and the equivalence principle, serving as abackground for the propagation of small fluctuations of the superfluidvacuum and the emergence of relativistic fields. In this formalism,microscopic particles are described by small-amplitude waves of thebackground superfluid that obey relativistic field equations in curvedspacetime with an induced metric, until they interact with ameasurement apparatus and manifest particle-like behaviour.Furthermore, the superfluid vacuum approach suggests an alternativemechanism for the emergence of the classical world from a quantumsubstrate. While decoherence may account for the classicality of systemsof many weakly interacting particles, strongly interacting matterscales up primarily by overlapping wavefunctions and formingcollective states. This naturally builds up the collective state'smass toward mesoscopic and macroscopic scales (maintaining theequivalence of gravitational and inertial mass due to the aforementionedspacetime structure), whereas quantum superposition eventually breaksdown.



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Sciforum-190335: Why is gravity in compatible with quantum mechanics? (OR) Can we hear the action of black-holes?

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In this talk, I demonstrate that the gravitational action or the Einstein-Hilbert action works like an analytical index that counts the distribution of the Quasi-Normal Modes of black-hole geometries. This implies that one cannot define an action principle for gravity, as extremization of analytical indicies has no meaning. This represents a fundamental contention with quantum mechanics which is explcitly a consequence of quantization of action i.e. Bohr-Sommerfeld quantization, Dirac quantization. An undefined action principle, hence, implies an undefined quantization procedure and consequently, incompatibility with quantum mechanics. Apart from the above physical consequence, the result also demonstrates how the Quasi-Normal modes encode geometric information and how we can see that from the gravitational action. Black-hole Quasi-Normal modes are completely analogous to the normal modes of a drum or a membrane and given that these are demonstrated to encode the geometry of the drum to an extent, the same must follow for black-hole geometries i.e. black-hole geometries must be analogous to drums. This connection compels us to ask - *Can we hear the action of black-holes?* just as Mark Kac was once compelled to ask - *Can we hear the shape of drums?*. Hence, presenting a novel relationship between analysis and geometry.



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Sciforum-190388: The Magnus Expansion for Non-Hermitian Hamiltonians

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The Magnus expansion offers a powerful, structure-preserving geometric method to express a time-ordered exponential operator solution—typical of time-dependent linear differential equations—as a single, ordinary operatorial exponential. This elegant representation possesses vital theoretical advantages, such as preserving the underlying Lie algebraic structure of the system while remaining an exact solution to the original differential equation. For any finite-dimensional Hermitian Hamiltonian governing a closed quantum system, the standard Magnus expansion inherently guarantees a manifestly unitary representation at every finite order of truncation, enforcing essential geometric conservation laws. However, this foundational symmetry completely breaks down when the underlying Hamiltonian is non-Hermitian, a regime increasingly vital for modeling open, dissipative, or topologically complex quantum networks. Under non-Hermitian configurations, standard Magnus generators cease to be self-adjoint, introducing unphysical norm inflation, numerical instabilities, and artificial exponential runaways that mask true phase dynamics.

In this work, we resolve these limitations by deriving a generalized, unitarized Magnus expansion framework that systematically restores and maintains the property of geometric unitarity for all bounded, finite-dimensional non-Hermitian Hamiltonians. Utilizing a dynamical metric normalization mapping inspired by right polar matrix decomposition, our formulation isolates coherent phase information from state-decay artifacts by consistently projecting the evolution manifold back onto a stable unitary subspace. We benchmark this framework against traditional expansions using two distinct numerical and analytical paradigms: a periodically driven Hatano–Nelson lattice exhibiting the non-Hermitian skin effect, and a driven $\mathcal{PT}$-symmetric two-level channel. The results demonstrate that our framework eliminates non-unitary drift down to machine precision, offering a robust tool for non-Hermitian quantum dynamics and topology.



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Sciforum-190526: Double Bose–Hubbard chain model and a transfer of Quantum steering

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We study the EPR quantum steering transfer along two chains of qubits. The chains do not interact with each other. The system under consideration is modeled by a Bose–Hubbard-type Hamiltonian that contains N anharmonic quantum oscillators arranged into two chains. We analyze the time evolution of the parameters introduced by Cavalcanti et al. [1], which describe the quantum steerability between two qubits in two cases. First, we are interested in the steering transfer along individual chains, and next, in the steering between two non-interacting chains. We show that if the first pair of qubits belonging to separate chains is prepared in a steerable state, then such steering can be transferred from the beginning of the chains to their end with good efficiency. Additionally, we show that:

* In cases of strong initial steering (where the initial steering between qubits is high), steering between chains plays the leading role. Moreover, the steerabilities between qubits belonging to the same chain are much weaker and so are less significant.

* In the weak initial steering case (where the initial steering between qubits is low), the steerabilities between qubits within the same chain dominate those between the two chains.

1. E. G. Cavalcanti, Q. Y. He, M. D. Reid, H. M. Wiseman, "Unified criteria for multipartite quantum nonlocality", Phys. Rev. A 84, 032115 (2011).



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Sciforum-190721: An Individual-Based Socio-Epidemiological Model for Crowd Evacuations and Pedestrian Crossings

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Introduction. This research extends individual-based crowd dynamics by integrating emotional and disease-related states into a microscopic socio-epidemiological model. Individuals are described by mechanical variables and an activity variable incorporating these states. Evaluating the model through evacuation and pedestrian crossing scenarios allows analyzing disease spread impacts and determining if emotional states significantly influence outcomes.

Methods. Using kinetic theory of active particles, each pedestrian is modeled using mechanical variables and a continuous activity vector representing the awareness level (u) and a disease-related status (SEIR/V). The mathematical model integrates ordinary differential equations based on the evolution of awareness, the law of mass action for infection dynamics, and directional forces (geometric and pedestrian-to-pedestrian interactions).

Results. Simulations show social dynamics critically impact epidemiological courses. In evacuations, low collective awareness causes crowding and higher exposure. In contrast, high awareness promotes distancing and reduces the transmission rate, though exit crowding makes infection inevitable. Notably, when the speed parameter depends on u, unaware pedestrians move faster, reducing evacuation time but potentially spiking infection rates exponentially. In crossings, maintaining social distancing is inherently difficult, reducing awareness effectiveness and highlighting the necessity of public health strategies like masking or vaccination.

Conclusions. This study demonstrates that social dynamics and compliance with public health protocols have a critical impact on the epidemiological trajectory. While collective awareness does not completely eliminate the risk of exposure in confined spaces or cross-flow, it does succeed in flattening the infection curve and protecting the most vulnerable populations. The model allows for the analysis of complex real-world scenarios, demonstrating that the incorporation of epidemiological and awareness variables provides the model with the necessary complexity to address multivariate dynamics.



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Sciforum-190735: Transformations of the 2-component BKP tau functions

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Associated with the infinite-dimensional Lie algebras b_{∞} and d_{∞}, the two-component BKP (2-BKP) hierarchy constitutes a significant class of integrable systems. This hierarchy accommodates the Novikov–Veselov equation and admits a concrete realization in the characterization of the total descendant potential for D-type singularities. In this paper, we reformulate the Lax equation of the 2-BKP hierarchy within Shiota's theoretical framework by introducing projection operations on mixed pseudo-differential operators, where scalar Lax operators parameterized by two independent differential operators ∂_1 and ∂_2 are adopted. Building on this, we establish the explicit (M_1,M_2)-reduction of the 2-BKP hierarchy. We then systematically construct tau function transformations for the 2-BKP hierarchy and demonstrate that such transformations exactly correspond to the Darboux transformations of the 2-BKP system, with the induced variations of the associated Lax operators explicitly derived. Moreover, we extend these transformation results to the (M_1,M_2)-reduction. We further prove that the additional symmetries of the 2-BKP hierarchy can be recovered as particular instances of the proposed tau function transformations. By iteratively implementing these Darboux-type transformations, we derive a series of Pfaffian identities for 2-BKP tau functions, which serve as generalized extensions of the classical addition formulae existing in previous references.



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Sciforum-190738: Generalized Wronskian Solutions of the Constrained KP Hierarchy

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The constrained KP hierarchy is characterized by the condition $(L^{k})_{＜0} = \sum_{i=1}^{m} q_i \partial^{-1} r_i.$ In this paper, we investigate the general form of gauge operators for solutions of the constrained KP hierarchy under successive Darboux transformations of both differential type $T_d(q)=q\partial q^{-1}$ and integral type $T_i(r)=r^{-1}\partial^{-1}r$. Starting from the trivial background $L^{[0]}=\partial$, we construct a chain of M type-I and N type-II transformations generated by multi-derivative families of eigenfunctions $f_i^{(0)},\dots,f_i^{(M_i k)}$ and adjoint eigenfunctions $g_j^{(0)},\dots,g_j^{(N_j k)}$, together with exponential-type functions φ_l, ψ_q. A key cancellation mechanism is identified: only the highest-order derivatives survive, while all lower-order terms are eliminated by subsequent gauge steps, yielding exactly m surviving channels that recover the initial constraint. For these channels we derive explicit expressions for $q_i^{[M+N]}$ and $r_i^{[M+N]}$ in closed determinant form, combining Wronskian-type and Grammian-type structures. The exclusion of specific gauge steps in the determinant entries is fully traced to the index of the generating function within each multi-derivative family, namely $M_1+\dots+M_i+i$ for type-I chains and $N_1+\dots+N_j+j$ for type-II chains. The transformed tau function $\tau^{[M+N]}$ and the normalized eigenfunctions $\rho_i^{[M+N]}=q_i^{[M+N]}\tau^{[M+N]}$, $\sigma_i^{[M+N]}=r_i^{[M+N]}\tau^{[M+N]}$ are then obtained for all three regimes $M＞N$, $M=N$, and $M＜N$, providing a unified description of the gauge-transformed constrained KP hierarchy.



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Sciforum-190858: Symmetries of field configurations on compact spaces, spinors and no-go theorems

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Most modern research into theories with more than four spacetime dimensions has taken place within the context of string theory and/or modern presentations of quantum field theories. They often involve making an ansatz for a Lagrangian, composed of fields described by fibre bundles and assumed to transform as multiplets of SU(N) gauge symmetries. Often, supersymmetry and/or quantisation of the theory are explicitly imposed. It is assumed that the Coleman-Mandula no-go theorem prevents non-trivial mixing of Lorentz symmetries and other symmetries.

By contrast, this work has been developed in a context in which higher-dimensional theories are rethought from first principles. The only fundamentals are the geometry of spacetime and its symmetry properties under transformations.

We consider diffeomorphisms on closed spacelike hypersurfaces, and the induced 'dragging action' on fields on these compact spaces. We use the action of translations and rotations on vector and scalar fields as examples.

Any fields on such a space can be decomposed into harmonics. These form multiplets under rotational diffeomorphisms.

We find that even classical fields or fields describing geometry can be decomposed into such harmonics, distinguished by their 'quantum numbers', which describe their transformation properties.

We show that if the three familiar spatial dimensions form a compact space, and there are six extra dimensions also forming a compact space, there are harmonics carrying the quantum numbers of a single generation of fermions in the Standard Model.

In such a theory, the relevant no-go theorem is not that of Coleman and Mandula, but that of O'Raifeartaigh. We find that the theory exploits a loophole in this theorem. The Poincaré group and the isometry group for the extra dimensions are embedded in an SO(10) group. However, this is only manifest in a non-physical 'decompactification limit'. Away from this limit, the extra symmetries are non-linearly realised.



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Sciforum-190867: DYNAMICAL CORRELATION IN BINARY MIXTURES OF BOSE–EINSTEIN CONDENSATES (BECs)

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This work intends to take account of dynamical correlations in binary mixtures of Bose–Einstein condensates (BECs), composed of two distinct species. In a preliminary analysis, we consider a purely one-dimensional mixture, where the anomalous (or off-diagonal) correlations are finite and do not require any renormalization.

Their behavior, examined within a variational framework using a gaussian density operator, corroborates a previous result indicating that the anomalous and noncondensate densities are of the same order and, therefore, any approach considering only noncondensate densities can only be viable at very low temperature (or zero temperature) or near the transition. This was demonstrated numerically by solving a set of self-consistent equations. Furthermore, in a binary mixture, a set of unprecedented correlations appears. Indeed, in addition to intraspecies correlations, we shed light on interspecies correlations, which depict the entanglement between the species.

In the vast literature, we noticed that these effects are generally omitted owing to the fact that, in many experiments, the entanglement between different species is considered to be marginal.

However, in this work, we show that this needs not be the case in a general context. These effects are, however, quite tiny and cannot be observed in situations where the mean field is dominant. Hence, we considered situations where the mean field is almost zero. This fortunately happens for a binary system when the interspecies interactions are attractive and almost neutralize the intraspecies ones. This leads to self-bound states named droplets, which were indeed discovered experimentally.

Based on a semi-classical approach, we determine variationally analytic expressions for both the anomalous and the noncondensate densities for one-, two- and three-dimensional systems. The anomalous averages and the noncondensate densities for a 1D homogeneous bose mixture are computed numerically.



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Sciforum-190882: Vibronic Coupling, Jahn–Teller Instability, and the Emergence of Spontaneously Broken Symmetry

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Spontaneous symmetry breaking (SSB) in molecular and condensed-matter systems is shown to originate fundamentally from electronic degeneracy and vibronic coupling. Because nuclear configurations are governed by electron–vibrational interactions, any structural transformation necessarily involves more than one electronic state, making vibronic coupling the essential mechanism that links electronic structure to nuclear geometry. Within this framework, the Jahn–Teller (JT) and pseudo-Jahn–Teller (PJT) effects provide the microscopic origin of instabilities in high-symmetry configurations.

Although the adiabatic potential energy surface (APES) is widely used to rationalize symmetry-lowering distortions, it becomes physically meaningful only when vibronic coupling is sufficiently strong to localize nuclear motion away from degeneracy points. The observability of symmetry breaking depends on the competition between tunneling dynamics and the probe's temporal resolution. A fast measurement can “catch the system” in a particular well, revealing a distorted configuration, whereas a slow probe averages over many tunneling events and reports an apparently high-symmetry structure.

An important part of this presentation is the emergence of SSB in crystals, where local vibronic instabilities become cooperative. In perovskites such as $BaTiO_3$, the collective interaction of many PJT-active centers deepens the effective wells on a macroscopic scale, suppresses tunneling, and produces a genuine phase transition characterized by long-lived broken symmetry. This cooperative behavior links local vibronic physics to macroscopic-order parameters.

Finally, conceptual parallels are drawn between vibronic SSB and the Higgs mechanism in particle physics, emphasizing that degeneracy (in a broad sense, exact or approximate) is the universal prerequisite for symmetry breaking across atomic, crystalline, and field-theoretic systems.



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Sciforum-191563: Variance of a multiple-species bosonic mixture

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At the limit of an infinite number of particles, when the product of the number of bosons times the interaction strengths is kept fixed (i.e., at the mean-field or Gross–Pitaevskii limit), trapped Bose–Einstein condensates and mixtures thereof are known to be 100% condensed [1,2,3]. Furthermore, the many-body energy per particle and intraspecies and interspecies densities per particle exactly coincide with the mean-field results. Nonetheless, there can be massive correlations at this limit, which are readily exposed by variances of observables [4]. Such correlations emerge from the occupation of even a single particle in an excited natural orbital of the reduced one-particle density matrix that interacts with the large number of bosons in the condensed mode, and originate from the fundamental difference between the many-body and mean-field wavefunctions at this limit [5]. The differences between the many-body and mean-field variances can be used to characterize and explore such correlations [6,7]. In this work, we address some uncharted possibilities and consequences for the variance of observables in a multiple-species bosonic mixture by employing a generic solvable multiple-species model [8,9]. We analyze the variance of the position, momentum, and angular-momentum many-particle operators in a multiple-species bosonic mixture and identify how various correlations express themselves in fluctuations of observables. We investigate intraspecies and, in particular, interspecies observables in the mixture and their uncertainty products. A multiple-species uncertainty product is found to be structureless at the mean-field level of theory, whereas, at the many-body level of theory, it exhibits intricate properties. Generalizations, e.g., to a time-dependent scenario [10], are envisioned.

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Sciforum-192055: A zero-range process with a dynamic edge-gating environment: exact reversible measure, load-coupled critical density, and metastability on dumbbell graphs

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We study a zero-range process (ZRP) on a finite connected graph G_0 = (V, E) coupled to a dynamic binary on-site environment. Each vertex v carries an activity l_v in {0, 1}; an edge {i, j} is open iff l_i l_j = 1, so the environment gates the active subgraph. In turn, l_v flips with rates that depend on the local particle number eta_v. The coupling is two-way: particles drive the environment and the environment selects the active subgraph. The environment is neither a second conserved particle species (as in two-species ZRP) nor quenched disorder; it is a non-conserved edge-gating field with feedback to the particles.

Result 1. For arbitrary positive rate functions alpha, beta and any ZRP jump rate u, the joint generator on the state space Z_+^V x {0,1}^V is reversible with respect to the explicit product measure

Pi(eta, l) ~ prod_v p(eta_v) (k_v)^{eta_v} [1{l_v=1} + (alpha/beta)(eta_v) 1{l_v=0}]

on the canonical sector, where p is the standard ZRP weight associated with u and k_v is the degree of v. The measure factorises as an eta-marginal times a state-dependent Bernoulli field for l.

Result 2. For affine coupling alpha(n) = A, beta(n) = 1 + n and jump rate u(n) = 1 + b/n with b $>$ 3, the eta-marginal admits a closed-form effective critical density rho_c(A) = [A(b-3) + (b+1)] / [(b-3) (A(b-2) + 1)], monotone decreasing in A and recovering the Evans value 1/(b-2) as A tends to infinity.

Result 3 (in progress). On the dumbbell graph, the Z_2 graph automorphism is preserved by the equilibrium measure but generically broken in the supercritical phase. Using the Beltran-Landim capacity method, we obtain the leading order of the inter-clique transition rate of the condensate as a function of (m, N, L, A).



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Sciforum-194863: Elements of Symmetry in the FDTD Method for Selected Problems of Computational Electrodynamics

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Introduction: The finite-difference time-domain (FDTD) method, based on Yee's staggered grid, is a cornerstone of computational electrodynamics. Its accuracy fundamentally depends on how discrete symmetries of Maxwell's equations are preserved under spatial–temporal discretization. This work investigates symmetry-related aspects of the classical Yee scheme in modeling plane-wave propagation and interface interactions.

Methods: We analyze the discrete dispersion relation of the Yee algorithm, derive FDTD analogs of Fresnel coefficients for dielectric and magnetic interfaces, and formulate effective boundary conditions emerging from the staggered placement of field components. The transition-layer model is introduced to interpret interface discretization as an implicit symmetry-breaking layer of width Δ_x.

Results: We demonstrate that the "magic" Courant condition $S_c = n_r$ minimizes but does not eliminate numerical dispersion errors, with residual asymmetries depending on the signal spectrum and source ramp-up. Furthermore, we establish that the ratio of numerical to exact wavenumbers is invariant under the transformation $n_r \leftrightarrow -n_r$, revealing a fundamental symmetry between right-handed (RHM) and left-handed (LHM) media. For interface coefficients, we prove that reflection errors δ_R remain invariant under impedance exchange $\eta_1 \leftrightarrow \eta_2$, whereas transmission errors δ_T exhibit directional asymmetry governed by the ordering of adjacent media. These symmetry properties are preserved for both dielectric and magnetic boundaries when impedance ratios are matched.

Conclusions: The study reveals that symmetry preservation and controlled symmetry-breaking in FDTD discretization govern accuracy at material interfaces. The derived error estimates and symmetry criteria provide practical guidelines for reliable modeling in metamaterials, nanophotonics, and other fields requiring precise interface treatment.



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Sciforum-195347: Holonomic Resonances as Emergent Signatures of a Trigonometric Topological Mapping in the Extended Alpha Group Matrix

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The Extended Alpha Group Matrix M(θ) is a 4×4 hypercomplex operator composed of coupled symmetric and antisymmetric sectors parameterized by trigonometric functions. In this work, we investigate the emergence of a trigonometric topological mapping generated by the competition between the cot(θ) and tan(θ) sectors of the matrix and its relationship with connectivity dynamics and holonomic resonances over the angular cycle $0 \leq \theta \leq 2\pi$.

For each angular value, a normalized trajectory is generated in a four-dimensional state space through successive applications of M(θ). Pairwise distances between trajectory points are used to construct graph representations of the dynamics, enabling the extraction of connectivity layers and the tracking of their evolution throughout the angular cycle.

To characterize structural interactions, connectivity layers are coupled through antisymmetric circulation operators, producing angular holonomic fluxes between graph levels. The accumulation of these fluxes defines a discrete absolute connectivity estimate C, obtained from the magnitude of the resulting circulation matrix. This quantity provides a global measure of connectivity dynamics.

Within this framework, the trigonometric topological mapping is defined as the angular correspondence Φ:S^1 → G, which associates each angular state of the Extended Alpha Group Matrix with a connectivity topology generated by the corresponding dynamical trajectory. Its topological nature stems from the continuous reorganization of connectivity structures driven by the interplay between the cot(θ) and tan(θ) sectors, giving rise to localized transition interfaces and emergent holonomic resonances.

The analysis reveals that the competition between the cot(θ) and tan(θ) sectors generates transition interfaces characterized by variations in connectivity, entropy, and holonomic behavior.

These findings support the interpretation that holonomic resonances are emergent signatures of global topological organization induced by the coupled action of the symmetric and antisymmetric sectors of M(θ), providing a novel framework for the study of connectivity-driven dynamical structures.



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Sciforum-198547: From Real Hilbert Symmetry to Complex One-Particle Structure: Real Commutants, Type I Descent, and Positive Energy

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Complex one-particle Hilbert spaces are often obtained from real phase or solution spaces by choosing a compatible imaginary unit. We treat this choice as a representation-theoretic invariant. For an orthogonal representation U on a real Hilbert space $H_{\R}$, with complexification V on $H_{\C}$ and canonical conjugation C, define the Real commutant\[\AR(U)=\{T\in B(H_{\C}):TV(g)=V(g)T\text{ for all }g,\ TC=CT\}.\]We prove that the symmetry-compatible orthogonal complex structures are exactly the skew-adjoint square roots of $-\Id$ in this real C^*-algebra. For central, equivalently disjoint, type~I direct-integral realizations of V, canonical conjugation determines a multiplicity-side antiunitary transport gauge class, and $\AR(U)$ is the algebra of decomposable multiplicity operators satisfying the corresponding transport relation. Abelian spectral orientations and compact Frobenius–Schur decompositions are recovered as scalar and atomic cases. For strongly continuous orthogonal flows, the sign of the Stone generator selects a canonical complex structure J on the nonzero-energy sector and factors the real generator as $A=JB=BJ$ with $B\ge0$; this positive-energy factorization characterizes J uniquely. With a compatible invariant symplectic form, the selected J yields the complex one-particle space, the pure gauge-invariant quasifree Weyl state, and, under a norm-equivalence hypothesis, the corresponding ground-state Fock representation. The stationary Klein–Gordon formula $J(q,p)=(\omega^{-1}p,-\omega q)$ is obtained as a direct example



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Sciforum-199147: Equations of motion on the extended Siegel–Jacobi space generated by a linear Hamiltonian

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Abstract: Starting from the case of an extended Siegel–Jacobi upper half-plane, as studied in previous publications [1,2,3], we present a study of the extended Siegel–Jacobi upper half-space of order n, with $n>1$. Starting from the associated Hamiltonian, which is linear in the generators of the Jacobi group, we write the general formulas for the components of thetotal energy function associated with it and then explicitely give their concrete forms. We show that the extended Siegel–Jacobi upper half-space endowed with a certain one-form and a symplectic two-form can be organized as an ACOS manifold, and also as a GTACOS manifold, a fact that helps us formulate the general relations for the equations of motion on it.

Deducing certain correspondences, the concrete expreessions for the equations of motion are computed. The results give a strong generalization of the previously studied case of the equations of motion on the extended Siegel–Jacobi upper half-plane, i.e., when n=1. In conclusion, the main contribution of this work consists of writing the energy functions and deducing the equations of motion on the extended Siegel–Jacobi upper half-space of order n for the coordinates (x,y,q,p,κ), where x,y are symmetric matrices, q,p are column vectors and κ is a real coordinate. The particular case of the extended Siegel–Jacobi upperhalf-plane (n=1) is treated separately, showing the correspondences between the formulas in the general case of $n>1$ and those for n=1. The definitions of ACOS (almost cosymplectic) and GTACOS (generalized transitive almost cosymplectic) manifolds are recalled, as they are essential to our endeavour.

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Sciforum-200105: Quantized Curvature Reconstruction via SU(2) Generators: Extending Absolute Unified Mathematics of Quantum Gravitation

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Mercury's relativistic perihelion precession, estimated at ~1.95 billion arcseconds over 4.54 billion years, corresponds to nearly 1,500 extra orbital revolutions. This anomaly, unexplained by Newtonian mechanics, remains a benchmark for testing Einstein's general relativity. Clifford algebra offers a deeper representation of the geometric symmetries of the Riemann curvature tensor, providing a basis for reformulating gravitational field equations within quantum frameworks. This study extends the Absolute Unified Mathematics of Quantum Gravitation framework introduced in 2024–2025. An integral algebraic framework is developed by embedding SU(2) basis elements () in the off-diagonal components of the metric tensor, where they generate spacetime symmetries and correspond to spinor structures. This demonstrates that Pauli matrices act as algebraic building blocks of spacetime geometry, enabling systematic study of curvature and symmetries. Newton's gravitational potential and Poisson's equation are derived through modified Christoffel symbols, linking algebraic operators to geometric curvature. The study further reconstructs the Riemann tensor and analyzes Pauli matrices in the spherical basis. The findings show that embedding SU(2) generators within the metric tensor reconstructs curvature structures and symmetries consistent with Einstein's framework, while extending toward quantum–gravitational interactions. Mercury's orbital precession anomaly is interpretable through these algebraic structures. In spherical coordinates, the operators (, ,∅) represent spin projections along local spherical directions. Since these directions vary with and ∅, the matrices exhibit orientation dependence rather than radial scaling. Along a fixed ray, changes in radial () do not alter direction, leaving the matrices unchanged. This directional dependence of spinors eliminates singularities in the framework, reinforcing its algebraic consistency. Embedding SU(2) generators within the metric tensor establishes a rigorous pathway for reformulating Einstein's field equations in terms of quantized curvature tensor fields, providing a testable foundation under extreme quantum–gravitational conditions. This reformulation enables quantum gravity applications in quantum computing software.



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Sciforum-200287: The equilibrium shape of a crystal

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Calculus of variations involves important minimization problems encoding physical phenomena; for example, in the formation of bubbles, the molecules permute themselves to form the least energy-efficient structure, and the result is a ball. Mathematically, this formation is the isoperimetric inequality. Several extensions of this inequality exist via a free energy encoding a potential g. Hence, minimizing this free energy with a mass constraint can generate a convex crystal. Thus, the minimizer, in some contexts, encodes the property such that if one takes two points in the set and connects them with a line segment, the line segment will likewise be in the set. The relation between minimizers and crystals was encountered by Willard Gibbs and Pierre Curie in the 19th century. In a more modern thermodynamics application, Almgren's problem investigates the formation of a convex minimizer assuming convexity of the sub-level sets. Interestingly, even in two dimensions, this is complicated. The full physics application is in three dimensions, and few results exist for all masses. My talk will address this problem and will illuminate the importance of symmetry. The methods include the first variation, geometric measure theory, and PDEs. The results discuss the existence and non-existence of crystals, their shape geometry, and stability. A few conclusions will be discussed regarding extending the theory.



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Sciforum-200470: Cross-couplings between a Weyl graviton and a massless conformal gravitino: a local BRST cohomological analysis

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The problem of constructing consistent cross-couplings between a Weyl graviton and thea massless conformal gravitino is investigated. The starting points are the free theory obtained as the direct sum of the linearized Weyl action and the free action describing the QQQ-gravitino. The analysis is performed within the antifield-BRST deformation framework, where consistent interactions are characterized as deformations of the solution to the classical master equation. The investigation is further supplemented by specific methods of local BRST cohomology. Under a set of strong assumptions, namely analyticity in the deformation parameter, spacetime locality, preservation of the original Poincaré symmetry, and the requirement that the interaction vertices contain at most four spacetime derivatives, all possible consistent deformations are systematically examined. The local BRST cohomological analysis shows that any candidate cross-interaction between the Weyl graviton and the conformal gravitino is either BRST-trivial or obstructed at higher orders of the deformation procedure. It follows that no nontrivial consistent cross-couplings can be introduced between the fields under consideration in four spacetime dimensions within the class of local deformations satisfying the above hypotheses. The obtained result thus provides a no-go theorem for local interactions between a linearized Weyl graviton and a massless conformal gravitino under the stated assumptions.



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Sciforum-201003: Complexity and Geometry of Potential Energy Surfaces: A Structural NP-Hardness Framework

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Potential energy surfaces (PESs) are a fundamental mathematical concept used to describe molecular systems. Despite the central role of searching for the global minimum in theoretical chemistry applications, there is still no general explanation for the associated computational difficulty. In this study, we present a theoretical framework linking the geometry of PES to computational complexity theory principles. We introduce formal mapping between continuous molecular configurations and discrete optimisation problems, constructing polynomial-time reductions from NP-hard problems, such as 3-SAT and frustrated Ising models, to the global optimisation of generic classes of PES. In this representation, bistable conformational degrees of freedom encode discrete variables, while suitable interaction terms reproduce the energy structure of the original combinatorial problems. This enables the intractability of global optimisation to be interpreted as a structural property of the geometry of the energy landscape. The proposed framework suggests that phenomena such as the exponential proliferation of local minima, energy frustration, fragmentation of the configuration space, and dynamics characterised by metastable trapping, emerge as geometric manifestations of the underlying computational complexity. This distinguishes between 'smooth' landscapes, which are associated with relatively tractable structures, and 'rugged' landscapes, in which the search for the global minimum is inherently difficult. This perspective unifies concepts from complexity theory, landscape theory, and the physics of disordered systems. It provides a possible basis for a 'complexity theory of molecular systems' and for developing exploration and optimisation strategies that consider the limits imposed by computational complexity.



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Session 2. High Energy Physics, Particles and Fields

Sciforum-184183: Vacuums at the Planck scale

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At the Planck scale, a generalized vacuum, as a quasi-Euclidean space with Planck length and symmetry, begins to present a series of new features, thereby defining a physical world with spontaneous particles and quantum constraints. Because of the Planck length, a vacuum is non-commutative with its corresponding Euclidean space, carrying uncertainty, UV cutoff, IR cutoff, and extremely small spaces named Planck units. Due to rotational symmetry, a Planck unit typically exists in two isolated states, the in situ state and the rotational/tangent state in low-dimensional spaces, until the 10D Planck unit connects these two states and the corresponding 10D vacuum becomes a non-trivial one. Further investigation has provided several results, including that the non-triviality lies in a (3+1)D subspace; two long-range interacting forces exist with constants of ~1/137.036 and ~1/1.628E+38; and the 10D Planck unit acts as a stationary electron and a Compton photon in its two allowed states, and it exhibits either an up quark or a down quark when it behaves in an anomalous state with broken dimensional structures. These results reveal the origin of the gravity-and-electromagnetism interaction, and also demonstrate that the entire first generation of Fermions stems from the quantum behavior of the 10D Planck unit. This exploration establishes a BSM approach to acquire the elemental particles of SM theoretically, aiming to lay the foundation for a new GUT.



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Sciforum-184730: Leading singularities in effective Kähler superpotential for general chiral N=1 superfield model

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Using the Bogoliubov–Parasiuk theorem and the BPHZ renormalization procedure, we derive differential equations that sum the leading ultraviolet divergences of the Kähler potential in a general N=1 supersymmetric chiral theory with an arbitrary Kähler potential and chiral superpotential. Our approach recovers the known renormalizable Wess–Zumino limit and naturally extends to non-renormalizable chiral interactions. The resulting equations are formulated covariantly in terms of the resummed Kähler metric and its associated curvature, allowing a systematic treatment of leading poles and leading logarithms. In some cases, the equations simplify drastically and can be analyzed in detail. For the Wess–Zumino model with a power-like superpotential, the partial differential equation reduces to a nonlinear ordinary differential equation exhibiting scale invariance, critical behavior, and, for certain exponents, log-periodic solutions for some types of chiral superpotentials. We also analyze a string-inspired no-scale model with constant negative curvature and a massive quadratic superpotential, showing that the same reduction to an ODE occurs. In all cases, solutions display universal power–law scaling at large parameters. The obtained equations provide a practical tool to sum quantum corrections to the effective Kähler potential in string-inspired inflationary models, where loop corrections often play a crucial role. We also extract the leading-log behavior of the effective potential in some supersymmetric models.



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Sciforum-188290: Consideration of flavor-related additive quantum numbers

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Why are the flavor quantum numbers of different quarks (including isospin I and its projection I3, as well as S, C, B*, and T, a total of six) different and cumbersome? Why are flavor quantum numbers not conserved under weak interaction? Why is hypercharge Y a combination of baryon number and some flavor quantum numbers, but appears alongside I3 in the Gell-Mann–Nishijima formula? Starting from the consideration of these questions, the fermion quantum number F was proposed. F is a flavor-related additive quantum number for fermions (including quarks and leptons), conserved under weak, electromagnetic, and strong interactions, and the relation Q/e = (F0 + F)/2 (where F0 is the unit electroweak charge, including baryon number and lepton number) applies to all fermions, making it more universal than the Gell-Mann–Nishijima formula (which applies only to quarks). The selection rules of weak interactions were previously only empirical, but now they can be derived from F conservation, and previously undetected selections can be explained by F non-conservation. Furthermore, it has been demonstrated that F/2 and F0 are equivalent to weak isospin (Iw3) and weak hypercharge (Yw) in electroweak theory. Thus, for flavor-related additive quantum numbers in electroweak and strong interactions, only two are needed, namely F and F0, and the Lagrangian has been reformulated using F and F0.



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Sciforum-189099: Precession of spherical orbits and observational constraints on black hole parameters from the jet of M87*

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Recent observations of the jet precession of M87* provide a new opportunity to probe strong gravity effects and the spacetime geometry around supermassive black holes. In particular, the observed precession period strongly suggests the presence of a rotating black hole surrounded by a tilted accretion disk, making jet precession a promising astrophysical probe of relativistic orbital dynamics. In this work, we investigate the Lense–Thirring precession of spherical orbits in rotating black hole spacetimes and explore its implications for tilted accretion disks and jet dynamics. Using the Hamiltonian formalism, we derive the equations of motion for spherical orbits in general stationary axisymmetric spacetimes and apply the formalism to Kerr–Newman and Kerr–Taub–NUT black holes.

We show that both the black hole charge and the NUT charge significantly affect the precession of spherical orbits. In particular, the black hole charge suppresses the precession angular velocity, whereas the NUT charge enhances it and further breaks the $\mathbb{Z}_2$ reflection symmetry of the orbits, leading to asymmetric orbital trajectories with respect to the equatorial plane. By modeling the motion of particles near the warp radius of a tilted accretion disk with spherical orbits, we use the observed jet precession of M87* to constrain the black hole parameter space, including the spin, charge, NUT charge, and warp radius, and discuss their implications for tilted accretion disk structures and possible naked singularity scenarios.

These results suggest that jet precession observations can provide useful insight into nontrivial black hole parameters and the underlying spacetime symmetry structure.



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Sciforum-189162: Nuclear Moments Violating Fundamental Symmetries: Testing Unification Theories and Searching for Dark Matter

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Measurements of nuclear moments that violate parity (P) and time-reversal invariance (T) provide critical tests of unification theories predicting CP violation and the matter–antimatter asymmetry observed in the Universe (baryogenesis). These T,P-violating moments include the electric dipole moment (EDM), Schiff moment, magnetic quadrupole moment, and octupole moment. T,P-violating nuclear moments are primarily generated by T,P-violating nuclear forces, with smaller contributions arising from the proton and neutron EDMs.

Oscillating T,P-violating nuclear moments can be induced through interactions with oscillating axion dark matter fields, making their measurement a promising method for dark matter detection. These moments exhibit collective enhancement in deformed nuclei. For example, the Schiff moment is enhanced by factors of 100–1000 in nuclei with octupole deformation, while the magnetic quadrupole moment is significantly enhanced in quadrupole-deformed nuclei.

The interaction of electrons with T,P-violating nuclear moments results in atomic and molecular EDMs, which are directly measurable. Nuclear EDMs are entirely screened in neutral systems, meaning atomic and molecular EDMs originate from interactions of electrons with nuclear Schiff and magnetic quadrupole moments. However, nuclear EDM screening can be circumvented in ions, non-stationary states, and experiments using oscillating electric fields.

Additionally, the nuclear anapole moment (a parity-violating magnetic moment) has been experimentally observed in ^{133}Cs. Ongoing experiments aim to measure the anapole moments of other nuclei, further advancing our understanding of fundamental symmetries and their violations.



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Sciforum-189173: Effects of Dark Matter and of Space–Time Variation in Physics Constants in Atomic Phenomena

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Fundamental constants—such as the fine-structure constant α, the strong-interaction scale, and particle masses—may vary in an expanding Universe. Spatial variation could help explain apparent fine tuning: we inhabit a region where the conditions permit life. Hints from quasar absorption spectra suggest a gradient in α, but decisive confirmation requires laboratory tests. Atomic clocks enable such tests and, through their exquisite stability, enable sensitive searches for new physics.

Interactions between dark matter and ordinary matter can induce temporal variation in constants. For low-mass bosonic dark matter produced after the Big Bang, the field behaves classically, yielding first-order effects in the coupling—an enormous advantage over traditional second-order responses. Using atomic clocks and Big Bang Nucleosynthesis data, existing bounds on scalar and axion dark-matter couplings to photons, electrons, quarks, and W, Z and Higgs bosons can be tightened dramatically**; our analyses improved previous limits by up to 15 orders of magnitude.**

Another four orders of magnitude in sensitivity may be achieved using nuclear clocks which were built recently. Atomic and nuclear clocks may also be used to search for violation of fundamental symmetries and testing of unification theories.

Separately from the ultralight sector, models of modified gravity with extra dimensions—such as the Arkani-Hamed–Dimopoulos–Dvali framework—admit extremely compact massive composite objects made from quarks or antiquarks, which may be dark-matter candidates. Detection strategies for these heavy composites will be discussed.

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Sciforum-189324: Symmetries and Kinematics of Spinning Test Particles Orbiting Kerr-Bertotti-Robinson Black Holes

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Introduction: The Kerr-Bertotti-Robinson (Kerr-BR) metric describes a rotating black hole immersed in an external uniform magnetic field, which is fundamentally characterized by its axial symmetry. Investigating the kinematic effects of spinning test particles orbiting this type of black hole offers a valid testbed to explore how background spacetime symmetries govern celestial dynamics under external uniform magnetic fields.

Methods: By employing the Mathisson-Papapetrou-Dixon (MPD) equations, we explore the dynamics of precessing orbits and characteristic equatorial orbital configurations. The underlying continuous symmetries of the spacetime enable the construction of conserved energy and total angular momentum for the spinning particles via the corresponding Killing vectors. A precise formulation for the angular frequency is analytically derived directly from the metric to properly encapsulate the spin-gravity coupling.

Results: Our analysis reveals that the external magnetic field significantly alters the topological structure of the effective potential and modifies orbital stability. Crucially, due to the nonspherical nature of the metric symmetry, a distinct breaking of parameter space symmetry is observed between the prograde (ISCOS) and retrograde (ISCOL) innermost stable circular orbits. While an augmented magnetic field necessitates an increased orbital angular momentum for prograde configurations, sufficiently strong fields completely suppress retrograde circular orbits in near-extremal regimes.

Conclusions: These findings uncover the profound impact of external fields and underlying spacetime symmetries on the trajectories of spinning particles. The analysis of these symmetry-preserving and symmetry-breaking dynamics provides valuable theoretical insights into celestial kinematics, offering refined mathematical templates for future horizon-scale observational tests.



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Sciforum-189337: The Non-Perturbative Partition Function of Refined Topological Strings on a Twisted Affine Line Bundle over CxT2.

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We compute the non-perturbative partition function of refined topological string theory on a twisted affine line bundle over (\mathbb{C}\times T^2), thereby extending recent result obtained in the unrefined setting to the more general case with two independent deformation parameters. Our analysis builds upon the correspondence between topological string theory and supersymmetric gauge theories and makes use of the instanton partition function of five-dimensional (U(1)) gauge theory with eight supercharges and a single adjoint hypermultiplet, commonly referred to as the $N=2^*$ theory. The gauge theory is placed on the (\Omega)-background with both deformation parameters turned on, providing a natural framework for the refined topological string.

Starting from the exact instanton partition function, we derive an integral representation suitable for investigating its non-perturbative structure. We then perform a detailed analysis of the associated integration contour. By deforming the original contour into one that surrounds the positive real axis while excluding the origin, we identify the relevant contributions arising from poles and obtain a non-perturbative completion of the refined partition function. This contour deformation provides a transparent interpretation of the non-perturbative sectors and their relation to the underlying geometry.

Furthermore, we discuss parallels between the resulting non-perturbative contributions and the Schwinger pair-production mechanism in quantum field theory.



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Sciforum-190210: Gauge-Invariant Obstruction and Particle Production of Ultra-light Axion-like Fields in Kerr-Spacetime Environments

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Ultra-light axion-like fields provide a theoretically well-motivated dark-matter candidate and a sensitive probe of quantum field theory in strong gravitational backgrounds. This talk develops a gauge-invariant framework for studying their nonlinear effects in rotating Kerr black-hole environments. The central question is whether a perturbative but spatially structured axion condensate can generate physical signatures that go beyond coordinate-dependent metric deformations and reveal genuine deviations from the Kerr paradigm. I first discuss how the stress-energy distribution of an ultra-light axion field modifies the Kerr geometry at the perturbative level. Particular emphasis is placed on the hidden symmetry structure responsible for the separability of geodesic motion and field equations in exact Kerr spacetime. I argue that the axion-induced deformation generically produces a gauge-invariant obstruction to this integrability structure, leading to measurable consequences such as orbital-frequency shifts, phase drift in extreme-mass-ratio inspirals, and deformation of photon trajectories near the black hole. The second part of the talk connects this geometric obstruction with quantum particle production. By quantizing field perturbations on the corrected rotating background, one can analyze how frame dragging, ergoregion dynamics, superradiant amplification, and non-adiabatic axion evolution contribute to particle creation in curved spacetime. This provides a unified theoretical bridge between ultra-light dark matter, gauge-controlled perturbation theory, and quantum field production in strong gravity. The proposed framework suggests that future gravitational-wave observations and horizon-scale black-hole imaging may offer complementary probes of axion-like dark sectors and nonlinear quantum effects around rotating black holes.



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Sciforum-190644: Extreme geometrical effects on relativistic isobaric collisions

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The chiral magnetic effect (CME) is a phenomenon that could explain the dominance of matter over antimatter in the universe. Initially, symmetric isobaric collisions of Ruthenium ($^{96}_{44}$Ru) and Zirconium ($^{96}_{40}$Zr) nuclei were considered ideal for eliminating possible background signals when considering the ratio, potentially yielding a CME signal. However, in experiments, there are still multiple non-CME signals that make the detection of small CME signals difficult. The spectator protons produce a huge magnetic field, and therefore, understanding the initial magnetic field generated in isobaric collisions is necessary for studying the possible chiral charge separation and its alignment with the participant plane. In this study, we perform a beam energy scan (BES) analysis for several Relativistic Heavy Ion Collider energies (7.7, 9.2, 11.5, 14.6, 17.3, 19.6, 27, 39, and 200 GeV) and Large Hadron Collider energies (2.76, 5.02, and 5.44 TeV) per nucleon using a Glauber Monte Carlo formulation to study their effect on the initial magnetic field produced in isobaric collisions, in order to obtain a better understanding of magnetic field evolution in such systems. We found that for most central to mid-central events, the ratio $\langle B_2 \cos 2(\psi_B - \psi_2)\rangle_{Ru}$ / $\langle B_2 \cos 2(\psi_B - \psi_2)\rangle_{Zr}$ increases with energy as expected, but in most peripheral events, the ratio for the highest-energy collisions moves below that of all other energies.



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Sciforum-190740: Origin of the Covariant Wigner Operator as a Quantum Amplitude in QCD

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The Wigner function in QCD has traditionally been interpreted as a quasiprobability density, but recent work in Koopman–von Neumann (KvN) mechanics suggests it is more naturally understood as a probability amplitude. KvN is a classical statistical theory formulated within the Dirac–von Neumann axioms, complete with wavefunctions, a complex Hilbert space, the Born Rule, Hermitian operators, the eigenvalue problem, etc. It establishes continuity between classical and quantum Hilbert spaces, which allows the nonrelativistic Wigner function to be derived naturally as a wavefunction on classical phase space rather than a quasiprobability distribution, resolving its seemingly pathological features. Here we extend this reinterpretation to relativistic fermionic dynamics. Extending the KvN framework, the quark Wigner operator emerges naturally as a phase space spinor, i.e., spinorial amplitude rather than density. The meaning of a spinor in the classical limit is discussed. This formulation provides a transparent conceptual explanation for the negativity and other nonclassical features of the Wigner function: these arise naturally from the amplitude interpretation rather than appearing as artifacts of a quasiprobability. The framework reproduces classical QCD dynamics in the appropriate limit, establishing clean continuity between relativistic quantum and classical descriptions. Because parton distribution functions, generalized parton distributions, and transverse momentum distributions all derive from the Wigner operator, this reinterpretation may carry direct phenomenological consequences. In upcoming work, we will explore the experimental implications of treating the quark Wigner operator as an isomorphism to a spinorial amplitude.



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Sciforum-190742: Phenomenology of Charged Higgs Bosons in 2HDM at Future Lepton Colliders

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In this presentation, I will synthesize comprehensive investigations into the phenomenology of charged Higgs bosons (H±H^\pmH±) within the framework of the Two-Higgs-Doublet Model (2HDM) at future high-energy lepton colliders. As a direct consequence of extending the Standard Model scalar sector, the discovery of H±H^\pmH± would provide unambiguous evidence for physics beyond the Standard Model and offer deep insights into the mechanism of electroweak symmetry breaking. We specifically focus on the comparative potential of muon (μ+μ-\mu^+\mu^-μ+μ-) and electron–positron (e+e-e^+e^-e+e-) machines, including proposed facilities such as the International Linear Collider (ILC) and the Compact Linear Collider (CLIC). Our analysis covers a broad spectrum of production mechanisms, including:

* Pair and Single Production: μ+μ-→H+H-\mu^+\mu^- \to H^+H^-μ+μ-→H+H- and μ+μ-→W∓H±\mu^+\mu^- \to W^\mp H^\pmμ+μ-→W∓H±.* Vector Boson Fusion (VBF): ℓ+ℓ-→νν¯H+H-\ell^+\ell^- \to \nu\bar{\nu}H^+H^-ℓ+ℓ-→νν¯H+H-, demonstrating that VBF becomes highly competitive with, and often surpasses, $2 \to 2$ processes at multi-TeV center-of-mass energies.* Associated Production: ℓ+ℓ-→τ+ντH-\ell^+\ell^- \to \tau^+\nu_\tau H^-ℓ+ℓ-→τ+ντH-, ℓ+ℓ-→tb¯H-\ell^+\ell^- \to t\bar{b}H^-ℓ+ℓ-→tb¯H-, and production associated with neutral Higgs bosons (S=h,H,AS = h, H, AS=h,H,A) or ZZZ bosons.

A key finding of our research is the significant enhancement in production cross-sections at muon colliders. This is primarily attributed to the exchange of neutral Higgs bosons and unique sss-channel and ttt-channel diagrams, which leverage the significantly larger Yukawa couplings of the muon (mμ/me≈207m_\mu/m_e \approx 207mμ/me≈207) compared to the electron. Furthermore, we explore how specific symmetries, such as the discrete Z2Z_2Z2 symmetry typically imposed to suppress flavor-changing neutral currents, influence the observed cross sections and branching ratios. We present updated results for various center-of-mass energies ranging from 500 GeV to 3 TeV, evaluating the sensitivity to different parameter spaces of the model. These findings highlight the critical role of muon colliders in exploring the scalar sector of the 2HDM with unprecedented precision and its implications for the underlying symmetries of nature.



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Sciforum-190779: Probing Charged Scalars at Photon Colliders: One-Loop Effects in the Inert Doublet Model

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We present a comprehensive full one-loop analysis of charged scalar pair production in photon–photon collisions, (\gamma\gamma \to H^{\pm}H^{\mp}), within the framework of the Inert Doublet Model (IDM). This process constitutes one of the most promising channels to directly probe the extended scalar sector predicted by the IDM at future high-energy photon colliders. Owing to the clean environment of photon-induced processes, this reaction offers a sensitive enviroment for investigating the properties of inert charged scalars and their interactions.

The calculation includes the complete set of electroweak virtual corrections together with the contributions from real photon emission required to ensure the cancellation of infrared singularities. The numerical analysis is performed consistently under current theoretical and experimental constraints, including perturbative unitarity, vacuum stability, electroweak precision observables, collider limits, and dark matter constraints. Several representative benchmark scenarios are considered in order to explore different regions of the IDM parameter space.

Our results show that radiative corrections play a crucial role and can significantly modify the tree-level predictions. In particular, the magnitude of the corrections is strongly correlated with the charged scalar mass and the trilinear scalar coupling. Depending on the center-of-mass energy and the chosen benchmark scenario, the corrections vary from moderate negative contributions to sizable positive enhancements, reaching values of approximately 60% at high energies and near specific threshold regions.

These findings demonstrate that higher-order effects are indispensable for precise theoretical predictions and for the reliable interpretation of future collider measurements. The process (\gamma\gamma \to H^{\pm}H^{\mp}) therefore emerges as a powerful probe of the IDM and of extended scalar sectors beyond the Standard Model.



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Sciforum-190785: Effective Partonic Entropy Suppression Near the QCD Crossover

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The QCD crossover at vanishing baryochemical potential and high temperature involves a smooth change from hadronic to partonic degrees of freedom. A phenomenological equation of state should therefore describe not only the activation of quark–gluon degrees of freedom, but also the fact that the plasma remains strongly interacting near the crossover.

In the present work, we consider an entropy-based crossover construction in which the total entropy density is written as a smooth combination of a hadronic baseline and an effective QGP contribution. The hadronic and partonic sectors are controlled by complementary crossover weights, while the non-ideal character of the partonic phase is encoded through a temperature-dependent entropy-suppression factor Rs(T). Once the entropy density is specified, the pressure, energy density, interaction measure, specific heat, and squared speed of sound are obtained from thermodynamic identities rather than fitted independently.

The suppression factor separates two physical effects: the gradual activation of partonic degrees of freedom and their delayed approach to an ideal QGP entropy. This construction gives a smooth entropy profile and allows derivative-sensitive observables, such as the speed of sound, to remain thermodynamically consistent. A representative comparison with lattice-QCD thermodynamic trends shows that the main crossover behavior can be captured by the effective release and suppression of degrees of freedom.

We conclude that entropy-level modeling provides a compact and physically transparent way to describe the QCD crossover. In addition, the effective suppression factor Rs(T) offers a simple representation of the non-ideal nature of the QGP near the transition.



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Sciforum-190805: Study of The Thermal Deconfining Phase Transition at Zero Baryochemical Potential for Different Hadronic Phase Configurations

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In this study, we investigate how the composition of the hadronic phase affects the thermally driven Deconfining Phase Transition in Quantum Chromodynamics (QCD) at zero baryochemical potential, from a confined Hadronic Gas (HG) phase to a deconfined Quark Gluon Plasma (QGP) phase. To describe this QCD transition in a finite volume, we work in a phenomenological statistical framework using a model which considers the coexistence of hadronic and QGP phases in a finite volume, together with the MIT bag model. Several configurations of the hadronic phase are examined, including a Hadronic Gas composed exclusively of pions, exclusively of kaons, or of a mixture of both mesons, as well as a configuration in which the Hadronic Gas is formed by nucleons, namely protons and neutrons. The study mainly focuses on the thermal behavior of several physical observables, referred to as response functions, in the vicinity of the transition region as the temperature varies. Through this analysis, we identify characteristic signatures associated with the phase transition and carry out a comparative investigation of the results obtained for the different hadronic phase compositions and their corresponding thermodynamic properties. These comparisons provide further insight into the role of hadronic constituents in determining transition behavior.



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Sciforum-190808: Recent Multi-Boson and Multi-Higgs results from LHC-ATLAS

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This talk will give a quick overview of recent ATLAS SM Multi-Boson measurement and Multi-Higgs measurement results including the highlight of recent inclusive diboson, VBS polarization with 1st evidence of at least one W boson longitudinally polarized, triboson observation, 1st ever Tri-Higgs search in the world, ATLAS+CMS Di-Higgs legacy combination, with emphasis of the deep connection of EWSB and Higgs physics: the structure of Higgs potential leads to the Electroweak Symmetry Breaking and the longitudinally polarized W/Z bosons couplings directly to the Higgs boson. To measure the multi-Higgs production will leads to a direct probe of Higgs self-coupling in trilinear and quartic Higgs coupling vertices effectively profiling th exact Higgs potential in its precision regard. The polarization measurements of W/Z boson in Vector Boson Scattering (VBS) processes offer a unqiue probe of Higgs unitarization scheme in the weak boson scattering topologies. The latest results from ATLAS reach new milestone in measuring such important physics porcesses and open new opportunities to the next stages of unifying electroweak interactions and Higgs mechanisms. The talk will not only show the experimental results, re-emphasize the physical milestones and impactful scientific values, but also examine the insights of beyond Standard Model new physics probes with Higgs and vector bosons.

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Sciforum-190855: Summation of logarithms in an arbitrary local quantum field theory

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The Bogolyubov–Parasyuk theorem yields a set of recurrence relations, independent of the specific model and renormalizability condition, which allows summing counterterms to the considered theory, as well as summing logarithmic quantum corrections in arbitrary order according to the perturbation theory. The renormalizability condition is irrelevant in this method because only the locality condition for the counterterms is used, which always has a local characteristic after subtracting divergent subgraphs in an arbitrary divergent graph.

The resulting equations for the scattering amplitudes are integro-differential equations, while for the effective potential, they are partial differential equations. Although a large set of previously obtained integro-differential equations exists, this report considers an example with an effective potential due to its simplicity and clarity. The simplicity of these equations leads to a simple analysis of these equations, which may be important in specific applications of this method. The talk demonstrates equations summing both leading and subleading counterterms and logarithms to an arbitrary scalar effective potential. Interestingly, the resulting equations are linear, and their limit to the renormalizable model yields different solutions relative to the renormalizable theory in the corresponding order. This discrepancy leads to a special method for summing logarithms in renormalizable theory, which allows for the simple extraction of a complete series of logarithmic quantum corrections.



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Sciforum-190859: The Impact of the Polyakov Effective Potential on the Deconfinement Transition of QCD Matter

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In this study, we analyzethe deconfinement phase transition in quantum chromodynamics (QCD) by combining the hadron resonance gas (HRG) model at low temperatures with the Polyakov-Nambu-Juna-Lacinio (PNJL) model near the deconfinement region considering two flavors of quarks. By using a switching function (SF), a connection between the HRG and quark-gluon plasma (QGP) phases via the SF ensures the continuity of the thermodynamic variables and, consequently, a smooth crossover from the HRG phase to the QGP phase. The PNJL model represents an extension of the Nambu-Juna-Lacinio (NJL) model by incorporating both quark chiral dynamics and the Polyakov loop, thus enabling a more realistic description of the QGP phase and its thermodynamic properties. In this analysis, emphasis is placed on the investigation of several important thermodynamic quantities—such as pressure, energy density, specific heat density, and the square of speed of sound—as functions of temperature at zero chemical potential. We focus on the role of the Polyakov effective potential near the deconfinement region and its impact on the thermodynamic behavior of the strongly interacting matter. The obtained results are compared with the available lattice quantum chromodynamics (LQCD) data, demonstrating that the combined HRG–PNJL framework provides a consistent description of the QCD crossover and reproduces key thermodynamic features around the transition temperature.



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Sciforum-190860: A Switching-Function Approach to Parametrizing the Temperature-Driven Deconfinement Phase Transition, at Zero Chemical Potential, in a Finite Volume

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In this work, we investigate the temperature-driven deconfinement phase transition for a strongly interacting system in a finite volume, at zero chemical potential. The transition is described from a low-temperature hadronic gas phase consisting of pions to a high-temperature quark–gluon plasma phase with 2+1 quark flavors. To model the occurring transition, we use a phenomenological switching-function approach, which continuously interpolates between the hadronic gas and quark–gluon plasma phases around the transition temperature. The pressure of the mixed system is constructed as a weighted combination of the partial pressures of the two phases, while the energy density is obtained through the standard thermodynamic relation involving the entropy density. From these quantities, we determine the squared speed of sound, $cs^2 = \partial P/\partial e$, which provides an important measure of the dynamical response of the medium across the transition region. The model parameters, including the transition temperature, the width of the transition region and the bag constant, are chosen to describe the thermal evolution of the system and to reproduce the smooth crossover behavior observed in finite systems, since finite-size effects are known to smooth out the sharp behavior expected in the thermodynamic limit. The obtained temperature dependence of the studied physical quantities is compared with lattice QCD results for (2+1) quark flavors. The comparison shows good qualitative agreement, and the obtained results indicate that the switching-function model provides a simple and effective framework for parametrizing the deconfinement phase transition in a finite volume and for describing the thermodynamic behavior of strongly interacting matter.



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Sciforum-190865: Shape Evolution and Ground-State Properties of the Nickel Isotopic Chain

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The systematic evolution of nuclear shapes within the nickel (Ni) isotopic chain is investigated using the framework of the relativistic Hartree–Fock (RHF) theory. By employing the density-dependent meson-exchange (DD-ME2) and density-dependent point-coupling (DD-PC1) parameterizations, this study self-consistently maps the structural transitions from the proton-rich to the neutron-rich frontiers. Various ground-state properties across the isotopic chain are evaluated, including total binding energies, charge radii, two-neutron separation energies, and the corresponding two-neutron shell gaps. A robust agreement is observed when comparing the calculated macroscopic observables with available experimental data from the AME2020 atomic mass evaluation, confirming the predictive power of the adopted RHF functionals. The results demonstrate that the ground-state deformation of these Ni isotopes evolves smoothly, correlating closely with the continuous and gradual changes observed in their underlying physical properties. Furthermore, analysis of the potential energy surfaces reveals a pronounced shape coexistence phenomenon in specific even–even isotopes. This behavior is characterized by the simultaneous presence of competing, energetically close prolate and oblate axial minima near the Fermi surface, indicating a complex interplay between driving shell effects and collective degrees of freedom. Ultimately, these self-consistent RHF calculations provide critical insights into the magicity of traditional shell closures and the onset of deformation in medium-mass nuclei, offering a more refined understanding of subshell evolution, pairing correlations, and the microscopic mechanisms that govern shape transitions in neutron-rich systems.



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Sciforum-191021: PHITS–ANN investigation of gamma and neutron shielding performance of recycled glass–cement composites reinforced with metal organic frameworks: Bi_2O_3, and B_4C

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This research investigates the radiation shielding performance of recycled glass–cement composites doped with Bi_2O_3 and B_4C and reinforced with metal–organic frameworks (MOFs) using PHITS Monte Carlo simulations combined with an artificial neural network (ANN) predictive model. The incorporation of MOFs is explored as a novel strategy to enhance both gamma-ray attenuation and neutron moderation, owing to their high surface area, tunable porosity, and ability to host high-Z metal centers. In addition to their shielding contribution, MOFs may improve the structural homogeneity of the composite matrix, offering multifunctional advantages. The interaction of photons and neutrons with the developed composites is evaluated in terms of the linear attenuation coefficient, mass attenuation coefficient, half-value layer, and effective removal cross-section over a wide energy range. The PHITS simulation results indicate that the synergistic combination of Bi_2O_3, B_4C, and MOFs significantly improves radiation shielding efficiency compared to conventional glass–cement matrices. Furthermore, the ANN model is trained on the simulated dataset to predict shielding parameters with high accuracy, reducing computational cost and enabling rapid optimization of material composition. The integration of simulation and machine learning provides an efficient framework for accelerated material design and performance prediction. The results demonstrate that MOF-reinforced recycled glass–cement composites exhibit promising multifunctional properties, making them strong candidates for advanced, sustainable shielding materials in nuclear technology, medical radiation protection, and other radiation-intensive applications.



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Sciforum-191062: Weak hadronic decays of B mesons into a pseudo-scalar (*P*) and axial-vector (*A*) meson

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Weak hadronic decays of bottom mesons have been investigated with a variety of theoretical frameworks. But we are still rather a long way from a model that can describe their occurrence in nature. Despite the theoretical diversity, we employ the topological quark diagram approach; this method provides a powerful tool for model-independent analysis. Axial-vector mesons remain one of the less explored sectors in Particle Physics, largely due to limited availability of experimental data. In this work we analyse axial-vector meson-producing meson decays. We employ a quark diagram scheme to analyse Cabibbo–Kobayashi–Maskawa enhanced and suppressed decay modes. Recent validation of predictions made by this technique motivated us to apply this technique to *p*-wave meson-producing decays. *SU (3)* flavour symmetry is used to relate the amplitudes extracted from various topologies of meson decays. The Group theory enables us to establish relations among amplitudes and branching fractions, based on symmetry groups. The relations where the branching of both the decays involved is known consolidate our method. The established relations allow us to study quark-level processes in these decays and to predict branching fractions of decay modes, which are yet to be observed. Our predictions may provide a useful platform to test theoretical models against the experimental data.



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Sciforum-191301: Large charge and perturbation theory

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In this talk, I will explore the applications of the semiclassical method, specifically focusing on the large charge expansion within the context of the asymptotically safe Litim–Sannino model. This model is significant in theoretical physics as it provides insights into quantum field theories that exhibit asymptotic safety, a concept that suggests the existence of a non-trivial ultraviolet fixed point.

To begin, I will delve into the perturbative aspects of the calculations associated with this model. This involves examining the behavior of the system under small perturbations and how these perturbations influence the overall dynamics and properties of the model. I will discuss the mathematical techniques employed to perform these calculations and highlight key results that emerge from this perturbative analysis.

Following this, I will identify the lowest-lying operator within the framework of the large charge expansion. This operator plays a crucial role in understanding the dynamics of the theory, particularly in relation to its symmetry properties and its implications for physical observables. I will also investigate the analytic structure of the large charge expansion itself, shedding light on how this structure informs our understanding of the model's behavior at different energy scales.

Furthermore, I will apply these findings to various proposed dualities at the critical point of the Litim–Sannino model. These dualities are essential for establishing connections between seemingly disparate physical theories and can provide deeper insights into their underlying structures. I will discuss how the results obtained from the semiclassical method and large charge expansion can be leveraged to test and refine these duality proposals.

Overall, this talk aims to provide a comprehensive overview of how semiclassical techniques and large charge expansions can enhance our understanding of the asymptotically safe Litim–Sannino model, with implications for broader areas in quantum field theory and theoretical physics.



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Sciforum-191461: Compact Representation of Particle-Collision Events for Physics-Informed Machine Learning

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We introduce a compact, physics-driven event representation, RMM-C46, designed to compress the high-dimensional rapidity mass matrix (RMM) into a low-dimensional, interpretable feature set suitable for physics-informed machine learning (ML) and quantum computing applications. The full RMM encodes detailed pairwise correlations among jets, b-jets, leptons, photons, and missing transverse energy but contains more than a thousand values per event, making it computationally heavy for large-scale training and incompatible with current low-qubit quantum devices. The proposed RMM-C46 input space for ML preserves the physical block structure of the RMM through aggregated invariant mass, rapidity difference, and transverse energy components, reducing the size of the original RMM by over an order of magnitude while maintaining interpretability. Applied to simulated proton-proton collisions at centre-of-mass energy of 13.6 TeV, these representations match or exceed the discriminative performance of the full RMM in both supervised and unsupervised ML tasks. Their compactness, stability, and physics transparency also make them naturally compatible with near-term quantum machine learning architectures. RMM-C46 provides a scalable, efficient, and quantum-ready alternative to the full RMM for next-generation collider physics analyses, enabling faster training, reduced memory consumption, and improved portability across diverse computational platforms. The framework also offers a promising pathway toward integrating interpretable collider observables with emerging hybrid classical-quantum analysis strategies.



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Sciforum-192265: String-Inspired Foam and Lorentz Violation: Reconciling Light-Speed Variation with Air Shower Observations

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Recent studies on the time-of-flight of gamma-ray burst (GRB) photons suggest an energy-dependent light-speed variation in vacuo, signaling a violation of Lorentz symmetry (LV) at the Planck scale. While generic (or field-theoretic) LV models with subluminal photons often lead to drastic, unobserved modifications to the electromagnetic (EM) shower development of cosmic photons in the Earth's atmosphere, we explore a string/D-brane-inspired model for quantum space–time foam that could reconcile these discrepancies and which may further provide an explanation for the persistent "muon puzzle" in ultrahigh-energy cosmic ray physics. By analyzing the formation length for pair creation—the most probable channel of a γ-ray's first reaction in air—we show that photon-initiated cascades can remain largely intact in our model, thereby avoiding stringent shower formation constraints while accommodating the subluminal delays of GRB photons. The inclusion of stochastic energy fluctuations inherent in our foam plays a key role in offsetting the shower anomaly effects as predicted by other scenarios.

Furthermore, we extend this mechanism to the case of extensive air showers (EASs) induced by primary hadrons. The analysis suggests that space–time foam effects may mildly suppress secondary EM subshowers, resulting in a reduction in the detected electron content, which implies a systematic underestimation in the reconstruction of the primary particle's energy under Lorentz invariance. Standard simulations of EAS obviously underestimate the actual muon densities in the presence of foam due to this energy bias, thereby offering a novel quantum-gravitational interpretation for the apparent muon "excess" reported by major observatories such as the Pierre Auger collaboration. Our results not only provide a unified way to understand the new physics behind the light-speed variation effect of cosmic photons from a string-theoretic point of view but can also, hopefully, shed light on the fast-expanding field of quantum-gravity phenomenology, deepening our understanding of space–time.



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Sciforum-193676: Symmetry-Driven Optical Response of GaAs: A First-Principles Density Functional Theory Investigation

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Introduction: Gallium arsenide (GaAs) is a direct-bandgap III-V semiconductor of exceptional technological importance, widely employed in optoelectronic devices, solar cells, and high-frequency electronics. Its zinc-blende crystal structure, governed by the Td point group symmetry, plays a fundamental role in determining its electronic and optical characteristics. Understanding how this underlying symmetry governs the optical response of GaAs is critical for the rational design of next-generation photonic and electronic materials.

Methods: In this work, we present a systematic first-principles investigation of the optical properties of GaAs using density functional theory (DFT) within the generalized gradient approximation (GGA) as implemented in the WIEN2k code. The electronic band structure and density of states are computed to characterize the symmetry of electronic states near the Fermi level. The optical response is evaluated through the complex dielectric function using the random phase approximation (RPA), from which the absorption coefficient, reflectivity, refractive index, and optical conductivity are derived.

Results: Our calculations reproduce the direct bandgap of GaAs at the Γ point, consistent with its Td symmetry-imposed band topology. The imaginary part of the dielectric function reveals characteristic optical transitions between valence and conduction bands, with prominent peaks in the visible and near-ultraviolet regions. The computed refractive index and reflectivity spectra are in good agreement with available experimental data.

Conclusions: This study demonstrates that crystal symmetry is a primary determinant of the optical behavior of GaAs. The DFT framework employed here provides a reliable and computationally efficient route for predicting optical properties of semiconductors, offering valuable guidance for the development of symmetry-engineered optoelectronic materials.



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Sciforum-194144: Zero-Equation: Unified Criterion for Real-Virtual Conjugate Symmetry and Necessary and Sufficient Condition for Symmetry Breaking

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In conventional physics, Noether's theorem establishes a one-to-one correspondence between continuous symmetries and conservation laws, yet it fails to interpret the breaking of discrete symmetries such as spacetime inversion and matter inversion. Experimental observations including parity non-conservation in weak interactions and CP violation in kaon decays have long lacked a unified criterion derived from first principles, remaining a critical unsolved puzzle within the Standard Model of particle physics.Based on the real-virtual dual-spacetime ontology of discrete spacetime theory, this paper puts forward the Zero-Equation centered on the rule that the aggregate physical quantity over real and virtual domains equals zero, extending symmetry definition from a sole real realm to real-virtual conjugate symmetry. We rigorously prove that breakdown of the Zero-Equation serves as the necessary and sufficient condition for all discrete symmetry breaking; spacetime and matter inversion symmetries are preserved if and only if $A_{real}+iA_{virtual}=0$.We further redefine conjugate transformations for discrete symmetries and formulate a generalized conservation law. The framework uniformly accounts for all observed symmetry-breaking effects in the Standard Model, attributing parity non-conservation and CP violation to unbalanced physical quantities across real and virtual domains. Complementing Noether's theorem, it separately governs conservation rules for discrete and continuous symmetries, offering an innovative mathematical-physical viewpoint toward unifying fundamental interactions.



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Sciforum-195625: Exotic and Rare Hadrons Across Flavor Sectors: Fragmentation, Symmetries, and Collider Phenomenology

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Exotic and rare hadrons, including tetraquarks, pentaquarks, and multiply heavy baryons, provide a unique laboratory for exploring the symmetries and dynamical mechanisms underlying hadron formation in QCD. Recent observations at the LHC, including the discovery of a family of all-charm tetraquark candidates and the establishment of doubly charmed baryons, have renewed interest in understanding the production mechanisms of multiquark and heavy-flavor states across different flavor sectors. Their production probes the interplay between perturbative and nonperturbative dynamics, offering new insights into the emergence of complex bound states beyond the conventional quark model. We present a unified fragmentation-based framework for the description of heavy and exotic hadron production, developed within the Heavy-Flavor Non-Relativistic evolution (HF-NRevo) scheme. This approach enables the consistent construction and evolution of fragmentation functions across energy scales while incorporating a systematic treatment of theoretical uncertainties. The framework reveals common structures and symmetry patterns linking quarkonia, tetraquarks, pentaquarks, and triply heavy baryons, thereby providing a coherent description of a broad class of hadronic systems. Publicly available fragmentation-function sets implemented in LHAPDF are discussed together with representative phenomenological applications at present and future colliders. Particular attention is devoted to the role of intrinsic heavy-quark components and to their impact on selected production channels. These studies establish a predictive and uncertainty-aware framework for heavy-flavor and multiquark phenomenology, highlighting how symmetry principles can guide the exploration of hadron structure, multiquark dynamics, and the production of rare states in high-energy collisions.



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Sciforum-196230: Improved Null Energy Condition

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Generalized entropy, defined as the sum of the area of a Cauchy-dividing surface in Planck units and the von Neumann entropy of the fields on one side of the surface, captures a deep connection between entanglement and geometry in semiclassical gravity. It is expected to satisfy fundamental thermodynamical conditions, notably Quantum Focusing Conjecture (QFC), which within certain limits reduces to quantum field theory inequalities. When we consider the limit in which the classical expansion is zero, it leads to the quantum null energy condition (QNEC). The QNEC is the quantum version of the null energy condition, and it has been proven in many cases.

Another limit of QFC, obtained through restriction to quantum extremal surfaces, leads to the Improved Quantum Null Energy Condition (INEC). The INEC is a statement relating the local energy density and the shape derivative of von Neumann entropy in Minkowski space-time. It is a stronger condition than the quantum null energy condition and includes the possibility of expanding or contracting geodesics. Using the properties of relative entropy and a modular Hamiltonian associated with null deformation of the sphere, we show that the INEC holds under an additional assumption relating the stress-energy tensor to the relative entropy. We show that the INEC holds at leading order in aperturbation of the thermal state on a cylinder, and reduce the general case to a monotonicitycondition on the cap entropy.



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Sciforum-198958: Chiral symmetry and scale invariance in the equation of state of strongly interacting matter

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We present an equation of state for strongly interacting matter aimed at exploring its thermodynamic behavior over a wide range of temperatures and baryon densities. Going beyond the standard mean-field approximation, the equation of state is derived from an effective Lagrangian of nucleons and mesons (σ, π, ω, ρ), with thermal field fluctuations systematically incorporated. A key feature of the framework is its consistent treatment of fundamental symmetries: it accounts for explicitly broken chiral symmetry and spontaneously broken scale invariance, with the latter governed by the dynamics of a dilaton field. After validating the role of thermal field fluctuations against lattice QCD results in the pure-gauge $SU(3)_c$ sector, we extend the analysis to the hadronic sector at finite temperature and baryon chemical potential. The resulting QCD phase diagram exhibits chiral symmetry restoration, appearing as a crossover at high temperature and evolving into a first-order phase transition at high baryon density. At the same time, the dilaton field induces a robust first-order transition associated with the restoration of scale symmetry. Finally, we implement this microphysically motivated equation of state to model compact astrophysical objects, evaluating the macroscopic structure, mass–radius relations, tidal deformabilities, and internal thermodynamics of both cold neutron stars and hot proto-neutron stars.



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Sciforum-199084: Investigating Groomed Jet Mass Spectra through Soft Drop Techniques in Proton–Proton Interactions

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The measurement of jet mass distributions in proton–proton collisions provides a powerful probe of Quantum Chromodynamics (QCD) dynamics and the internal structure of hadronic jets. However, the jet mass observable is highly sensitive to soft wide-angle radiation, underlying-event activity, and non-perturbative effects, which complicate precise theoretical predictions. In this work, we investigate the application of the Soft Drop grooming algorithm to the calculation of the resummed jet mass distribution in proton–proton processes. Soft Drop systematically removes soft, large-angle emissions from reconstructed jets according to a momentum-sharing criterion characterized by the grooming parameters (z_{cut}) and (β), thereby reducing contamination from non-global and non-perturbative contributions. Using factorization techniques within perturbative QCD and effective field theory methods, the groomed jet mass spectrum is decomposed into hard, collinear, and soft components, enabling the resummation of large logarithms of the dimensionless jet mass variable $(\rho = m_J^2/(p_T^2R^2))$ to all orders in the strong coupling. The resulting predictions achieve improved perturbative convergence and reduced theoretical uncertainties compared with ungroomed jet mass calculations. We present the structure of the resummed cross-section, discuss the role of Soft Drop in modifying the soft radiation phase space, and highlight its impact on logarithmic accuracy and sensitivity to hadronization effects. The study demonstrates that Soft Drop-groomed jet mass observables provide a theoretically robust framework for precision QCD studies and for comparisons with experimental measurements at high-energy proton–proton colliders such as the CERN Large Hadron Collider.



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Sciforum-199412: Black Hole Entropy Beyond the Wald Term in Nonminimally Coupled Gravity: A Covariant Phase Space Decomposition

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We study the entropy of static, spherically symmetric black holes in diffeomorphism-invariant theories with nonminimal matter–curvature couplings, using the covariant phase space formalism. For regular bifurcate Killing horizons, the Iyer–Wald construction gives the standard Wald entropy. If a matter field cannot be smoothly extended to the regular bifurcation surface, however, the entropy-sector horizon surface charge variation can contain finite contributions that are not included in the Wald entropy density. In the representative obtained by directly varying the action, and after ordinary non-gravitational boundary terms have been separated into the work sector, we decompose the entropy entering the first law of black hole thermodynamics as \(\SH=\SW+\Sone+\DeltaS\). Here, \(\SW\) is the Wald entropy, \(\Sone\) is the non-Wald part of the entropy-sector Noether charge, and \(\DeltaS\) is the remaining integrable part of the entropy-sector horizon surface charge variation. Applying this criterion to Kalb–Ramond, bumblebee, and extended Gauss–Bonnet black holes, we find that the regular Kalb–Ramond branch has \(\SH=\SW\), the bumblebee branches yield either \(\Sone=0\) with \(\DeltaS\neq0\) or a cancellation between \(\Sone\) and \(\DeltaS\), and the Weyl-vector extended Gauss–Bonnet examples require both corrections. This provides a direct test of whether the Wald density is sufficient or whether the full horizon surface charge variation is required.



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Sciforum-200362: Entanglement between field modes mediated by a fluctuating conducting wall in a two-cavity system

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We consider a system of two cavities separated by a conducting wall free to move; two massless one-dimensional scalar fields are defined in each sub-cavity. The movable wall is bound to its equilibrium position by a harmonic potential and its mechanical degrees of freedom are described quantum mechanically. The presence of the movable wall, and its quantum position fluctuations, yields an effective interaction between the field modes in each sub-cavity and between the wall and the two scalar fields that is described by a simple generalization of the Law effective Hamiltonian. This Hamiltonian is linear in the wall operators and quadratic in the field operators of each sub-cavity.

We first evaluate, up to the second order in the interaction, the system interacting ground state and the reduced density operator for the field quantities in the two sub-cavities. In order to quantify the entanglement between field observables in the two sub-cavities, we then evaluate the negativity associated with two field modes, one in each sub-cavity, as well as the multimode negativity. We find that the motion-induced interaction generates entanglement between field degrees of freedom in the two sub-cavities, an effect that is absent when all the three walls are fixed in space. We also evaluate the magnitude of this entanglement for values of the system parameters (the mass and oscillation frequency of the central movable wall, and its distances from the two external fixed walls) that are accessible in present and foreseeable experimental implementations.

Finally, we address the problem of renormalization of the movable wall mass due to its interaction with the two scalar fields at both sides of it.



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Sciforum-200385: Quark Families represent the discrete symmetry groups of the 4-D Regular Polytopes

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We propose that the quark families of the Standard Model (SM) of particle physics represent the 4-D regular polytope discrete symmetries, the 5-cell = (333), 16-cell = (334), 24-cell = (343), and the 600-cell = (335), a unique group for each family. Earlier we established that the 3 lepton families represent the 3 discrete symmetry binary subgroups of SU(2), i.e., 2T = (332), 2O = (342), and 2I = (352), for the 3-D regular polyhedrons because we could calculate their correct 3 mixing angles relating mass states to flavor states without needing a free parameter. By assigning these 4 discrete symmetry groups to identify 4 quark families in 4-D, we can predict the known mixing angles for the known 3x3 CKM submatrix of a 4x4 mixing matrix. If the 3 color charges for each quark are identified as the 3 pairs of rotation planes in 4-D, exact color symmetry is explained. Then the 8 gluons for the color interaction would represent 8 specific 4x4 matrices in agreement with the SM. And quarks are defined specifically in 4-D, so quarks cannot exist in 3-D, and quark confinement follows directly.

But quantum chromodynamics (QCD) dictates a self-contained world consisting of only quarks and gluons, so why do leptons exist? Because only the first quark family with its (333) discrete symmetry has the eternal K5 graph. All the other quark family discrete symmetries represent 4-D graphs that must decay according to Kuratowski's theorem in graph theory. Therefore, because gluons can only change the color charge and cannot mediate quark decays, the weak interaction bosons and leptons are required. In conclusion, when leptons and quarks represent our assigned discrete symmetries, their physical propertes can be related to specific geometrical properties.



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Sciforum-200956: Description of the prolate nuclei within deformation-dependent mass and energy-dependent Davidson potential formalisms

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In this work, we generalize our earlier formalism, which couples an energy-dependent Davidson potential with a deformation-dependent mass term, from the γ\gammaγ-unstable regime to axially symmetric, prolate-deformed nuclei, treated within the framework of the Bohr Hamiltonian. The approach is tested on a set of 62 nuclei, spanning 13 actinides and 49 rare earth isotopes. Four free parameters enter the description and are determined by a least-squares fit to measured energy spectra. We focus in particular on the N=90N = 90N=90 isotones ^{150}Nd, ^{152}Sm, ^{154}Gd, and ^{156}Dy, which are widely regarded as the prime empirical realizations of the X(5) critical point symmetry. The calculations yield a marked improvement relative to earlier treatments, most notably by correcting the well-known overestimation of β\betaβ-band-level spacings that affects the standard Davidson potential. Examining the resulting effective potentials, we identify clear fingerprints of criticality: the wells associated with the $0^+_{\beta}statesbecomedeeper,whiletheminimaofthegroundand states become deeper, while the minima of the ground and statesbecomedeeper,whiletheminimaofthegroundand\gamma$ band-head states grow more widely separated. The model further reproduces $B(E2)$ transition ratios reasonably well, albeit with some overestimation of interband transition strengths. Overall, our findings indicate that jointly incorporating energy dependence and a deformation-dependent mass provides a richer and more consistent description of collective nuclear behavior throughout transitional regions, and points toward broader applications in nuclear structure studies.



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Session 3. Applied Physics

Sciforum-188479: Rhodonea-type Laser Scan Patterns Generated by a Pair of Rotational Risley Prisms

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We report our study on the symmetries of scan patterns generated by laser scanners with a pair of rotational Risley prisms [https://doi.org/10.3390/sym15020336]. Such rhodonea curve types have been approached using the approximation of small angles of optical prisms or by employing rather complicated (although exact) approaches. In contrast to both these research avenues, we developed a graphical method for simulating such patterns with a mechanical design program, CATIA V5R20 (Dassault Systems, Paris, France) [*https://doi.org/10.3390/app11188451*]. This method offers both the advantage of generating exact scan patterns and of simplicity. The presentation starts with the fundamentals of Risley prism-based laser scanners. One of the four possible configurations is chosen and its basic equations are derived. The scan patterns are analyzed with regard to the characteristic parameters of the system: angles of the prisms, refractive indexes, rotational velocities, and the two Marshall parameters (i.e., k, ratios of the prism angles, and M, ratio of the rotational velocities). The symmetries of the scan patterns are studied and repetitive parts are identified (i.e., *structures of symmetry*). We show how such structures simplify the process of generating the scan patterns, thereby shortening the simulation time. Rules of thumb are extracted for generating patterns that may be appropriate for certain applications.



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Sciforum-189729: Comparative Seismic Performance of Symmetrical Mixed RC–Steel and Reinforced Concrete Buildings

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Reinforced concrete (RC) buildings are among the most common structural systems in seismic regions, and current design codes provide well-established provisions for their analysis and detailing under strong earthquake loading. However, in practical applications, mixed structural configurations are frequently used, in which an RC substructure supports an upper steel superstructure (SS) of similar geometry. Despite their widespread use, mixed RC–steel systems are not explicitly addressed in existing seismic design codes, highlighting the need for further investigation.

In this study, the seismic performance of mixed RC–SS in-plan symmetrical systems is evaluated through nonlinear time-history analyses using a set of strong earthquake records. For comparison purposes, geometrically similar RC frames with symmetric layouts are also examined in the 3D space under the same excitations. The numerical models incorporate material nonlinearity to realistically capture the inelastic response of both reinforced concrete and steel components.

Selected significant response parameters, e.g., interstory drift and peak floor acceleration, are examined to assess the structural performance of the investigated symmetrical models. Response plots of dimensionless parameters are shown to obtain an objective comparison between mixed and conventional RC structures.

The results provide insight into the influence of symmetric mixed construction on seismic response, revealing differences in stiffness, ductility, and overall performance compared with traditional RC systems. The findings contribute to a clearer understanding of the behavior of symmetric mixed RC–steel structures and may support the future development of seismic design provisions that explicitly address such building typologies.



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Sciforum-190668: Symmetry-Reduced Optimal Control of Quadrotor Dynamics via Noether's Theorem

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Modern quadrotor unmanned aerial vehicles (UAVs) rely extensively on numerical trajectory optimization and path-planning algorithms for navigation in complex environments. While these approaches emphasize feasibility, collision avoidance, and control constraints, they often overlook underlying physical symmetries in the governing dynamics, leading to redundant computations. This work introduces a symmetry-aware trajectory optimization framework that employs principles from classical mechanics, specifically Noether's theorem, to exploit the rotational invariance of quadrotor dynamics about the yaw-axis. By identifying yaw as a cyclic coordinate in the Lagrangian formulation, a conserved quantity corresponding to angular momentum about the inertial vertical axis is derived using Noether's theorem and enforced as a constraint. This eliminates two redundant input dimensions in the system's state vector, replacing yaw-dynamics with algebraic expressions that leverage symmetry-conserved constants and omit yaw-heading rotations. This yields a reduced-order model that preserves the behavior of the system. To evaluate the proposed framework, physics-based models are implemented using CasADi, where both full-order and symmetry-reduced models are formulated as nonlinear optimal control problems and then solved using the Acados multiple-shooting OCP solver. CasADi is then used for solution verification against physical constraints. A multi-objective cost function is considered, incorporating energy expenditure, control effort, and total travel time. Comparative analyses are performed to assess solution quality and computational efficiency across both formulations versus existing solutions. Comparative results indicate that the proposed framework achieves improved computational efficiency relative to conventional formulations, operating within memory requirements of approximately 1–2 GB of RAM. The symmetry-reduced model is expected to improve algorithmic speed by reducing system dimensionality and eliminating redundant dynamics. A practical consideration of the proposed framework is the reduction of direct control over heading, making the approach most suitable for scenarios where yaw-tracking is not a primary requirement. Extensions of the framework could incorporate hybrid strategies to resolve this constraint.



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Sciforum-190764: Theoretical study of the electronic and magnetic behavior of Ca-Doped Ni–Zn Ferrites

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IntroductionThe cubic spinel structure gives Ni–Zn ferrites low cost, simple synthesis, and favorable magnetic properties [1]. Ca and Zn substitution in nickel ferrites has attracted considerable attention because of its strong influence on magnetic behavior. Some studies further suggest that low concentrations of substituted ions may induce superparamagnetic features in nanosized ferrites because of reduced magnetic anisotropy [2]. On the other hand, increasing Ca content can decrease saturation magnetization, since non-magnetic Ca^{2+} ions weaken the A–B superexchange interactions within the spinel lattice [3].

MethodsThe characteristics of nickel-ferrite and their influence on Ca doping, together with atomistic modeling and electronic-structure analysis using density functional theory, were explored. The interaction between ions and electrons was described by the projector augmented wave method. To achieve reliable results, a plane-wave basis set with a suitable energy cutoff and a dense Monkhorst–Pack k-point grid was employed for the electronic structure calculations.

DiscussionThe electronic structure calculations reveal that Ni0.32Zn0.58Ca0.10Fe2O4 exhibits semiconducting behavior with a band gap of 1.08 eV. The absence of electronic states is observed at the Fermi level, confirming its non-metallic nature. The relatively narrow band gap is expected to facilitate charge carrier excitation, making this ferrite more favorable than conventional ferrites for electronic applications.

ConclusionThe theoretical results show that Ca-doped Ni–Zn ferrites possess stable semiconducting behavior together with favorable magnetic characteristics. The work illustrates that calcium incorporation is an effective approach for tuning the electronic and magnetic performance of Ni–Zn spinel ferrites for potential multifunctional material applications.

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Sciforum-190766: DFT Investigations on the Thermoelectric and Electronic Properties of $Ni_{0.40}Zn_{0.60}Fe_2O_4$

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IntroductionThe electronic properties of materials are important for understanding charge transport behavior and distinguishing between metals, semiconductors, and insulators through their band-gap characteristics. These properties strongly influence optical, thermodynamic, and thermoelectric performance, making them essential for applications such as transistors, sensors, and solar cells [1-3].Methods We applied a theoretical approach to study the structural and electrical properties of nickel ferrite. Theoretical approaches such as Density Functional Theory using with spin generalized gradient approximation was employed. Discussion The purpose of this review is to provide a brief overview of the Hubbard-U approach and examine how its incorporation into conventional DFT affects the electronic and structural characteristics of the Ni0.40Zn0.60Fe2O4 system, together with its thermoelectric behavior. ConclusionsIn this work, the electronic and thermoelectric properties of Ni0.40Zn0.60Fe2O4 spinel ferrites were investigated using DFT. The electrical transport properties are directly influenced by the energy band structure of the material. We have explored Zn doping in NiFe2O4 materials and the dopant concentration in Ni1-xZnxFe2O4 is varied (x=6%). The calculated electrical conductivity (σ) for the investigated compounds shows different behavior due to the particular charge concentration. In addition, the obtained DOS, electronic structure, and thermoelectric results are consistent with available experimental data.

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Sciforum-190798: A Fused Approach of Lie Group SE2(3) and Hill-Clohessy-Wiltshire Optimization for Relative Navigation of a Group of Small Objects.

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This research presents a performance analysis of a hybrid navigation approach combining Lie group theory and the Hill Clohessy Wiltshire (HCW) equations for managing a swarm of small spatial objects. Our study addresses the challenges of precise relative navigation by leveraging the geometric structure of the $SE_2(3)$ Lie group coupled with the linearized orbital dynamics of the Hill Clohessy Wiltshire model. Using the MATLAB environment, we developed a unified mathematical model that integrates complex rigid transformations and orbital proximity constraints, providing a robust framework for trajectory tracking and collision avoidance between objects. The research methodology in this work was structured around a rigorous three-step approach. First, we formulated the navigation state on $SE_2(3)$ to naturally capture the coupling between rotation and translation of objects. Secondly, we merged this framework with Hill Clohessy Wiltshire optimization to linearize relative perturbations around a reference orbit, thus enabling optimal state estimation. Finally, this merged approach was validated by a high-fidelity simulation involving a multi-object deployment, taking into account sensor measurement errors and environmental perturbations. Our results demonstrate that while the traditional Hill Clohessy Wiltshire model excels in simple proximity calculations, merging it with optimization on $SE_2(3)$ significantly reduces singularity errors and improves navigation convergence during wide angle maneuvers.



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Sciforum-192879: Microstructural Anisotropy Induced by Laser Thermal Gradients in 316L Stainless Steel: Implications for Symmetry and Grain Evolution

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Laser processing introduces highly localized thermal gradients that significantly influence microstructural evolution in metallic materials. Although grain refinement and microstructural modification in laser-processed 316L stainless steel have been widely reported, these phenomena are rarely interpreted from the perspective of symmetry and anisotropy. In this work, the evolution of grain morphology in laser-processed 316L stainless steel is investigated to understand how directional heat flow alters microstructural organization across different regions of the material. Microstructural observations reveal spatial variations in grain size and morphology resulting from non-uniform thermal exposure during laser processing. Regions located closer to the laser-affected zone experience different thermal histories compared with regions farther away, leading to distinct grain evolution characteristics. These variations demonstrate that the imposed thermal gradient creates preferential directions for heat transport and microstructural development, producing anisotropic features within the processed material. **The novelty of this study lies in interpreting laser-induced grain evolution through a symmetry-based framework. Rather than considering microstructural changes solely in terms of grain growth or refinement, the work examines how directional thermal gradients reduce microstructural uniformity and promote anisotropic structural organization. This approach provides a new perspective for understanding the relationship between heat flow, microstructural evolution, and symmetry modification in laser-processed materials.** The findings contribute to a deeper understanding of structure–processing relationships in laser-based manufacturing and highlight the importance of symmetry considerations when designing advanced processing strategies for stainless steel components. The proposed framework may also be extended to other laser-manufactured and additively manufactured metallic systems where thermal-gradient-driven microstructural evolution governs material performance.



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Sciforum-194263: Symplectic Symmetry Preservation in Relativistic Particle Dynamics via Physics-Informed Neural Network Integrators

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The long-term numerical integration of relativistic charged particle dynamics in intense laser-plasma fields poses a fundamental challenge: conventional explicit integrators break the symplectic symmetry of the underlying Hamiltonian structure, leading to systematic energy drift over extended simulation times. This symmetry violation reflects a failure to preserve the phase-space geometry dictated by Liouville's theorem and the canonical Poisson bracket structure of relativistic mechanics.

We present a physics-informed neural network (PINN) framework explicitly designed to enforce symplectic symmetry as a hard architectural constraint. The network is trained to approximate the flow map of the relativistic Hamiltonian system, with the symplectic condition incorporated directly into the loss functional alongside the equations of motion residuals and initial condition penalties. This approach preserves the time-reversal and area-preserving symmetries of Hamiltonian dynamics by construction, in contrast to standard PINN formulations where physical constraints are imposed only softly. The method is validated for relativistic electrons in the fields of plane monochromatic waves and ultrashort pulsed laser beams, using rapidity-spectral Lorentz coordinates as the dynamical variables. Energy conservation errors are compared across classical integrators, standard symplectic methods, and the proposed PINN-symplectic scheme over timescales spanning many oscillation periods. The PINN-symplectic integrator demonstrates bounded energy error growth consistent with near-conservation of the modified Hamiltonian, outperforming both standard PINN and explicit Runge–Kutta approaches. These results demonstrate that embedding continuous symmetry principles into neural network architectures yields physically consistent, long-time-stable simulations of relativistic systems — a finding of direct relevance to laser-driven particle acceleration, spin-polarized beam modeling, and relativistic plasma diagnostics at next-generation facilities.



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Sciforum-198807: Adaptive restoration of wavefront symmetry in turbulent atmospheric optical channels: an applied physics perspective on control algorithms

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Atmospheric turbulence acts as a physical symmetry-breaking agent on propagating optical fields, distorting the spatial phase distribution and destroying the radial symmetry essential for efficient focal-plane energy concentration. In free-space optical links, these symmetry violations manifest as asymmetric beam wander, focal-spot broadening, and degraded coupling into receiving apertures, directly impacting signal fidelity and link stability. From the standpoint of applied physics, adaptive optics provides a closed-loop instrumental solution that forcibly restores the wavefront symmetry by applying spatially distributed phase corrections via a deformable mirror, thereby recovering the coherent focusing condition and maximizing energy throughput.

This experimental study compares two physically distinct control paradigms for symmetry restoration in a 45 mm laser beam focused through 20 μm and 10 μm apertures—conditions that model the high aperture sensitivity of practical optical receivers. The photodetector signal behind the aperture serves as the direct metric of focal-region symmetry, with higher transmitted energy indicating better restoration of the ideal, rotationally symmetric focal spot. A deterministic hill-climbing algorithm sequentially adjusts mirror electrodes based on the measured signal gradient. This approach achieves near-complete recovery of focal symmetry, delivering transmission efficiencies of approximately 94% (20 μm) and 90% (10 μm), albeit requiring about 5,800 control steps and 8 minutes of convergence time. In contrast, a stochastic Bernoulli perturbation algorithm, which probes multiple spatial modes simultaneously, converges to a slightly lower symmetry-restoration level (~90% and 82%, respectively) but within only 500 steps and approximately 10 seconds.

These results reveal a fundamental applied-physics trade-off: deterministic control maximizes structural symmetry recovery under quasi-static turbulence conditions, while stochastic methods offer robust, rapid adaptation for dynamically decorrelating channels. This quantitative characterization provides a practical basis for designing adaptive optical instruments that balance correction fidelity against temporal bandwidth, directly addressing the real-world challenges of turbulence mitigation in optical sensing and communication systems.



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Sciforum-199132: Symmetric vs. asymmetric PON architectures: a comparative analysis of power budgets and reach for next-generation access networks

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The rapid growth of bandwidth-intensive services (4K/8K video, cloud gaming, IoT) demands a fundamental upgrade of the access network segment, commonly known as the "last mile". Among the competing solutions, passive optical networks (PONs) are considered the most scalable and cost-effective. However, the evolution from asymmetric GPON (2.5/1.25 Gbit/s) to symmetric XGS-PON (10/10 Gbit/s) and further to 25G-PON (25/25 Gbit/s) raises a critical question: how does the symmetry of the physical layer affect the reach and power budget?

In this work, we develop a comprehensive analytical model for the optical power budget of PON systems. We calculate the maximum achievable link length as a function of the splitting ratio (1:2 to 1:64), considering all key loss components: fiber attenuation (0.35 dB/km), connector losses (0.5 dB), splicing losses (0.1 dB per splice), and a 3 dB engineering margin. The transmitter power is taken as +5 dBm, and the receiver sensitivity as –28 dBm, yielding an overall power budget of 33 dB. Our results reveal that both GPON and XGS-PON provide sufficient optical power for a standard 20 km reach at a 1:32 split ratio. However, the symmetric XGS-PON architecture demonstrates an 8× higher upstream capacity (10 Gbit/s vs. 1.25 Gbit/s) while maintaining the same physical reach as asymmetric GPON when using identical optical components. The emerging 25G-PON standard is discussed qualitatively as an intermediate step toward multi-gigabit services; its detailed power budget analysis is beyond the scope of this work.

We conclude that symmetric PON solutions, particularly XGS-PON, represent an optimal balance between physical layer performance and future upgrade paths. The preservation of reach and power budget while increasing capacity eightfold makes XGS-PON the preferred choice for operators seeking a sustainable long-term evolution strategy. Our findings provide a quantitative framework for network planning and technology selection in urban and suburban access networks.



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Sciforum-199140: Symmetry breaking in fiber optic connector endface geometry: correction via controlled manual polishing

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The performance of fiber optic networks critically depends on the physical contact between mating connector endfaces. The three-dimensional geometry of the ferrule endface—particularly the radius of curvature (ROC), apex offset, and fiber height—dictates the quality of this contact. Symmetrical geometry ensures uniform pressure distribution and low insertion loss (IL), while asymmetrical defects lead to increased signal attenuation and back-reflection.

This study investigates the effectiveness of secondary manual polishing techniques in restoring the symmetry of the polished surface. Multimode patch cords with FC/PC connectors (OM2 fiber, 50 µm core) were divided into control groups exhibiting either excessive apex offset (mean 304 µm) or surface damage. An interferometer (Data-Pixel DAISI V2) was used to quantify the 3D surface parameters before and after polishing.

A novel combined polishing technique, employing circular motion with a deliberately biased pressure vector, was developed to induce localized abrasive wear. This method acts directly against the vector of the observed apex offset, effectively “re-centering” the spherical surface of the ferrule. The physical mechanism relies on the controlled breakdown of geometric symmetry to re-establish a uniform contact pressure profile.

The combined method demonstrated superior efficiency. In 80% of cases, the median apex offset was reduced from 304 µm to 41 µm. For samples with surface damage, the mean IL decreased by a factor of 4.5 (from 0.18 dB to 0.041 dB). The study reveals that while the technique is highly effective, it cannot correct defects originating from glue or fiber breakage, confirming that such flaws are fundamental limitations set by manufacturing processes and can be identified through 360° mechanical rotation tests.



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Sciforum-200109: Symmetry and Quantum Entanglement in the Rotational Dynamics of a Molecular Dimer under an External Electric Field

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Ultracold polar diatomic molecules coupled via dipole–dipole interaction have been proposed as promising candidates for implementing quantum logic gates and for controlling chemical reactions in ultracold gases. In particular, the interplay between intermolecular interactions and external fields provides a versatile framework for manipulating rotational dynamics and entanglement.

In this work, we investigate the rotational dynamics and entanglement properties of a dimer formed by two identical linear polar molecules coupled via dipole–dipole interaction and subjected to an external electric field. A linearly switched-on electric field is considered, and a general prototype of linear polar molecule is assumed. The molecules are allowed to rotate in three dimensions and are described within the Born–Oppenheimer and rigid-rotor approximations.

We analyze the symmetry properties of the system, which allow us both to simplify the problem and to identify a set of general relations governing the rotational dynamics. We explore the dependence of the rotational motion and entanglement on the switching time of the field and on the spatial configuration of the molecular dimer. Finally, we discuss the coupling mechanisms associated with dipole–dipole and field–molecule interactions, and analyze the time-dependent wave function to provide a physical interpretation of the different dynamical regimes arising from various molecular arrangements.



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Sciforum-200417: Designing High-Performance Broadband Mirrors: A Multiscale Mathematical Framework for Multilayer Thin-Film Optimization

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Thin-film optical coatings, including single or multilayer packs characterized by their refractive indices, absorption coefficients, and thicknesses, allow accurate management of the reflectance, transmittance, and absorptance. This work reports on the design of dielectric thin-film coatings aimed to build high-reflectance mirrors with broadband spectral yield that have the capability of direct relevance to optical instruments such as spectrophotometers, laser resonators and cavities, optical metrology setups, and hyperspectral and multispectral instruments that are essential in optical imaging and astronomical observations. Commonly used metallic reflectors—fabricated from silver, aluminum, or gold—show crucial limitations involving unavoidable optical absorption, exposure to oxidation under adverse environmental circumstances, and wavelength-dependent reflectivity. Most dielectric multilayer reflectors avoid these disadvantages by proposing low absorption, widening the effective lifetime, and better environmental stability. A series of mirror configurations, each containing a substrate with sustaining periodically regular manner of low- and high-refractive-index dielectric layers, is suggested and numerically estimated after mathematical modeling that contains all parameters for effectively describing the proposed design. The calculated spectral reflectance is compared with previously reported experimental data to justify the proposed design methodology. Results display that period alternating, linked with adjacent stack designs, significantly increases the high-reflectance bandwidth while maintaining the low-loss and stability benefits underlying to all-dielectric designs.



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Sciforum-207068: Enhanced Targeted Therapeutics: Investigating DNA Coverage Density Effects on the Structural Dynamics of DNA-Functionalized Gold Nanoparticles for Precision Nanomedicine

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The functionalization of gold nanoparticles (AuNPs) with DNA represents a major advance in nanomedicine, offering new opportunities to improve the specificity, efficiency, and controllability of diagnostic and therapeutic systems. This approach combines the distinctive physicochemical, optical, and surface properties of AuNPs with the molecular recognition capabilities, structural programmability, and biological relevance of DNA. Consequently, DNA-functionalized AuNPs can be engineered as hybrid nanostructures that interact selectively with biological targets and respond to specific molecular environments. In this study, the design, structural organization, and behavior of DNA-modified AuNPs are investigated through a complementary combination of molecular simulations and experimental approaches. Particular attention is devoted to the influence of DNA surface coverage on chain conformation, intermolecular interactions, spatial distribution, and packing efficiency at the nanoparticle interface. The results indicate that variations in DNA grafting density can promote favorable conformational rearrangements and lead to a more organized and stable nucleic acid shell. These changes may affect target accessibility, hybridization efficiency, colloidal stability, and the overall biological performance of the resulting nanoconjugates. By clarifying the relationship between surface composition and DNA organization, this work provides a deeper understanding of the structural principles governing DNA–AuNP assemblies. Such knowledge may facilitate the rational design of nanoscale platforms for targeted gene delivery, controlled gene regulation, molecular diagnostics, biosensing, and precision treatment strategies. Overall, the findings highlight the versatility of DNA-functionalized AuNPs and support their continued development as adaptable tools for personalized medicine, targeted therapeutics, and the diagnosis and treatment of complex diseases.



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Session 4. Astronomy, Astrophysics and Cosmology

Sciforum-157977: AI-Based Image Reconstruction and Physical Parameter Estimation of Black Holes

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Black holes are among the most enigmatic and fascinating objects in the universe, characterized by extreme gravitational fields that prevent any direct observation of their interiors beyond the event horizon. The groundbreaking work of the Event Horizon Telescope (EHT) collaboration led to the first-ever images of black holes, marking a milestone in observational astrophysics. However, these images suffer from limited resolution due to the sparse sampling inherent in very long baseline interferometry (VLBI) techniques. This research proposes an integrated artificial intelligence framework designed to enhance the resolution of black hole images while simultaneously estimating key physical parameters such as mass, spin, and accretion properties.

The methodology leverages state-of-the-art convolutional neural networks (CNNs) and generative adversarial networks (GANs) to perform super-resolution image reconstruction. The models are trained on large datasets of simulated black hole images, which are generated using advanced general relativistic magnetohydrodynamic (GRMHD) simulations that capture the complex dynamics around black holes. Following the image enhancement stage, a regression-based neural network model predicts the black hole's physical parameters directly from the super-resolved images.

This novel approach bridges the gap between sparse observational data and high-fidelity imaging, enabling automated, accurate estimation of astrophysical properties. It represents a significant step forward in black hole research, offering astrophysicists a powerful tool to decode the structure, dynamics, and fundamental properties of these cosmic phenomena.



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Sciforum-164417: Induced Curvature Singularities in Modified Theories of Gravity

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Modified theories of gravity, while offering potential solutions to long-standing cosmic puzzles such as dark energy and the hierarchy problem, can introduce highly complex spacetime structures. This work investigates the emergence of "induced curvature singularities" at finite radii in static, spherically symmetric geometries within these extended theories. Unlike the event horizons of regular black holes, which are merely coordinate singularities, these induced singularities represent genuine curvature divergences where the Kretschmann scalar K tends toward infinity ($K \to \infty$).

These divergences typically stem from non-General Relativity (non-GR) terms that are present in the field equations. In this study, we explore the precise mathematical conditions required for their formation, identifying their physical mechanisms, such as poles in effective mass functions and the appearance of branch points in the metric solutions. Our analysis further explores the profound physical implications of these structures, specifically focusing on geodesic incompleteness for both timelike and null observers, as well as the presence of unbounded tidal forces that would lead to spaghettification.

By distinguishing these true singularities from standard coordinate horizons, we emphasize the crucial role of metric function regularity and the violation of effective energy conditions. This investigation aims to provide a diagnostic tool for testing the viability of modified gravity models in the strong-field regime. Our results suggest that certain modified gravity frameworks may be physically untenable due to the unavoidable formation of these singular surfaces at finite distances from the source.



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Sciforum-179354: Studying Hyperon Puzzle in a Unified SU(3)–SU(6) Baryon–Meson Coupling Scheme

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We investigate the influence of hyperons on the equation of state (EOS) and global properties of neutron stars using the relativistic mean-field (RMF) framework. To address uncertainties in hyperon–meson interactions, we adopt a unified baryon–meson coupling scheme grounded in SU(3) flavour symmetry and partially broken SU(6) spin-flavour symmetry. Within this scheme, vector meson coupling constants are determined through SU(3) symmetry relations and parameterised by a single free parameter, α_v, while scalar couplings are constrained to reproduce empirical hyperon potential depths at nuclear saturation density.

Within this symmetry-guided framework, we construct the EOS for dense matter, incorporating the full baryon octet and leptons under conditions of β-equilibrium and charge neutrality. The resulting EOS is applied to the Tolman–Oppenheimer–Volkoff equations to determine neutron star mass–radius relations. Our findings indicate that the presence of hyperons softens the EOS, in agreement with the established hyperon puzzle, and decreases the maximum mass of neutron stars. The appearance of hyperons, especially the Λ baryon, occurs at densities several times higher than nuclear saturation density and substantially alters the internal composition of massive stars.

The dependence of stellar properties on the coupling parameter α_v is further analysed. Lower values of α_v result in a stiffer equation of state and higher maximum masses, which partially alleviates the hyperon puzzle. The model successfully reproduces current astrophysical constraints, including the existence of two-solar-mass neutron stars and established bounds on tidal deformability.

Overall, the unified SU(3)–SU(6) coupling scheme provides a consistent and less arbitrary framework for describing hyperonic matter in neutron stars, thereby offering deeper insight into the role of strangeness in dense astrophysical environments.



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Sciforum-184194: Gauge-Controlled Breakdown of Kerr Integrability in Weakly Non-Kerr Spacetimes

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The Kerr geometry is distinguished not only by stationarity and axisymmetry, but also by a hidden tensorial structure responsible for the Carter constant and the complete integrability of geodesic motion. This talk presents a perturbative and gauge-controlled framework for studying how these manifest and how hidden symmetries break down in weakly non-Kerr spacetimes. Instead of starting from a particular phenomenological deformation, the analysis is formulated directly at the level of symmetry restoration: whether the stationary and axial Killing vectors, the Killing-Yano tensor, and the associated rank-two Killing tensor can be consistently continued under a small metric perturbation.

The central result is a hierarchy of symmetry obstruction. Manifest symmetry breaking controls the drift of the energy and axial angular momentum, while hidden-sector obstruction diagnoses the possible failure of the Carter-type invariant even when stationarity and axisymmetry remain intact. This distinction is essential for relativistic orbital dynamics, because a perturbation may preserve the usual conserved charges but still destroy the separability structure that organizes Kerr geodesics.

I will discuss how these obstruction tensors can be projected onto relativistic orbits and used to classify weak environmental deviations from Kerr, including stationary–axisymmetric perturbations that isolate the hidden sector. The resulting framework provides a geometrically transparent route toward understanding frequency-map drift, resonant structures, and long-time phase accumulation in extreme-mass-ratio inspirals. It therefore connects black-hole perturbation theory, hidden symmetry, and future precision tests of the Kerr paradigm via gravitational-wave observations.



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Sciforum-188342: Study of anisotropic compact stars in modified gravity with Bose–Einstein dark matter density profile

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This work presents a novel anisotropic compact star model in the framework of modified f(Q) Gravity by incorporating a Bose–Einstein condensate dark matter density profile together with a bag-model-type equation of state. The stellar configuration is developed in a static spherically symmetric spacetime using the teleparallel formalism, where the gravitational interaction is governed by torsion instead of curvature. The interior spacetime is smoothly matched with the exterior Schwarzschild solution through the standard junction conditions, leading to well-behaved metric potentials throughout the stellar interior. The physical properties of the model are analyzed in detail for several realistic compact objects, namely LMC X-4, Her X-1, 4U 1538-52, SAX J1808.4-3658, and Cen X-3. It is observed that the energy density, radial pressure, and tangential pressure attain their maximum values at the stellar core and decrease monotonically toward the surface, ensuring regularity and physical acceptability of the solutions. The anisotropic factor remains positive inside the compact configuration, indicating the presence of a repulsive anisotropic force which enhances the stability and allows the existence of more massive stellar structures. Furthermore, the model satisfies the null and strong energy conditions, confirming the physically viable nature of the matter distribution. The equation-of-state parameters remain within acceptable bounds throughout the stellar interior. Stability analysis through the causality condition and adiabatic index demonstrates that the obtained stellar configurations are stable and free from physical singularities. The derived mass–radius relation and compactness factor also fall within observationally accepted limits, establishing that the proposed Bose–Einstein dark matter-inspired compact star model in modified gravity provides a realistic and stable description of ultra-dense astrophysical objects.



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Sciforum-188548: Gravitational Instantons and Chiral Reduction in Spin(4) Gauge theory of spacetime.

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\textbf{Abstract:}In this talk, the Spin(4) gauge formulation of spacetime with a Cartan Khronon field is presented. As motivation, the relation between Euclidean and Lorentzian descriptions is recalled as a Wick-rotation prescription relating two dynamically equivalent descriptions of the same underlying field configurations, once a time direction is selected by the Cartan Khronon. Within this setting, the Spin(4) theory provides a gauge-theoretic description of spacetime in which the coframe structure emerges from the covariant derivative of the Khronon field.\\

The full theory involves both self-dual and anti-self-dual sectors of the Spin(4) connection. A symmetry-breaking pattern is then discussed, in which the distinction between time and space selects a preferred structure and is accompanied by a chiral reduction of the gauge sector. This transition can be viewed as a passage from a Palatini-like Spin(4) phase to a self-dual (Ashtekar-like) formulation, in which only the self-dual sector governs the gravitational dynamics and reproduces the standard general relativistic behavior.\\

Within this chiral phase, gravitational instantons provide natural test configurations, as their (anti-)self-dual nature confines them to a single sector of the Spin(4) decomposition, allowing one to probe whether this sector alone reproduces the gravitational dynamics. This perspective is illustrated through the example of the Eguchi–Hanson gravitational instanton, which is presented as a first explicit instanton solution in this framework.\\

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Sciforum-189072: Scalar Field Reconstruction and Anisotropic Evolution of Barrow Holographic Dark Energy in Bianchi Type-V Spacetime

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This research investigates the cosmological trajectory of the late-time universe within the framework of modified gravity, specifically incorporating Barrow Holographic Dark Energy (BHDE). By adopting the Hubble horizon as the cosmic infrared cut-off, we derive the modified Friedmann equations influenced by the Barrow exponent). This quantum-gravitational exponent (Δ) effectively accounts for the intricate, fractal-like deformations of the black hole event horizon inspired by quantum gravity principles. To establish a robust theoretical connection with mainstream dark energy frameworks, we perform a formal mathematical correspondence and reconstruction between the BHDE model and standard scalar field descriptions, specifically focusing on quintessence, k-essence, and dilaton field paradigms. Unlike conventional isotropic studies that rely entirely on standard homogeneous models, we explore these evolutionary dynamics within an anisotropic Bianchi Type-V spacetime framework. This approach allows us to critically analyze how early-universe initial anisotropy influences or persists into late-time cosmic acceleration. To rigorously determine the physical viability and stability of the reconstructed scalar potentials, we systematically evaluate the classical Energy Conditions, including the Null, Weak, and Strong Energy Conditions. Furthermore, we utilize Statefinder diagnostic parameters {r, s} to distinctly differentiate this modified holographic model from the standard concordance Lambda Cold Dark Matter (ΛCDM) paradigm. Our final results focus on identifying the stable, physically permissible parameter space for the Barrow exponent(Δ) that guarantees a smooth topological transition from an early decelerated phase to a perpetually accelerating FLRW universe. Ultimately, this comprehensive study provides a rigorous mathematical bridge between modified spacetime geometry, quantum-corrected horizon thermodynamics, and the underlying holographic nature of the dark sector.



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Sciforum-189148: The Evolution of dark energy in a gas of thin closed tubes of a massless scalar field, radially changing their size.

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Observational indications of a possible complex two-phase evolution of dark energy obtained within the framework of DESI DR2 actualize the search for cosmological models capable of realizing the transition through the phantom barrier and, at the same time, capable of eliminating the theoretical problems associated with the violation of the weak energy condition in the phantom regime. In addition to multicomponent scalar field models and modified gravity models, the two-phase evolution of dark energy can also be realized by “simple” (one-component) scalar field models that are based on the standard General Theory of Relativity, provided that these models can simultaneously realize several mechanisms of accelerated expansion. Models with this property include the gas of thin tubes of a massless scalar field (TToMSF). In the simplest version, the homogeneous phase of TToMSF gas is formed by tubes that, on a small time scale, either move without changing their shape and size, or radially increase (radially decrease) their size. In this work, we investigated the influence of the gravitational field of a thin closed tube of a massless scalar field, radially changing its size, on its local environment in the gas. In particular, we showed that locally the accelerated expansion is influenced by such parameters as the shape of a thin tube, its thickness, and the relative position of the tubes in the gas.



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Sciforum-189150: Asymptotics of the gravitational field for a homogeneous phase of a gas of thin tubes of a massless scalar field.

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The problem of the cosmological constant and the fundamental nature of dark energy remain among the key challenges for modern physics. Observed anomalies, in particular, cosmological tensions within the standard LCDM model related to large-scale structure growth rates and the pronounced heterogeneity of the visible Universe, require the development of radically new theoretical approaches. In this work, we research the concept of a gas of thin tubes of a massless scalar field (TToMSF) as a promising alternative within dynamical dark energy and unified dark fluid models. The TToMSF gas possesses an interesting property: its equation of state parameter w depends on the density of the medium. In the rarefied limit, the gas exhibits behavior characteristic of dark energy, whereas compression drives the system into a stiff matter state. Such dynamics make it possible to realize local equations of state that can mitigate existing cosmological contradictions. Furthermore, local gravitational interaction within the TToMSF gas can generate particles with an effective non-zero rest mass, allowing the TToMSF gas to be classified as a unified dark fluid model. In this study, we analyzed the temporal evolution of the geometry of the gravitational field generated by the homogeneous phase of the TToMSF gas. In this work, the asymptotics of the gravitational field is understood as its behavior under long-term time averaging. We have shown that as a result of such time averaging, the metric tensor of the homogeneous TToMSF gas phase tends toward flat Minkowski spacetime. The Minkowski solution as the time asymptotics confirms the dynamic stability of the homogeneous TToMSF gas phase and forms a fundamental basis for subsequent analysis of local perturbations.



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Sciforum-189165: Dark matter across scales: Ultralight fields, Bose-star signatures, massive compact composites, and giant resonance in dark matter–matter oscillations

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Axion- and dilaton-like fields induce stochastic variations in atomic transition frequencies. We develop a statistical framework for clock searches, demonstrating that higher-order moments of frequency fluctuations (variance, skewness, kurtosis, correlators) encode distinctive patterns generated by ultralight dark matter. This approach significantly extends the discovery potential of clock experiments.

Astrophysical implications of self-gravitating bosonic configurations (“Bose stars”) are also analysed. Their interaction with interstellar gas can trigger localised chemical enrichment and seed high-metallicity molecular clouds. These processes offer novel observational pathways for probing bosonic dark matter on macroscopic scales.

A recently identified absorption–ionisation mechanism further broadens the accessible mass range for direct searches in underground laboratories. Dark matter–nucleus scattering, previously assumed incapable of producing detectable signals at low energies, can ionise atoms even for particle masses below 100 keV. This opens a new experimental window long considered unreachable.

Matter and dark matter in galaxies represent two main components linked by the gravitational interaction. Collisions of galaxies may create an offset between the centers of mass of these components. We focus on possible relative oscillations of the matter in a dark matter halo. This two-fluid model is somewhat similar to the "giant dipole resonances" in nuclei. Such oscillations should manifest themselves as anomalies in the velocities of stars in the galaxy, such as density waves and runaway stars, which have orbit periods in resonance with oscillations.

Models of modified gravity with compact extra dimensions naturally generate strongly singular potentials at short distances. These may stabilise exceptionally dense composite objects formed from quarks or antiquarks, providing another class of dark matter candidates with rich phenomenology and potential astrophysical signatures.

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Sciforum-189414: Nanohertz Gravitational-Wave Signatures from the Supermassive Black Hole Binary Candidate in 3C 454.3

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We investigate the orbital dynamics and gravitational-wave (GW) properties of the blazar 3C 454.3 as a candidate supermassive black hole binary (SMBHB). Nearly six decades of multi-frequency radio monitoring reveal a persistent quasi-periodic modulation with a period of 2.3 yr in the observer frame. Interpreting this modulation as a Doppler-boosted signature of orbital motion or jet precession, and applying a relativistic transformation with Lorentz factor $\gamma = 5$, we derive an intrinsic orbital period of ≈31 yr in the source frame.

Using Keplerian and relativistic precession models, we estimate component masses of $M \approx 2.4 \times 10^{10} M_\odot$ and $m \approx 6.8 \times 10^{9} M_\odot$, corresponding to an orbital separation of $a \approx 4.6 \times 10^{17}$ cm (~0.15 pc). The system therefore lies firmly in the nanohertz GW regime accessible to pulsar timing arrays (PTAs). The predicted GW frequency is $f_{GW,obs} \approx 1.1 \times 10^{-9}$ Hz, while the characteristic strain amplitude reaches $h_0 \approx (7–9) \times 10^{-16}$, making 3C 454.3 a promising PTA-band source.

The GW-driven inspiral timescale is $t_{GW} \approx 5.7 \times 10^{4}$ yr. The expected orbital period decay is extremely small, $|\dot{P}| \approx 2 \times 10^{-4}$ yr yr^{-1}, corresponding to only a few days of cumulative drift over 60 yr. Such a variation is masked by the intrinsic variability of Doppler-boosted jet emission and cannot be detected in current electromagnetic data.

We show that the extreme chirp mass and low GW frequency of 3C 454.3 make it a potentially significant contributor to the anisotropy of the stochastic nanohertz GW background. The source lies far below the sensitivity range of LISA and LGWA, confirming that it is a PTA-only target throughout its inspiral evolution. Overall, 3C 454.3 emerges as one of the most promising high-mass SMBHB candidates for future PTA observations.



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Sciforum-190508: Axion propagation in a gravitational wave background

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Axion-like particles provide a natural bridge between hidden-sector physics, dark matter, and symmetry principles beyond the Standard Model. They arise in theories with spontaneously broken global symmetries and interact with electromagnetism through the pseudoscalar invariant $aF_{\mu\nu}\tilde F^{\mu\nu}$. This coupling links axion dynamics to the electromagnetic parity-odd invariant $E\cdot B$, making axion electrodynamics a useful framework for studying how symmetry structures are modified by external fields. We investigate how weak gravitational wave backgrounds affect axion--photon dynamics in compact-object environments without introducing new fundamental interactions.

Starting from the covariant axion-photon effective action, we derive the coupled evolution equations governing the axion and electromagnetic sectors in the presence of an external perturbative background. We then specialize the system to the case of a weak excitation propagating along the z direction and analyze the resulting modifications of the axion-photon interaction terms. The equations are expanded to leading order in the perturbation amplitude, and a reduced one-dimensional model is constructed to isolate the role of the pseudoscalar source term E_iB^i in the coupled dynamics.

The external perturbation induces time-dependent corrections to the axion evolution operator and generates modifications of the electromagnetic sector. For a strictly one-dimensional axion configuration, $a=a(t,z)$, the leading-order correction to the axion propagation operator cancels identically. Nevertheless, axion dynamics remain sensitive to the perturbed environment through the electromagnetic fields, whose evolution modulates the pseudoscalar interaction term E_iB^i.

These results show that axion--photon interactions can develop coherent, time-dependent features driven by external perturbations through the standard pseudoscalar coupling alone. The analysis provides a framework for investigating axion dynamics in realistic high-energy and astrophysical environments, and clarifies how anisotropic perturbations can influence axion propagation, axion-photon conversion, and the generation of electromagnetic signals.



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Sciforum-190628: Observational constraints on a cosmological model in f(R,G,T) gravity

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The current acceleration of the universe is one of the most important issues to explain in cosmology today. The most widely accepted model is the ΛCDM model in general relativity, but this model also has several unresolved issues. Thus, a lot of research is expended in looking at models in modified gravity theories that can solve some of these unresolved problems. In this work, we examine a Friedmann–Lemaitre–Robertson–Walker cosmological model in f(R,G, T) gravity, where R is the Ricci scalar, G is the Gauss–Bonnet invariant, and T is the trace of the energy momentum tensor. We assume a particular form of the deceleration parameter and apply observational constraints on the model. The Cosmic Chronometer and Pantheon datasets are used in a Markov Chain Monte Carlo (MCMC) study to ascertain the model's parameters. A viable model that satisfies observational limitations and shows a shift from deceleration to acceleration is produced. Analysis and illustrations are provided for several cosmographic characteristics, including the deceleration, jerk, and state-finder parameters. A comparison with the conventional ΛCDM model is made. Lastly, the equation of state and energy conditions are covered. We learn more about the dynamics and evolution of the universe by examining this model.



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Sciforum-190646: Resolving the Hubble Tension via Time Reparametrization Invariant Scaling Symmetry

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Introduction: The persistent discrepancy between local measurements of the Hubble constant (H_0) and those inferred from the Cosmic Microwave Background (CMB)—known as the Hubble Tension—remains a fundamental crisis in modern cosmology. While many attempts to resolve this issue rely on ad hoc models involving exotic fields or novel particles, a powerful guiding principle is needed to narrow the boundless space of possibilities. Recently, Courbin and Maeder (2026) demonstrated that the Scale Invariant Vacuum (SIV) paradigm offers a natural solution.

Methods: This work utilizes Time Reparametrization Invariant Scaling Symmetry (RISS), a framework that is mathematically equivalent, but physically distinct, to SIV. Instead of introducing new physical entities, we consider a time-dependent conformal unit rescaling, (dt, dr) →λ(t)(dt, dr), which maintains a constant speed of light. This transformation is alternatively considered as a Weyl scaling of the scale factor and lapse function, (a, N) → λ(t)(a, N), yielding a self-consistent augmentation of the standard Friedmann–Lemaître–Robertson–Walker (FLRW) equations.

Results: We show that the time-dependent unit rescaling almost exactly matches the modern treatment of CMB physics. The resulting augmented cosmological equations reveal that the observed H_0 tension is an artifact of treating time units as static across cosmic epochs. Because the global gravitational potentials differ significantly between the decoupling era and the current epoch, the effective unit of time has evolved.

Conclusions: Consequently, the H_0 tension is naturally resolved not by changing the intrinsic expansion rate of the universe, but by properly accounting for the conversion of time units between the early and late universe. RISS provides an elegant, symmetry-based solution to a premier cosmological puzzle without invoking dark sectors or exotic physics.



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Sciforum-190802: Modified Gravity Derived from the Thermodynamics of a U(1) Gauge Symmetry

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Jacobson (1995) showed that Einstein's field equations arise as a thermodynamic equation of state from the Clausius relation on local causal horizons. Building on this, we present a framework, Timeflow Gravity, formulating gravitation as an emergent thermodynamic phenomenon driven by the wave mechanics of a U(1) spacetime medium. Spacetime is modeled as a medium carrying a U(1) phase field φ, with the flow of time identified as the field's phase evolution.

Imposing energy and information conservation partitions the field into a coherent, structure-forming sector and an incoherent thermal sector. A variance-sum decomposition of the phase gradient yields a wave-mechanical efficiency function μ governed by the coherence angle θ between matter and the vacuum. Applying the Clausius relation to the partitioned energy flux, we modify Jacobson's derivation to obtain a state-dependent field equation, in which macroscopic curvature is sourced by the ensemble-averaged squared phase gradient.

The field equation decomposes into three components: classical matter, a repulsive cosmological background, and a wave-mechanical interference cross-term. In coherent, rotationally supported galaxies the framework reproduces the universal Radial Acceleration Relation and MOND-like dynamics without dark matter, and it explains the apparent absence of dark matter in dispersion-dominated systems. We present a strictly falsifiable prediction: the intrinsic scatter of the Radial Acceleration Relation correlates directly with the kinematic disorder of the baryonic source, parameterized by $\cos\theta$—a signature distinguishing this framework from both ΛCDM and standard MOND, testable with existing SPARC data.



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Sciforum-190806: The Nuclear Physics of Exotic Compact Objects

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The term Exotic Compact Objects (ECOs) refers to astrophysical bodies whose internal structure and composition differ fundamentally from those of standard compact objects predicted within conventional astrophysics, such as white dwarfs, neutron stars, and black holes. More generally, ECOs are hypothetical ultra-dense astrophysical configurations that may exhibit bulk properties similar to those of black holes and neutron stars, while being governed by fundamentally different underlying physics and matter composition. Examples of such objects include strange stars, hybrids stars, quarkyonic stars, boson stars, gravastars, wormholes, fuzzballs, dark-matter stars, and other speculative compact configurations proposed in extensions of standard astrophysical and gravitational theories. In the present talk, we shall focus on the class of compact objects that includes quark stars and hybrid stars, as well as configurations whose composition consists of a combination of baryonic and dark-matter components. We shall demonstrate under which assumptions, and possibly conjectures, recent astrophysical observations of compact objects can be interpreted. In particular, we will discuss the source XTE J1814-338, with a mass of $M=1.2_{-0.05}^{+0.05}\ M_{\odot}$ and a radius of $R=7_{-0.4}^{+0.4}\ {\rm Km}$. We will also examine the properties of HESS J1731-347, which has a mass of $M=0.77_{-0.17}^{+0.20}\ M_{\odot}$ and a radius of $R=10.4_{-0.78}^{+0.86} \ {\rm Km}$. In addition, we will consider the recent observations of XTE J1810-189, which suggest extreme predictions for both the mass and radius of the compact object. We shall demonstrate how the combination of recent astrophysical observations, together with the information that can be extracted from the detection of gravitational waves, may shed light on the physics of these objects and substantially advance our understanding of the physics of ultra-dense matter.



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Sciforum-190811: Modeling the Transition from Hadron Gas to Deconfined Quark Matter via the Albright–Kapusta–Young Switch Function

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In the present study, the transition from hadronic matter to deconfined quark matter is investigated using the switching function proposed by (Albright et al., 2014). This function has an exponential form, S(μ) = exp[-(μ0/μ)r], and enables smooth interpolation between the two phases. The analysis is performed at zero temperature (T = 0), thereby neglecting superconducting effects. Under this assumption, the switching function reduces to the form introduced by (Blaschke et al., 2022). The adopted framework aims to describe a continuous transition from hadronic matter to deconfined quark matter, in contrast to the Maxwell construction, which typically introduces discontinuities or sharp features in the pressure.

The hadronic equation of state (EoS) employed in modeling hybrid stars is based on the (Divaris et al., 2024) model, which depends on the parameter η and is constrained by nuclear matter properties such as the slope parameter Land the incompressibility parameter K0. The quark matter EoS in the deconfined phase is taken from the model of (Blaschke et al., 2022) and is characterized by the parameters a4, Δ, and Bef f . A set of ten hadronic EoSs is combined with a quark matter EoS, considering three distinct quark matter parameter sets. This approach allows for a systematic investigation of how the parameter η influences the hadronic EoS, while the parameters a4, Δ, and Bef f govern the properties of the quark phase.



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Sciforum-190830: Machine and Deep Learning Regression Techniquesfor the Inverse Stellar Structure Problem

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IntroductionUnderstanding the equation of state (EoS) of dense nuclear matter remains a key challengein nuclear astrophysics, with direct implications for the structure and observable propertiesof neutron stars and quark stars. Despite strong constraints from nuclear theory, heavy-ioncollisions, and astrophysical observations, the high-density regime of the EoS is still subjectto significant uncertainties. This motivates the development of inverse approaches that infermicroscopic properties from macroscopic observables.**Methodology**In this work, we investigate the inverse stellar structure problem by reconstructing theEoS from mass–radius relations using a data-driven framework. A comprehensive set ofphysically consistent equations of state is constructed covering hadronic and quark matterregimes. For the low-density sector, we employ HLPS2/HLPS3 and tabulated Com-pOSE equations of state for cold neutron stars. The high-density extension depends on theunderlying low-density baseline: for HLPS-based models, we use multimodalparametrizations, while for CompOSE-based models, we apply speed-of-sound parametriza-tions, ensuring thermodynamic consistency across the matching density region. Quark mat-ter is modeled using MIT bag and color-flavor-locked (CFL) equations of state.The inverse mapping is performed using tree-based regression models (Decision Trees,Random Forest, Gradient Boosting, and XGBoost) and Deep Neural Networks with (at least)three hidden layers. Model optimization and fine-tuning are carried out using GroupKFoldcross-validation combined with Grid Search.**Results**The reconstruction of the EoS from mass–radius data is found to be robust across allmodel classes. XGBoost and the Deep Neural Network models consistently achieve the highest ac-curacy, while the ensemble tree methods provide stable intermediate performance. Remarkably, theuse of only eight mass–radius points yields comparable accuracy to denser sampling, indicatingstrong information efficiency in the inverse problem.**Conclusions**Our results demonstrate that inverse modeling techniques provide a powerful and efficientframework for constraining the EoS of dense matter. The approach offers a complementarymethodology to traditional theory-driven studies and highlights the capability of modernregression techniques to extract physical information from sparse astrophysical observations.



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Sciforum-190846: Ground-state properties of astrophysical waiting-point nuclei under extreme magnetic fields

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Extreme astrophysical environments, such as magnetars and core-collapse supernovae, are characterized by super-strong magnetic fields approaching 10^{18} G, which radically modify the quantum states of nuclear matter. In this study, we explore the structural response of astrophysically relevant finite nuclei to these intense fields, with a specific focus on critical waiting-point nuclei involved in r- and rp-process nucleosynthesis. Covariant Density Functional Theory (CDFT) is utilized to evaluate key bulk observables, including charge radii, deformations (β_2), and total binding energies. To quantify model dependence and ensure theoretical reliability, self-consistent calculations are carried out using two independent parametrizations, the non-linear NL3* and the density-dependent DD-ME2 effective interactions, implemented within an axially symmetric, constrained relativistic mean-field framework. The explicit inclusion of the magnetic field breaks time-reversal symmetry and lifts Kramer's degeneracy, prompting a dramatic reorganization of the nuclear single-particle energy levels. Specifically, we identify critical level crossings at the Fermi surface that manifest at field strengths of B~10^{17}-10^{18} G. This microscopic phenomenon triggers abrupt structural phase transitions, compelling transitional nuclei to shift from spherical configurations into oblate or prolate ground states. Additionally, we systematically study the evolution of the nucleon density distributions and the neutron–proton density difference as a function of the magnetic field. The systems exhibit a marked enhancement in total binding energy driven by the strong spin-alignment of nucleons. With both effective interactions yielding consistent structural and energetic shifts, we conclude that these magnetically induced modifications could significantly impact stellar weak interaction rates. Consequently, incorporating rigorous microscopic nuclear structure models is essential for the accurate simulation of nucleosynthetic pathways.



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Sciforum-191939: Reconstruction of Cosmic Expansion History through a Logarithmic Deceleration Parameter Parametrization

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The current expansion of the universe is baffling researchers in their search to find an acceptable explanation without problems. Despite a lot of research in the area, there is still no acceptable model. Hence, much effort is being made to study alternative models or modified theories. In this work, we introduce a novel cosmological framework based on a logarithmic parametrization of the deceleration parameter, designed to capture the dynamical evolution of the universe with enhanced analytical elegance and physical consistency. The proposed parametrization allows the complete reconstruction of the cosmic expansion history in a closed analytical form, providing significant theoretical tractability and deeper insight into the late-time accelerated expansion scenario. To assess the viability of the model, we perform stringent observational constraints on the free parameters by employing a comprehensive joint analysis of the latest cosmological datasets, including Cosmic Chronometers (CC), Pantheon Type Ia Supernovae (SNe Ia), Baryon Acoustic Oscillation (BAO) measurements, and the R19 prior on the Hubble constant. Utilizing these observational probes, we reconstruct the key dynamical and kinematical parameters governing cosmic evolution and determine the cosmic transition redshift with high precision, thereby offering a robust description of the transition from decelerated to accelerated expansion. In addition, our model adds to the search for a proper explanation of the current expansion of the universe.



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Sciforum-194147: Spacetime–Mass Equivalence Principle: A New Gravitational Field Equation and Its Cosmological Applications

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Einstein's general relativity interprets gravity as a geometric effect of spacetime curvature and has achieved great success in explaining classical gravitational phenomena. However, it fails to reveal the dynamical nature of gravity and encounters fundamental barriers to unification with quantum mechanics.

Based on spacetime–matter symmetry, this paper proposes the Spacetime–Mass Equivalence Principle. Its core axioms are: space and time are equal in status; mass is the total curvature integral of finite-scale compactified spacetime; mass corresponds to first-order spacetime curvature, while energy corresponds to second-order curvature; gravity arises from dynamical interactions between spacetime curvature configurations, rather than a pure geometric effect.

By this principle, gravity stems from local spacetime compactification. The first-order curvature matches the Newtonian gravitational field, satisfying g(r) = c^2 K(r) . The first-order field carries curvature energy and induces self-gravitation corrections. The radial self-gravitation is a single term; including tangential self-coupling, temporal self-coupling, spacetime cross-coupling and relativistic dynamical corrections yields a total second-order effect 6 times the radial value, fully consistent with the post-Newtonian approximation of general relativity.

The study derives a new recursive gravitational field equation with infinite self-consistent cyclic corrections. Its weak-field second-order approximation accurately reproduces two key observations: the 43 arcseconds per century residual precession of Mercury's perihelion, and the 1.74-arcsecond starlight deflection by the Sun.

This framework resolves the r→0 gravitational singularity within classical topology without quantum gravity assumptions. It also accounts for dark energy density, dark matter origins, cosmic expansion and early cosmic inflation, laying a new foundation for unified field theory.



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Sciforum-196472: Gravitational-wave polarization modes and stability analysis in Weyl geometry gravity

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We investigate the polarization modes and stability properties of gravitational waves in Weyl geometry gravity on a Minkowski background endowed with a nonvanishing Weyl gauge field. Starting from the Weyl-invariant gravitational action, we perform a linear perturbation analysis and construct gauge-invariant variables through scalar–vector–tensor decomposition. This framework enables a systematic classification of the propagating degrees of freedom and their associated gravitational-wave polarizations. Our analysis shows that the tensor sector contains the two standard plus- and cross-polarization modes, both propagating at the speed of light, as in general relativity. Although the vector sector possesses a propagating dynamical degree of freedom, it does not generate observable vector polarization modes. The scalar sector, in contrast, features a mixture of breathing and longitudinal modes associated with a single scalar degree of freedom. This scalar mode exhibits superluminal propagation and an intrinsic damping of amplitude during propagation, both originating from the background Weyl gauge field. We further estimate the observational prospects of detecting this scalar signal and discuss the conditions under which it may be distinguishable in future gravitational-wave observations. To assess the theoretical consistency of the theory, we derive the quadratic actions for tensor, vector, and scalar perturbations and analyze their stability properties. While the tensor and vector sectors are free from ghost and Laplacian instabilities, a Hamiltonian analysis reveals the presence of an Ostrogradsky ghost in the scalar sector. These results clarify the distinctive gravitational-wave signatures of Weyl geometry gravity and provide a theoretical basis for testing this framework with future multi-messenger and gravitational-wave experiments.



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Sciforum-196610: Running Vacuum framework: a unification paradigm for the expansion history of the Universe

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The concordance ΛCDM model, based on a rigid Λ-term for the entire cosmic history, is not a fundamental model for the cosmological evolution since there is no theoretical explanation for Λ. In our expanding Universe, an evolving Λ-term with the expansion is intuitively much more reasonable, especially if a fundamental theoretical reason can be given for it. The running vacuum model (RVM) framework can provide such an explanation. It is a novel paradigm for the overall description of the universe based on quantum field theory (QFT) in curved spacetime. Quantum effects are computed and appropriately renormalized using an off-shell adiabatic produce. These effects induce a renormalized vacuum energy density (VED) which is a function of the Hubble rate H and its time derivatives. The RVM predicts that in the current universe the VED evolves very slowly with the expansion, changes being proportional to the square of the Hubble rate times as small (calculable) parameter which acts as a $|beta$-function coefficient of the running. This fact provides a possible fundamental origin of dark energy (DE), conceived as dynamical vacuum energy. In the very early universe, on the other hand, the vacuum fluctuations of the quantized matter fields induce higher (even) powers, e.g. $\sim H^4$, capable of triggering fast inflation in a very short period and leading to an automatic graceful-exit into the standard FLRW radiation era. The mechanism of `RVM-inflation' is genuinely different from Starobinsky inflation. It does not require an `inflaton' field since inflation is brought about by pure QFT effects on the geometric background. The RVM appears to be a unified QFT framework for dark energy and inflation, with successful phenomenological implications; in particular, it predicts that the observed hints of dynamical DE could just be running VED, therefore of fundamental origin.



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Sciforum-196974: Neutron Star Cooling in Covariant f(Q) Gravity

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We present a study of the thermal evolution of neutron stars in covariant f(Q) gravity, extending our previous analyses of neutron star structure within this framework (Alwan, 2025). Stellar structure profiles are obtained from a modified Tolman–Oppenheimer–Volkoff solver incorporating three representative f(Q) models (logarithmic, exponential, and quadratic) and subsequently used as input to the NSCool thermal evolution code. Since f(Q) gravity modifies the underlying spacetime geometry, its effects propagate into the neutron star cooling behavior through changes in stellar structure rather than through direct modifications to the microphysical processes governing heat transport and neutrino emission. We find that these structural modifications naturally give rise to distinct slow, threshold, and rapid cooling regimes, whose boundaries shift relative to the general relativistic case depending on the specific f(Q) model and its parameters. The logarithmic and exponential models allow for shifts in either direction depending on the choice of model parameters, whereas the quadratic model produces shifts in only one direction, indicating that the f(Q) gravity sector produces characteristic features in the thermal evolution of neutron stars that are absent in general relativity. We also compare the resulting cooling curves with available observational data from isolated neutron stars (Beznogov & Yakovlev, 2015).



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Sciforum-198043: Anisotropic Transition Cosmology Driven by Bulk Viscosity and Gaussian Expansion Dynamics

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In this paper, we have investigated an anisotropic LRS Bianchi type-I cosmological model filled with a bulk viscous fluid in the framework of f(R, T) gravity. We have consider the linear functional form f(R, T)=R+2λT,

where λ denotes the matter–geometry coupling parameter. To obtain the exact cosmological model solutions, a novel Gaussian-type transition ansatz, $e^{\int q/a\, da} = 1/[a\sqrt{\alpha}+\beta e(-ma^{*2})]$,

where α, β, and m are positive constants, is introduced. The proposed ansatz provides a smooth description of the cosmic transition from an early decelerated expansion phase to the present accelerated epoch. Exact expressions for the deceleration parameter, Hubble parameter, expansion scalar, shear scalar, energy density, effective pressure, bulk viscosity coefficient, jerk parameter, and statefinder diagnostics and transition time are derived and analyzed. The physical behavior of the model shows that the energy density remains positive throughout the cosmic evolution, while the bulk viscous effects generate a negative effective pressure that drives late-time acceleration. The anisotropy decreases with cosmic time, indicating a gradual approach toward isotropy. Furthermore, the model evolves asymptotically toward a de-Sitter phase and reproduces the standard ΛCDM statefinder limit (r,s)=(1,0). Therefore, the obtained model provides a physically viable and observationally consistent framework for studying transition cosmology and accelerated expansion in modified gravity.



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Sciforum-198843: Quasinormal Spectrum in Six-Dimensional Thick Branes

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Quasinormal modes provide a characteristic spectral description of open systems, encoding how perturbations dissipate and relax under physically relevant boundary conditions. They have attracted broad interest in gravitational wave physics. In this work, we investigate the quasinormal modes of a six-dimensional thick brane model and explore how the higher-dimensional brane geometry affects the dynamics of gravitational perturbations and other test fields. These modes can be explained as either metastable gravity wave bound states or decaying massive gravitons from different perspectives. Compared with conventional five-dimensional thick brane scenarios, the six-dimensional setup possesses a richer transverse structure, which can modify the effective perturbation potential and lead to distinctive spectral behavior. We analyze the perturbation equations in this background and compute the corresponding quasinormal spectrum. Particular attention is paid to the relation between the brane configuration, the effective potential, and the distribution of quasinormal frequencies. Our results show that the six-dimensional thick brane geometry can leave clear imprints on the quasinormal spectrum, influencing both the oscillation and damping properties of gravitational perturbations. These findings provide a dynamical spectral perspective on higher-dimensional braneworld models and may offer useful insights for future studies of braneworld holography, gravitational localization, and the stability of thick brane configurations.



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Sciforum-198889: Neural Network and Machine Learning Methods for Distinguishing Hadronic and Quark Compact Stars

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Compact stars offer a powerful window into the behavior of matter at extreme densities, although their internal structure is still not fully understood. Distinguishing hadronic neutron stars from quark stars is particularly difficult, since the two classes can produce similar mass–radius relations and may therefore be indistinguishable using standard observables alone.In this study, we employ machine-learning techniques and neural networks to classify compact stars on the basis of their mass, radius, tidal deformability, Love number, and central pressure.The analysis is based on 39,920 stellar configurations derived from 3,992 equations of state, comprising 2,048 hadronic models and 1,944 quark-matter models described within the MIT bag and Color–Flavor-Locked approaches. Several supervised learning algorithms, including Random Forest, XGBoost, Decision Tree, and Support Vector Machine, are tested and compared with a feedforward neural network. Under ideal noise-free conditions, all models show excellent performance, with XGBoost, SVM, and the neural network achieving accuracy, F1-score, and AUC values close to one. Feature-importance and ablation studies indicate that the Love number plays a central role in the classification process, emphasizing the diagnostic value of tidal-response observables. The robustness of the models is further examined by introducing Gaussian uncertainties in mass and radius and by reconstructing tidal deformability through a compactness–deformability relation including intrinsic scatter. While observational uncertainties degrade the performance of some classifiers, XGBoost and the neural network retain strong predictive capability. These results demonstrate that machine learning can provide an effective framework for identifying the nature of compact stars and underline the importance of accurate tidal-response measurements for constraining the dense-matter equation of state.



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Sciforum-198909: A Consistency Criterion for Reconstructive Cosmological Models

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Reconstructive methods are widely used in modified gravity cosmology to avoid the difficulty of finding exact analytical solutions. These methods, however, contain an implicit assumption. The kinematic ansatz for the Hubble parameter and the dynamic equations of motion determine the transition to accelerated expansion through independent procedures. There is no guarantee that these two approaches yield consistent results. In this work, we introduce the phase coherence criterion, defined as Δz=|zq-zω|\Delta z = |z_q - z_{\omega}|Δz=|zq-zω|. Here, zqz_qzq is obtained from the kinematic condition q(zq)=0q(z_q) = 0q(zq)=0, and zωz_{\omega}zω is obtained from the dynamic condition ωdyn(zω)=-1/3\omega_{\mathrm{dyn}}(z_{\omega}) = -1/3ωdyn(zω)=-1/3 on the equation of state. We demonstrate the criterion on the generalized Einstein–scalar–Gauss–Bonnet model using three independent observational datasets. These are Cosmic Chronometers, BAO from SDSS and DESI, and Pantheon+ with SH0ES calibration. The exact analytical solution obtained via Noether symmetry yields Δz∼10-3\Delta z \sim 10^{-3}Δz∼10-3 across all datasets. This value is two orders of magnitude below the observational threshold derived from Pantheon+ precision. In contrast, the hybrid reconstructive ansatz produces Δz≈0.69\Delta z \approx 0.69Δz≈0.69. This exceeds the threshold by a factor of five. The criterion is straightforward to evaluate and requires no new observational data. It is compatible with any modified gravity theory that admits an effective fluid interpretation within the flat FLRW framework.



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Sciforum-200042: Feasibility Analysis of Holographic Dark Energy Based on Generalized Entropy

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In this study, we investigate the feasibility of generating holographic dark energy based on different infrared cut-offs under different generalized entropy functions. Specifically, we analyze six dark energy models formed by combining three-parameter generalized entropy and five-parameter generalized entropy with three infrared cut-offs. These three cut-offs are the Granda-Oliveros (GO) cut-off, the GO+1/H (GO-H) cut-off, and the GO+H$^{\cdot}$\ddot HH$^{\cdot\cdot}$ (GO-ddH) cut-off, all of which belong to the Nojiri–Odintsov cut-off category. We perform a Bayesian Markov Chain Monte Carlo (MCMC) analysis to constrain the cosmological parameters using the Pantheon+, DESI, and Compressed Planck likelihood datasets. Furthermore, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) statistical tools are also used to compare preferences between models. The results show that the data does not favor any of the six models as much as the \LambdaCDM model. Although the maximum log-likelihood of some models is greater than that of the \LambdaCDM model, the extra parameters will cause ΔAIC $>$ 0. Among these six models, the three-parameter entropy with GO and the five-parameter entropy with GO-ddH are significantly better than the other models, with ΔAIC values of 3.16 and 18.37, respectively. The other four models all have a ΔAIC greater than 100, and our dataset also makes it difficult to distinguish them. We also investigate the evolution of the dark energy equation of state. The results show that, except for the three-parameter entropy combined with the GO model (which exhibits quintessence-like behavior), all other models display phantom crossing (inverse phantom crossing) phenomena.



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Sciforum-200173: Validity of Generalized Second Law of Thermodynamics in Rastall Cosmology: Generalized Chaplygin Gas with Tsallis Entropy.

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In this study, we analyze the cosmological dynamics and thermodynamic features of the universe using Rastall gravity and the generalized Chaplygin gas (GCG) model, as well as Tsallis entropy. The goal of this analysis is to better comprehend the role of modified gravity and non-extensive thermodynamics in explaining the observed accelerated expansion of the universe. The behavior of the effective equation-of-state parameter is investigated in depth, and the results show that the model can describe both quintessence-like and phantom regimes at various periods of cosmic evolution. To investigate the universe's expansion history, two types of scale factors, power-law and hybrid, are explored. Among them, the hybrid scale factor provides a more realistic picture of the universe by accurately replicating the shift from an early decelerated phase to the current accelerated phase of expansion. Tsallis entropy is used to investigate the validity of the generalised second law of thermodynamics. The research shows that the generalized second rule holds true throughout the evolutionary history of the universe, showing that the proposed model is thermodynamically consistent. Furthermore, observational restrictions are applied using the most recent Hubble parameter H(z) data. Confidence contour analysis is used to define the model parameters' allowable ranges and evaluate their compliance with observations. The acquired results indicate that the suggested Rastall gravity model, when combined with generalized Chaplygin gas and Tsallis entropy, provides a realistic and observationally compatible framework for characterizing the large-scale evolution and accelerated expansion of the universe.



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Sciforum-200260: Quasinormal modes of charged black holes surrounded by fluid in Rastall gravity

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We investigate the quasinormal modes of charged black holes surrounded by a dust-like field in Rastall gravity. The background spacetime is described by a static and spherically symmetric charged black hole solution, in which the Rastall parameter modifies the radial dependence of the surrounding-field contribution. We focus on the asymptotically flat branches of the solution and analyze the axial perturbations of the system. Due to the presence of the black hole charge, the electromagnetic and gravitational perturbations are coupled, leading to a two-component Schrödinger-like master equation with an effective potential matrix. The quasinormal frequencies are computed by imposing the ingoing boundary condition at the event horizon and the outgoing boundary condition at spatial infinity, and the resulting eigenvalue problem is solved numerically by a spectral method.

We study in detail how the fundamental quasinormal modes depend on the Rastall parameter, the surrounding-field parameter, and the black hole charge. In the small-λ asymptotically flat branch, the real oscillation frequency and the damping rate exhibit systematic variations as the Rastall parameter is increased. In the large-λ branch, when the sign of the surrounding-field parameter is chosen consistently with the energy condition, the damping rate can display a non-monotonic behavior. We also examine the dependence on the surrounding-field strength and on the charge, the latter controlling both the background geometry and the electromagnetic-gravitational coupling strength. These results show that the quasinormal spectrum encodes the combined effects of Rastall gravity, the surrounding dust-like field, and the charge-induced coupling of perturbations.



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Sciforum-200262: Does the 4+4 alternation persist in recent solar cycles?

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The solar hemispheres exhibit asymmetries in both timing and intensity of their activity. Investigations of these asymmetries have revealed a long-term hemispheric phase lag. Specifically, a 4+4 Schwabe-cycle variation has been identified: during four consecutive solar cycles the northern hemisphere leads in time, while during the subsequent four Schwabe cycles the southern hemisphere takes over the leading role. This alternating pattern has been observed up to Solar Cycle 23. Whether it also persists in the most recent cycles remains an open question.This phenomenon has previously been studied using several sunspot databases, including the Greenwich Photoheliographic Result, the Debrecen Photoheliographic Data, and several historical sunspot observations by Schwabe, Staudacher, and Spörer. In the present study Solar Cycles 24 and 25 are investigated using the Solnechnie Dannie and SOON databases. Therefore, hemispheric solar asymmetry can be examined over a period spanning nearly 250 years.Unlike most previous studies, we characterize hemispheric solar cycles by their centers of mass and calculate the temporal and intensity differences between the northern and southern hemispheric centers of mass.Our results indicate that Solar Cycles 24 and 25 appear to conform to the 4+4 alternation rule. Since similar periodic behavior has been identified in other solar phenomena, this variation may represent a long-term property of the solar dynamo and may be related to the physical background of the Gleissberg cycle.



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Sciforum-200295: Scalar quasinormal modes of rotating black holes in low-energy Horava gravity

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We investigate the quasinormal modes (QNMs) of a massless scalar field perturbing a rotating black hole solution in the low-energy sector of Horava–Lifshitz gravity. In this background, the radial and angular parts of the perturbation equation generally can not separate, preventing a direct reduction to a one-dimensional Schrödinger-like equation. To overcome this problem, we adopt two distinct approximation schemes. The first one expands the equations in powers of the rotation parameter a up to cubic order, which restores separability and yields standard QNM frequencies. The second one treats the Lorentz-violating parameter l as a small quantity and uses the biorthogonality of spheroidal harmonics to expand the angular dependence, thereby obtaining an effective radial potential from which the QNM spectrum can be computed. We compare the QNM frequencies derived from both methods and find excellent agreement in the regime where both a and l are small. However, because current astrophysical observations place stringent upper bounds on l, while black hole spins can be appreciably large, the second approach—valid for small l but arbitrary a—is more suitable for confronting realistic astrophysical data. Our analysis thus provides a practical framework for testing Horava gravity through black-hole spectroscopy in the era of gravitational-wave astronomy.



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Sciforum-200329: Bulk viscosity effects on dissipative tidal deformability of neutron stars

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In our study, we examined the impact of the bulk viscosity of dense nuclear matter containing neutrons, protons, and electrons on the tidal deformation of neutron stars. The bulk viscosity considered in the present approach originates from the direct Urca process, which acts as a restoring mechanism for chemical equilibrium. The bulk-viscous dissipation that occurs affects the dissipative tidal deformability of binary neutron stars, especially in the late phase of inspiral, which encodes the non-equilibrium properties of neutron star matter and the equation of state. By using a phenomenological nuclear model, we employed different equations of state under various temperature values for neutrino-transparent matter and for oscillation frequencies of baryon density relevant to neutron stars; we examined the aspects of the dissipative tidal deformability so that its dependence on the equation of state could be understood better. The ability to measure the parameters that are encoded in the tidal dissipative deformability by the current terrestrial gravitational wave detectors, as well as future new-generation ones, offers an extra tool for the understanding of dense nuclear matter and its characteristics. Our findings show that, under specific conditions, such effects can be potentially measurable, posing a strong motivation for further theoretical work.



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Sciforum-200410: A Curvature–Elasticity Criterion for Viable Unified F(R) Cosmology

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The reconstruction freedom of F(R) gravity makes it possible to reproduce a wide range of cosmological expansion histories; however, reproducing a background evolution is not sufficient to guarantee physical viability. A unified cosmological scenario connecting an early quasi-de Sitter inflationary stage, a radiation/matter-dominated epoch, and a late-time accelerated attractor must also satisfy ghost freedom, scalaron stability, suppression of matter-era instabilities, and consistency with a general-relativistic intermediate regime. In this work, we formulate these requirements through two dimensionless curvature functions: U(R), which characterizes the normalized slope of the Einstein-frame scalaron potential, and E(R), which measures the logarithmic curvature response of the effective gravitational coupling. This construction provides a compact phase-space representation in the curvature plane spanned by potential control and gravitational elasticity. We show that viable unified F(R) cosmologies correspond to continuous trajectories in the (U,E)-plane linking three physically distinct regimes: a high-curvature slow-roll window, an intermediate general-relativistic corridor with a heavy scalaron, and a low-curvature stable de Sitter attractor. Models that fail to preserve this trajectory structure typically develop scalaron instabilities, matter-era deviations, or unstable late-time behavior. The proposed curvature–elasticity criterion offers a concise diagnostic for separating physically admissible unified F(R) models from formally reconstructed but unstable backgrounds, while also providing a useful framework for constructing ghost-free models that connect Starobinsky-like inflation with late-time dark-energy acceleration.



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Sciforum-200414: A Gauss–Bonnet Mass-Drift Degeneracy Criterion for Apparent Phantom Crossing

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Apparent phantom crossing in late-time cosmology does not necessarily require a fundamental phantom fluid. It may also emerge from an incorrect separation of dark-matter and dark-energy components when the reconstructed expansion history is interpreted under the standard assumption of conserved cold dark matter. In this work, we formulate a model-independent degeneracy criterion for apparent phantom behavior in the presence of a geometrically induced drift of the effective dark-matter mass. We consider a dark-matter density of the form $\rho_\chi(a)=\rho_{\chi 0}a^{-3}\mu(a)$, where $\mu(a)=m_{\rm eff}(a)/m_{\rm eff}(1)$, and allow the mass-drift function to be sourced by a Gauss–Bonnet response, $m_{\rm eff}=m_{\rm eff}(\mathcal{G})$, with $\mathcal{G}=24H^2(H^2+\dot H)$. If the same background expansion is reconstructed by assuming $\rho_\chi\propto a^{-3}$, the inferred dark-energy density is shifted by the mismatch between the true and assumed dark-matter dilution laws. We derive the resulting apparent equation of state and show that a positive mass-drift index, $\Xi(a)=d\ln\mu/d\ln a$, can drive $w_{\rm app}<-1$ even when the underlying dark-energy component remains non-phantom. The proposed criterion provides a compact diagnostic for distinguishing genuine phantom evolution from geometrically induced dark-sector misidentification. This framework is relevant for Gauss–Bonnet cosmologies, effective dark-energy reconstruction, and regular phantom-like scenarios without introducing a fundamental phantom degree of freedom.



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Sciforum-200440: Lorentz Violation without Gravitational Ringdown Modification in Einstein-Bumblebee Black Holes

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Introduction: Lorentz symmetry breaking is expected to leave dynamical imprints on black holes, and gravitational ringdown provides one of the most direct probes of such effects. A key question is therefore whether Lorentz-violating couplings necessarily modify the gravitational quasinormal-mode spectrum. In Einstein-Bumblebee theory, this issue must be addressed by perturbing both the metric and the bumblebee field consistently.

Methods: We study the full linear perturbations of a Schwarzschild-like black hole in Einstein-Bumblebee theory. By imposing the fixed-norm constraint and treating the metric and bumblebee perturbations on the same footing, we develop a hierarchical decoupling procedure for the coupled gravito-vector system.

Results: The coupled system reduces to four decoupled master equations. In each parity sector, the perturbations separate into a Schwarzschild-like gravitational sector and a Lorentz-violating Maxwell-like vector sector. Although Lorentz violation changes the effective propagation speed of the vector modes, the gravitational master equations retain exactly the Regge-Wheeler and Zerilli forms. As a result, the gravitational ringdown spectrum remains unmodified, and the odd- and even-parity gravitational perturbations remain strictly isospectral. The reconstruction formulas show that Lorentz-violating effects are still present, but are encoded in the relation between the master variables and the original metric-bumblebee perturbations. The Maxwell-like vector sector, by contrast, exhibits nontrivial quasinormal-mode shifts that cannot be obtained by a simple rescaling of the Schwarzschild electromagnetic spectrum.

Conclusions: These results show that Lorentz violation can affect black hole perturbations without modifying the gravitational ringdown spectrum. The mismatch between the gravitational and vector propagation speeds may provide a possible timing signature of bumblebee vector dynamics in multi-messenger observations.



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Sciforum-200568: Determination of the Connection in Metric-Affine Gravity

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Curvature is not the only geometric quantity that describes gravitational interactions; torsion and non-metricity also serve as alternative descriptions. Nevertheless, in general relativity, the connection is uniquely determined in a spacetime endowed exclusively with curvature, whereas the same does not hold for gravitational theories that involve only torsion or only non-metricity. Spherically symmetric metric solutions of f(T) gravity theory obtained in pure tetrad formulation are inconsistent in different coordinates. The covariance of the f(T) theory can only be ensured by considering spin connections. Meanwhile, different ways of determining the spin connection also give rise to different branches of covariant f(T) theory. In f(Q) gravity, a majority of studies have been conducted within the coincident gauge. Notably, in the case of a spatially flat universe, there exist three branches of connections that satisfy cosmological symmetry, with the coincident gauge being merely one of them. It is therefore worthwhile and interesting to study the determination of the connection and the relationship between the different branches in metric-affine gravity. We provide a comprehensive summary and comparative analysis of the various theoretical branches of torsional gravity and non-metric gravity, which arise from different choices of affine connection. We identify two types of background-dependent and classical correspondences between these two theories of gravity.



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Sciforum-201001: QNMs and Shadows of Charged Scale-Dependent Black Holes

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In this talk, we present a detailed analysis of the quasi-normal modes (QNMs) and optical shadows of charged black holes within the framework of scale-dependent gravity. These models introduce an explicit dependence of the geometric parameters on the radial scale, which modifies the properties of the event horizon and the asymptotic behavior of spacetime compared to the standard Einstein–Maxwell solutions.We compute the full spectrum of QNMs for different types of perturbations using the higher-order Wentzel–Kramers–Brillouin (WKB) method and the eikonal approximation in the large angular momentum limit. Both techniques yield complex frequencies (ω=ωR+iωI\omega = \omega_R +i \omega_Iω=ωR+iωI) with high precision, revealing the impact of the charge and scale dependence on the stability and relaxation timescales of the black hole.Additionally, we determine the black hole shadow by calculating the unstable photon orbits and the critical radius of the photon sphere. We analyze the deformation and size of the shadow as functions of the charge and scale parameters, and evaluate its observability with next-generation instruments such as the Event Horizon Telescope. All results are systematically compared with the classical Reissner–Nordström black hole, allowing us to quantify the deviations introduced by scale dependence. We show that both the QNMs and the shadows exhibit observable differences that could serve as probes to distinguish these modified solutions from the predictions of general relativity.



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Session 5. Algebra and Geometry

Sciforum-155221: Involutions and Lagrangian Submanifolds in Higgs-Bundle Moduli Spaces of Curves

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In this work, we study geometric properties of moduli spaces of Higgs bundles over Riemann surfaces of genus 2 through the action of natural involutive symmetries. Higgs bundles, introduced by Hitchin in his foundational work on self-duality equations, are pairs consisting of a holomorphic vector bundle and a holomorphic section of its endomorphism bundle tensored with the canonical bundle. The moduli space of such objects carries a natural hyperkähler structure. Every genus-2 Riemann surface is hyperelliptic and therefore admits a canonical involution with six fixed points. This symmetry induces an automorphism of the moduli space of Higgs bundles by pullback. Understanding the geometric properties of fixed-point loci under such automorphisms provides insight into the global structure of these moduli spaces and connects to the theory of real structures in algebraic geometry. According to the results of Schaposnik, the fixed-point locus is Lagrangian with respect to the natural symplectic structure on the moduli space, meaning that the symplectic form vanishes identically on tangent vectors to the fixed-point set. This Lagrangian property connects to mirror symmetry and the geometric Langlands program, where Lagrangian submanifolds correspond to branes and special coherent sheaves. Our main result provides explicit distance estimates relating the hyperkähler distance between a Higgs bundle and its involution image to computable norms of the Higgs field. The proof combines variational methods for path length estimation with the Hitchin–Kobayashi correspondence to control connection variations through the linearized self-duality equations. These distance estimates provide quantitative tools for measuring how involution symmetries deform the hyperkähler geometry, with potential applications to the study of geodesics and metric completeness.



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Sciforum-162141: Characterizing Rings via Domination in Compressed Zero-Divisor Graphs and Applications to Network Optimization

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Compressed zero-divisor graphs provide a symmetry-preserving representation of the algebraic structure of finite commutative rings by merging vertices with identical annihilator behavior. This compression highlights structural regularities and reduces graph complexity while retaining essential algebraic information. Although domination parameters have been widely studied in classical graph theory, their role in characterizing rings through compressed zero-divisor graphs remains largely unexplored. This work investigates how domination numbers and related variants encode algebraic properties of rings and how these parameters can be translated into efficient models for real-world network optimization.

Methods:
For several families of finite commutative rings, we construct compressed zero-divisor graphs using equivalence classes defined by identical annihilator sets. We compute classical domination, Roman domination, and double Roman domination numbers and analyze how these values correlate with ring-theoretic features such as annihilator chains, idempotent decompositions, and local ring behavior. Exact results are obtained through structural lemmas and combinatorial reductions, supported by computational verification using integer linear programming formulations.

Results:
The study shows that domination parameters in compressed zero-divisor graphs serve as effective invariants for distinguishing ring structures. Rings with symmetries induced by annihilator equivalence display predictable domination patterns, while irregular annihilator distributions yield unique domination signatures. Furthermore, compressed graphs significantly reduce computational complexity without loss of domination-relevant information.

Conclusion:
The results demonstrate that domination in compressed zero-divisor graphs provides a novel framework for characterizing finite rings and for modeling optimal resource deployment in symmetric network environments. These findings bridge algebraic graph theory with practical optimization, offering new perspectives for analyzing highly structured networks.



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Sciforum-169405: ON THE HIGHER NASH BLOW-UP DERIVATION LIE ALGEBRAS OF ISOLATED HYPERSURFACE SINGULARITIES

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Introduction: Isolated hypersurface singularities arise naturally in singularity theory and in mathematical physics through deformation theory and related invariants. Since the classical Yau derivation Lie algebra does not always separate simple (ADE) singularities, this paper investigates higher Nash blow-up local algebras and the Lie algebras of their derivations as finer analytic invariants.

Methods: Let $(V,0)\subset (\mathbb{C}^s,0)$ be an isolated hypersurface singularity defined by a polynomial F. Using the higher-order Jacobian matrix $\mathrm{Jac}_n(F)$ and the ideal J_n generated by its maximal minors, we define the higher Nash blow-up local algebra \[M_n(V):=\mathcal{O}_s/\langle F, J_n\rangle. \] For each k, we study the derivation Lie algebra $L_k(V):=\mathrm{Der}(M_k(V))$. To obtain computable Lie-theoretic data, we construct a generalized Cartan matrix $C_k(V)$ from the nilradical of $L_k(V)$ via maximal tori and the associated root space decomposition, following a Mostow-type structural framework.

Results: Concentrating on the case $k=2$, we explicitly compute $C_2(V)$ for all ADE hypersurface singularities, including the infinite families A_k and D_k and the exceptional types E_6, E_7, and E_8. We prove that $C_2(V)$ distinguishes analytic isomorphism classes within the ADE category: for ADE singularities X and Y, one has $C_2(X)=C_2(Y)$ if and only if $X \cong Y$ analytically.

Conclusions: These results confirm, for $k=2$, a Lie-theoretic characterization of ADE singularities through derivations of higher Nash blow-up algebras. The invariant $C_2(V)$ refines the classical derivation-based approach and suggests a systematic route to separating broader classes of singularities.



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Sciforum-181562: Symmetrically Fuzzy β-Connected and Disconnected Sets in Fuzzy Topological Spaces

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In fuzzy topology, symmetry plays a central role in analyzing generalized forms of connectedness. This study investigates the symmetric properties associated with fuzzy β-connectedness and fuzzy β-disconnectedness in fuzzy topological spaces. Let (X, τ) be a fuzzy topological space. Two non-empty fuzzy sets A and B in X are called fuzzy β-separated if $A \wedge \beta cl(B) = 0_X$ and $\beta cl(A) \wedge B = 0_X$. A fuzzy set $A \leq X$ is called symmetrically fuzzy β-disconnected if both A and its complement A^c can be expressed as unions of two fuzzy β-separated sets; otherwise, A is symmetrically fuzzy β-connected. Several symmetry-preserving properties are established: the complement of a symmetrically fuzzy β-connected set is also symmetrically fuzzy β-connected, and the union of two symmetrically fuzzy β-connected sets that are not completely disjoint remains symmetrically fuzzy β-connected. Furthermore, if $f: X \to Y$ is an M-fuzzy β-continuous map satisfying $f^{-1}(\beta cl_Y(B_i)) = \beta cl_X(f^{-1}(B_i))$ for appropriate fuzzy sets B_i, then the preimage of a symmetrically fuzzy β-disconnected set is also symmetrically fuzzy β-disconnected, demonstrating that symmetry is preserved under fuzzy continuous mappings. In general, the intersection of two symmetrically fuzzy β-connected sets is not guaranteed to be symmetrically fuzzy β-connected; however, it is preserved when one set is contained in the other. These findings highlight the fundamental importance of symmetry in understanding and analyzing fuzzy β-connectedness, providing a rigorous framework for symmetric decomposition, connectivity analysis, and the structural characterization of fuzzy sets. Overall, this study contributes to a deeper comprehension of the interplay between symmetry and connectivity in fuzzy topological spaces, offering valuable theoretical tools for further research and applications in fuzzy mathematics.



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Sciforum-184547: Topological Structures of the Alpha Group via Monte Carlo Simulations

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We present a Monte Carlo dynamical simulation of the Alpha Group, a novel algebraic–topological framework in which the geometry of space emerges from the structure of the operator, M(θ), defined through the imaginary component, μ, and a projective division mechanism. The operator encodes two nontrivial nodal structures that influence the interaction dynamics and resulting topology. By evolving a normalized four-dimensional state vector through repeated iterations of M(θ)M(\theta)M(θ) with small stochastic perturbations, we obtain a trajectory that reveals the intrinsic structure of the underlying space.

The Monte Carlo simulation is performed over 80,000 steps near θ=π/2, producing a high-resolution dataset corresponding to an emergent three-dimensional subspace. The resulting trajectory is analyzed using graph-based methods. Three-dimensional visualizations in transverse planes and two-dimensional projections (XY, XZ, YZ) are used to identify structural centers, ranges, and statistical characteristics. Topological features are extracted via proximity graphs, where 3-cliques (triangles) define 2-simplices, and homology is computed using the Euler–Poincaré relation $\chi = V - E + F = \beta 0 - \beta 1 + \beta 2$.

The results show a single connected component ($\beta 0=1$), the presence of nontrivial one-dimensional cycles ($\beta 1>0$), and persistent two-dimensional cavities ($\beta 2>0$). The Euler characteristic is consistent, confirming internal topological coherence. The Monte Carlo analysis provides computational evidence that the emergent geometry of the Alpha Group is non-Riemannian and dynamically generated, with a homological structure influenced by the imaginary sector μ. These findings suggest that space may exhibit a dynamic, parameter-dependent topology, opening new directions for mathematical and physical investigations of complex geometric structures.



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Sciforum-188528: From Smith Theory to the Dold–Thom Theorem: A Unified Study of Symmetry in Algebraic Topology

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Symmetry in topological spaces, expressed through continuous group actions, underlies some of the deepest invariants in algebraic topology. This work presents a unified framework connecting fixed-point theory, prime-order group actions, equivariant cohomology, symmetric products, and covering symmetries, areas that have traditionally been developed independently. All results are developed within the category of compactly generated Hausdorff G-spaces and equivariant maps, where G is a finite topological group acting continuously on a topological space X (with G taken as a compact torus $(S^1)^n$ in the localisation result). The principal tools are the Borel construction $EG \times_G X$, Smith exact triangles over $\mathbb{Z}/p\mathbb{Z}$-module complexes, the infinite symmetric product functor, and the monodromy action of the fundamental group on the universal cover. We present several fundamental results linking fixed-point theory, Smith theory, equivariant cohomology, symmetric products, and covering spaces. In fixed-point theory, the Lefschetz Fixed-Point Theorem is proved and refined equivariantly, showing that a non-zero Lefschetz number forces a G-invariant fixed-point set. In Smith theory, the Smith Inequality bounds the mod-p homological dimension of X^G relative to X, and the Smith–Floyd Theorem yields sharp Lusternik–Schnirelmann category estimates for free $\mathbb{Z}_2$-actions. For equivariant cohomology, an Equivariant Künneth Formula and a Localisation Theorem show that torus-equivariant cohomology localises to fixed-point data after inverting the augmentation ideal. The Dold–Thom Theorem is proved in full, identifying $\pi_n(SP^\infty(X))$ with $H_n(X; \mathbb{Z})$ via the Eilenberg–Steenrod axioms. Finally, the Galois Correspondence between subgroups of $\pi_1(B)$ and connected covering spaces is established, with the Deck Group Isomorphism as its centerpiece. These results demonstrate that equivariant symmetry acts as a unifying principle across algebraic topology. Natural directions for further work include Smith theory for non-abelian actions, motivic equivariant cohomology, and symmetric products of non-simply connected spaces.



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Sciforum-189804: Skeletal Polyhedra, Geometric Graphs, and Crystallographic Groups

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The study of highly symmetric geometric structures in ordinary 3-dimensional Euclidean space has a rich history tracing back to the early days of geometry. Over the course of time, new notions of discrete polyhedral structures have intrigued scientists and brought to light new exciting figures intimately related to finite or infinite groups of isometries.

A new graph-theoretical, or skeletal, approach to polyhedral structures was pioneered by Branko Grunbaum in the 1970's and built on Coxeter's extensive work on polyhedra, polytopes, and symmetry. Skeletal polyhedra or polyhedral complexes in ordinary space are viewed not as 3-dimensional solids but rather as finite or infinite periodic geometric graph, linearly embedded into space and equipped with an overarching polyhedral super-structure imposed by the faces.

This new definition was deliberately designed to rid the theory of polyhedra of the psychological block that membranes must be spanning the faces to give a surface. In the new framework, the familiar 3-dimensional cube is viewed as a graph with eight vertices, twelve edges, and six 4-cycles in the graph distinguished as faces, namely the boundary cycles of the usual square of the cube. The level of symmetry of skeletal polyhedra or complexes is determined by distinguished transitivity properties of their geometric symmetry groups, which are either finite groups of isometry and infinite crystallographic groups of isometries.

These skeletal figures in ordinary space exhibit fascinating geometric, combinatorial, and algebraic properties and include both new and well-known structures. We discuss approaches to the classification for some of the most prominent classes of skeletal figures including the regular, chiral, or uniform polyhedra. The talk gives an introduction to this area.



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Sciforum-190165: Diagonalizability Conditions for a (2,1)-Band Matrix of order 4 with a Prescribed Spectrum

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Introduction: This work studies the inverse eigenvalue problem for a (2,1)-band matrix of order four. The analysis demonstrates that the transformation matrix is nonsingular if and only if the eigenvalues are pairwise distinct. We ask ourselves what is the minimum number of entries that we can nullify so that it is diagonalizable with an eigenvalue of algebraic multiplicity two.

Methods: Using diagonal similarity transformations, the original matrix family is reduced to a normalized form with unit superdiagonal entries. This normalization preserves the spectrum while reducing the number of effective parameters. We study the characteristic polynomial, and the coefficient comparison yields a nonlinear algebraic system involving the matrix coefficients.

Results: If one eigenvalue has algebraic multiplicity two, the matrix cannot remain diagonalizable while preserving all nonzero band entries. Motivated by this obstruction, we investigated the minimum number of entries that must be nullified in order to admit a diagonalizable matrix with a repeated eigenvalue. For four eigenvalues, one of them double and the others single, the symbolic construction produces a nonsingular transformation matrix T, which establishes the existence of a complete basis of eigenvalues.

Conclusions: We have studied the minimum number of entries that must be set to zero in order to recover diagonalizability in the presence of a double eigenvalue. Explicit symbolic constructions were obtained. In particular, a parametric family of matrices satisfying the prescribed spectral conditions was derived together with a closed-form diagonalizing transformation matrix.



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Sciforum-190524: Fibonacci Tower Vector Fields: Algebraic Structure, Stability and Finitely Generated Abelian Groups

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This paper investigates the underlying algebraic structures and asymptotic stability properties of a specialized class of complex exponential towers indexed by Fibonacci numbers. We formally define the mathematical tower set $\vec{T}_{\text{Fib}}$ composed of iterative exponentiations of complex variables $z = re^{it}$, where the corresponding phase parameters are explicitly constrained to the ordered Fibonacci sequence. We demonstrate that under appropriate algebraic restrictions, multi-tower collections of these structures can be modeled effectively through finitely generated modules over commutative rings. To evaluate the precise asymptotic behavior of these complex systems, we utilize eigenvalue-based stability criteria to analyze the maximal rate-of-change within the associated tower vectors rigorously. Furthermore, we establish a novel theoretical connection between the structural constraints of these algebraic groups and bounded periodic behavior, which functions analogous to classical Burnside-type properties. By doing so, we provide a rigorous classification of the underlying group representations. Additionally, we analyze the structural transitions that occur within the network topology when external parameters fluctuate dynamically in real-time scenarios. Finally, the practical utility, convergence, and robustness of the proposed mathematical framework are comprehensively demonstrated by applying these algebraic tower representations to actively stabilize the curvilinear motion within an advanced, engineering-based vibrating dynamical system model under external perturbations.



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Sciforum-190532: Gaussian Balancing-Like Sequences and Their Asymptotic Geometry

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In this study, we investigate complex extensions of balancing-like sequences. These sequences are defined by second-order linear recurrence relations with complex initial conditions and depend on a real parameter $A > 2$. We first derive explicit Binet-type formulas, generating functions, and matrix representations for sequences. Several fundamental identities, including Cassini, Catalan, d'Ocagne-type, and mixed identities, are also established.

A central aim of this study is to examine not only the algebraic structure of these sequences but also their geometric behavior in the complex plane. Using the dominant root of the characteristic equation

$\lambda^2 - A\lambda + 1 = 0$,

we show that the terms of these sequences become asymptotically aligned along the same ray in the complex plane. More precisely, their arguments converge to a common limiting direction determined by the dominant characteristic root. Furthermore, the angular deviation from this limiting direction decreases exponentially.

These results show that the proposed extensions preserve many classical algebraic properties of balancing-type sequences while also exhibiting a well-defined asymptotic geometry. Thus, the study provides a bridge between recurrence sequence theory, complex valued sequences, and discrete dynamical behavior.

The methods of the study rely on characteristic-root analysis, Binet-type formulas, generating functions, and companion matrix representations. These tools allow us to obtain closed forms, prove algebraic identities, and analyze the asymptotic direction of the sequences in the complex plane.



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Sciforum-192051: Universal invariants and value-reflection symmetry in the lattice of R1-precomplete subclasses of (k+1)-valued logic

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Post's lattice fully describes the closed classes of Boolean functions, but its (k+1)-valued analogue is, in general, intractable: the lattice of clones in P_k has the cardinality of the continuum for every k >= 2. Within the well-defined sublattice of R_1-precomplete subclasses inside the class T_01 of functions preserving both 0 and k, we identify a structural invariant which is uniform in k and which we interpret as a universal property of the R_1-closure.

We work within the Pol-Inv Galois, restricted to the R_1-closure operator. The chain {0, ..., k} carries a natural value-reflection involution x -> k - x; its induced action on the subclasses of T_01 is analysed orbit by orbit. The R_1-closure-equivalence classes are enumerated via a matrix-logic representation.

We obtain three results. (i) For every k >= 2 and every n >= 1, the number of R_1-closure equivalence classes of T^(n) equals 2^(2^n - 1), independently of k. (ii) The cells of the structural decomposition satisfy |A_J| = (k+1)^(n - |J|). (iii) A matrix-logic reduction T_sim = P_2 holds, yielding a two-valued shadow of the (k+1)-valued structure. All three statements were cross-verified by exhaustive enumeration for k = 2, 3.

The k-uniformity of the count 2^(2^n - 1), combined with the value-reflection involution, provides a symmetry-type invariant of the lattice. Open directions include the complete orbit poset under the value-reflection involution and a possible polyhedral, in particular tropical, interpretation of the R_1-closed sublattice.



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Sciforum-193774: Relative De Rham cohomology in diffeological spaces

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Diffeological spaces extend the category of smooth manifolds and provide a flexible framework for studying geometric and topological structures on spaces that may fail to be manifolds in the classical sense. In recent years, De Rham cohomology has been successfully developed in this setting, revealing new connections between differential forms, smooth maps, and categorical constructions.In this communication, we present recent developments concerning De Rham cohomology and its relative version for diffeological spaces. We discuss two approaches to relative De Rham cohomology associated with smooth maps and examine their fundamental properties. Particular attention is paid to the difficulties that arise from the absence of partitions of unity in general diffeological spaces, a feature that distinguishes this theory from its classical counterpart.Our approach is based on categorical methods. Instead of using open coverings and their intersections, we work with generating families of plots together with pull-back and join constructions. These tools provide a natural framework for studying relative cohomological structures and allow classical ideas to be reformulated in the diffeological context.Within this setting, we obtain a new viewpoint on Mayer–Vietoris type sequences and analyze the conditions under which long exact sequences in cohomology can be constructed. We also discuss the existence of relative cup products and show how these constructions extend familiar cohomological operations beyond the category of smooth manifolds.The ideas presented in this communication are based on recent joint work concerning relative De Rham cohomology and related cohomological constructions in diffeological spaces.



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Sciforum-194342: The Dihedral Stabilizer of the Lam–Postnikov Triangulation of the Hypersimplex

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The standard unimodular triangulation of the hypersimplex , equivalently the Stanley–Sturmfels sorted, type A alcove, and Lam–Postnikov circuit triangulation, is a central object in algebraic and polyhedral combinatorics. Motivated by a recent conjecture of Clarke and Kölbl on equivariant Ehrhart theory, we determine its exact coordinate stabilizer. We study the triangulation through its sorted one-skeleton and use the cyclic alternation criterion for sorted pairs of -subsets. For , this graph encodes the crossing relation of joining segments in a convex -gon with cyclically ordered vertices: two disjoint joining segments and cross exactly when and form a sorted edge for any admissible common core of size . This reconstruction forces every coordinate permutation preserving the triangulation to preserve the crossing relation on disjoint joining segments. Since the boundary edges of the -gon are precisely the joining segments crossing no disjoint joining segment, such a permutation must preserve the cycle graph and is therefore dihedral. Conversely, every dihedral symmetry preserves cyclic alternation and hence the flag-sorted complex. We conclude that the coordinate stabilizer is for , while the simplex cases and have full coordinate stabilizer . The result proves dihedral maximality in the non-simplex range, supplies the necessary endpoint correction, and shows that the cyclic order is recoverable, up to reversal, from the naturally labeled sorted one-skeleton of the canonical hypersimplex triangulation.



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Sciforum-195160: A pedagogical note on the algebraic instability of the Pythagorean relation for exponents greater than two.

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This paper investigates the algebraic behavior of the generalized Pythagorean relation for integer exponents greater than two, where the base values belong to a primitive Pythagorean triple. While the classical identity at exponent two represents a perfect geometric and numerical balance, the behavioral dynamics of this relation at higher-order powers remain less explored pedagogically. By defining a ratio function, we analyze the scaling behavior of the hypotenuse term against the corresponding sum of the legs. Through rigorous inequality and analysis, we demonstrate that this sequence is strictly increasing for all exponents greater than or equal to two. This behavior confirms an escalating numerical instability wherein the dominant hypotenuse term exponentially outpaces the growth rate of the base powers. A numerical illustration utilizing the primitive triple (three, four, five) is provided to visualize the acceleration of this divergence. Ultimately, our findings contextualize the unique mathematical stability of the traditional Pythagorean theorem as an isolated state of structural balance. This structural analysis provides an accessible entry point for understanding non-linear exponential systems within classical number theory frameworks, demonstrating the immediate onset of functional divergence once the base equilibrium is disrupted. Furthermore, by evaluating the continuous limits of this system, we illustrate how elementary calculus and analytical bounds can be applied to illuminate behavior typically reserved for advanced modular forms. This unified approach offers students a robust pedagogical framework for transitioning from static geometric constraints to dynamic algebraic functions.



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Sciforum-198391: Chiral polyhedra from AGL(1, *q*)

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An abstract polyhedron is a partially ordered set characterized by four properties which altogether generalize the face lattice of a Euclidean polyhedron. Abstract polyhedra (with finite faces) are maps on closed surfaces, and most maps on surfaces are abstract polyhedra. The \emph{regular} abstract polyhedra (that is, those with a flag-transitive automorphism group action, hence maximal "rotational" and "reflective" symmetry) in particular have received the most attention. Chiral polyhedra, on the other hand, possessing maximum rotational symmetry without reflective symmetry, are trickier to construct, and their groups are evidently rarer among groups of a given order. Unlike regular polyhedra (which are isomorphic to their mirror images), chiral polyhedra always arise in mirror–image pairs called "enantiomorphs." In this talk, we present Angelone & Schulte's construction of chiral polyhedra whose automorphism group is a subgroup of the general affine group over a field. We also show that our construction is the only way that chiral polyhedra can be derived from the affine group, and that no chiral ddd-polytopes can be constructed from the affine group for ranks d≥4d \geq 4d≥4. The polyhedra arising from this construction end up being minimal under the covering relationship between chiral polyhedra, and we leverage this fact to show that all groups of chiral polyhedra of a certain type are extensions of this affine-group construction.



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Sciforum-199411: Automorphism Groups and Multiscale Symmetry Breaking in a Rooted Watershed Tree: An Algebraic–Geometric Study of the Funa Drainage Network

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Introduction: Drainage networks can be represented as rooted planar trees, making it possible to study their branching structures through algebraic and geometric concepts of symmetry. This study models the Funa drainage network in Kinshasa as a rooted spatial tree and investigates its exact automorphisms, approximate branch symmetries, and multiscale symmetry breaking.

Methods: The drainage network was represented as a directed rooted tree, with the outlet as the root, river reaches as edges, and sources and confluences as vertices. Exact symmetries were identified through rooted-subtree isomorphisms and computation of the automorphism group. Approximate symmetry at each binary confluence was quantified using a composite index, combining topological similarity, branch-length similarity, and angular similarity. Symmetry was then analysed across hierarchical scales defined by the number of contributing sources and Strahler stream order.

Results: The validated tree contains 66 vertices, 65 edges, 33 sources, and 32 binary confluences. Eight confluences, representing 25 percent of all confluences, exhibit exact topological symmetry. Seven correspond to exchanges between terminal-source branches, while one represents a higher-order branched symmetry. The rooted automorphism group contains 256 elements. Twenty-one exact rooted-subtree isomorphism classes were identified, including seven recurrent classes. No symmetry remained exact after strict metric and angular constraints were imposed. The mean approximate-symmetry score was 0.528, while the source-weighted global score was 0.405, producing a symmetry-breaking index of 0.595. Symmetry decreased significantly with increasing hierarchical scale, with a Spearman correlation coefficient of minus 0.737 and a probability value of 0.0000015.

Conclusions: The Funa network exhibits recurrent local automorphisms within a globally asymmetric hierarchy. The proposed framework provides a reproducible algebraic and geometric method for analysing symmetry, subtree equivalence, and scale-dependent symmetry breaking in hydrographic networks.



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Sciforum-200337: Symmetric Circle Configurations from Regular Skeletal Polyhedra

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Point-circle configurations provide a natural generalization of classical point-line configurations and connect incidence geometry with the theory of symmetric polyhedra. In this work, we investigate highly symmetric point-circle configurations arising from regular skeletal polyhedra in Euclidean three-space. Our goal is to understand how geometric and combinatorial symmetry can be transferred from polyhedral structures to circle configurations. This work can lead to the investigation of restrictions on geometric realizations of combinatorial configurations.

We introduce the E-construction, a procedure that associates a circle configuration with a polyhedron through the neighborhoods of its edges. For each edge, the adjacent vertices in the surrounding faces determine a candidate block, and coplanarity conditions are analyzed to determine when these blocks can be realized as circles. The construction is applied systematically to the complete classification of regular skeletal polyhedra, including finite, planar, blended, and pure infinite examples. Symmetry properties are studied through the action of the corresponding polyhedral symmetry groups.

The E-construction produces a large family of finite and infinite point-circle configurations that inherit the full symmetry of the underlying regular polyhedra. In most cases, the resulting configurations are point-circle transitive and consist of congruent circles. A complete classification is obtained for the regular skeletal polyhedra that satisfy the required coplanarity condition. In contrast, chiral polyhedra fail this condition and therefore do not yield point-circle configurations through the E-construction. The study also reveals relationships between Petrie-dual polyhedra and their associated configurations.

Regular skeletal polyhedra constitute a rich source of highly symmetric point-circle configurations in three-dimensional space. The results establish a unified framework linking polyhedral symmetry, incidence structures, and geometric configurations, and suggest several open directions concerning the classification of point-circle transitive configurations in Euclidean 3-space.



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Sciforum-200624: Group-Equivariant Fuzzy Topological Vector Spaces: A New Framework via Symmetric Group Actions

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The theory of fuzzy topological vector spaces, initiated by Katsaras and Liu, requires algebraic operations to be fully fuzzy continuous but does not account for any underlying symmetry structure present in many mathematical and physical contexts. This paper introduces Group-Equivariant Fuzzy Topological Vector Spaces, a new algebraic-topological framework that systematically incorporates the action of a symmetry group G into the structure of fuzzy topological vector spaces by equipping a vector space X with a Lowen fuzzy topology T and a linear group action ϕ: $G \to \mathrm{GL}(X)$, requiring both the fuzzy topology and the algebraic operations to be G-invariant and G-equivariant respectively, with classical results extended using invariant neighbourhood filters, Baire category arguments, and categorical methods. We establish that every group-equivariant fuzzy topological vector space admits a G-invariant fuzzy neighbourhood base at the origin, the fixed-point subspace X^G inherits a classical fuzzy topological vector space structure, a G-equivariant fuzzy open mapping theorem holds under completeness assumptions, the orbit space X/G carries a natural quotient structure, the category of G-equivariant fuzzy continuous linear maps is complete and cocomplete, and new symmetry-induced separation axioms G-T_0, G-T_1, G-T_2, strictly stronger than classical fuzzy separation axioms, are introduced and characterised. Examples on finite-dimensional Euclidean spaces with orthogonal group actions, the real line with two-element group symmetry, and bounded sequence spaces with permutation symmetry illustrate the theory, while counter-examples confirm that G-equivariance is a strictly non-trivial condition. The proposed framework strictly generalises classical fuzzy topological vector spaces, recovered as the special case $G = \{e\}$, opening new directions in symmetric fuzzy functional analysis, equivariant spectral theory, and invariant approximation theory.



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Session 6. Mathematical Analysis and Statistics

Sciforum-173683: Efficient and Accurate Variational Inference for Multilevel Threshold Autoregressive Models in Intensive Longitudinal Data

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Recent technological advancements have facilitated the collection of intensive longitudinal data (ILD), comprising repeated measurements from the same individual. The threshold autoregressive (TAR) model is commonly utilized to capture the dynamic outcome process in ILD, with autoregressive parameters varying based on outcome variable levels. For ILD data from multiple individuals, multilevel TAR models have been proposed, with a Bayesian approach recommended for parameter estimation. However, fitting multilevel TAR models can present computational challenges. In this study, we introduce a mean field variational Bayes (MFVB) algorithm as an alternative to traditional Bayesian inference. By optimizing to approximate posterior densities, variational Bayes aims to find the best approximation within a defined set of distributions rather than asymptotically converging to the true posterior. Simulation results demonstrate that our MFVB algorithm is significantly faster than the standard Markov Chain Monte Carlo (MCMC) approach. Additionally, the simulation results reveal that increasing the number of individuals or the number of time points improves the accuracy of parameter estimates using the MFVB method, suggesting that sufficient data are critical for accurate parameter estimation, especially for complex models like multilevel threshold autoregressive models. When the algorithm was applied to real-world data, it significantly outperformed MCMC in speed and accuracy. In summary, the MFVB algorithm offers a notably faster and consistent alternative to MCMC for intricate and large-scale inference in intensive longitudinal data models, underscoring its potential as a swift and efficient tool for data analysis.



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Sciforum-175070: Approximate Bayesian Inference for Functional Mixed Effects Regression with Partially Observed Functional Covariates

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Functional covariates are increasingly collected over time, such as in environmental monitoring studies. In practice, these covariates are often partially observed due to irregular measurement schedules, sensor failures, or temporal aggregation. Although functional mixed-effects regression has been extensively studied for longitudinal data, existing Bayesian approaches, such as variational Bayes (VB) and Markov chain Monte Carlo (MCMC), assume fully observed functional covariates, thereby overlooking the pervasive issue of functional data incompleteness. Recent methodological developments have addressed partially observed functional data under both continuous and randomly scattered missingness, primarily through reconstruction or descriptive tools such as functional depth measures. However, these approaches are not embedded within functional mixed-effects regression models and therefore do not propagate reconstruction uncertainty into inferential procedures. To bridge this gap, we develop a unified modeling framework for a functional mixed-effects regression model with partially observed functional covariates that jointly reconstruct incomplete functional trajectories and perform inference. This integration enables simultaneous evaluation of imputation quality, functional coefficient estimation, and model-based inference. Through simulation studies across varying numbers of subjects and repeated measurements, we compare VB and MCMC in terms of estimation accuracy and computational efficiency. The results show that the reconstructed functional covariates preserve key features of the underlying trajectories and that VB produces estimates comparable to MCMC while achieving substantial reductions in computation time. We further illustrate the proposed method using real environmental data, modeling weekly water levels with daily temperature curves as functional covariates and evaporation levels as scalar covariates, demonstrating its practical and computational efficiency.



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Sciforum-176017: Symmetries and paradoxes in Bayesian inference

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Building on Harold Jeffreys' rule for contructiong priors and Edwin Jaynes's ideas on transformation groups and priors, we discuss the role of symmetries in Bayesian inference and their role in the selection of prior distributions in Bayesian inference. Within objective Bayesian approaches, transformation groups may provide formal rules for selecting priors in a given problem. We show that in some cases dependence on the choice of transformation group may lead to paradoxes, such as Bertrand's paradox. This paradox concerns 'random' chords drawn across a circle, a failure of the principle of indifference, and what we mean by 'random'. In other cases we show that transformation groups may ameliorate paradoxes or difficulties, such as the Jeffreys-Lindley paradox. This paradox concerns the foundations of statistical inference, as differences between Bayesian and frequentist inferences are shown to grow as sample size increases. Using transformation groups, we find priors representing complete indifference to sample size in the problem, which, if succesful, would resolve the paradox. Unfortunately, it leads to an improper scale-invariant prior. We show, however, that a truncated scale-invariant prior delays the dependence on sample size, which could be practically significant. Lastly, we shed light on the paradox by relating it to the fact that the scale-invariant prior is improper.



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Sciforum-184254: A foundational analysis of local functional calculus

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We introduce the local functional calculus, a unifying framework for local functional operators built on arbitrary continuous functions. This framework subsumes many known functional and fractional calculus operators (conformable and non-conformable) and, more generally, any local derivative or integrator with dependent scaling. As a special case, we develop the local Euler fractional calculus with the exponential kernel, which yields a particularly elegant calculus. For both the general functional and the Euler fractional cases, we prove the fundamental properties: linearity, product rule, continuity, Rolle's theorem, the mean value theorem, and the fundamental theorem of calculus. Crucially, we derive the correct chain rule, which rectifies an error present in the earlier literature on generalized conformable derivatives. The local functional calculus thus provides a solid, self-contained foundation for a wide class of local functional calculus operators, extending far beyond the traditional fractional setting. It is easy to show that this work can be extended and applied in statistics. I will present recent advancements in mathematical and cosmological frameworks, drawing from my key studies. Noticing the need for advanced math, I will introduce and explain the exploration of novel tensor-based approaches from Advancing Tensor Theories (ATTs) and Advanced Manifold-Metric Pairs (AMMPs), offering new tools for modeling. I will introduce and explain functorial methods in Functors of Actions Theories (FATs), bridging mathematical rigor with cosmological phenomena and providing a novel way to think about mathematical analysis and its applications in statistics.



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Sciforum-190456: Finite Mixture Models as a Framework for Symmetry Breaking in Statistical Distributions: Theory, Estimation, and Applications to Asymmetric Data

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Symmetry is a foundational concept in mathematics and statistics, yet real-world data frequently exhibit heavy tails, multimodality, and skewness that classical symmetric models fail to capture. Finite mixture models (FMMs) offer a principled framework for characterizing such symmetry-breaking phenomena, representing complex asymmetric distributions as convex combinations of simpler component distributions. This work presents a unified treatment of FMMs with emphasis on their role in modeling asymmetric and heterogeneous data. We establish theoretical foundations, including identifiability conditions under which decomposition is unique and the characterization of approximable asymmetric densities. Parameter estimation is developed within the Expectation–Maximization (EM) framework, addressing convergence, local optima, and component degeneracy. Special attention is given to skew-normal and skew-t mixtures, which provide additional flexibility beyond Gaussian mixtures for parsimoniously representing skewed data. The connection between mixture models and latent variable representations is also discussed, highlighting how unobserved group membership variables formalize the notion of hidden symmetry. Model selection is addressed through information-theoretic criteria and likelihood ratio testing under non-standard conditions. These developments are illustrated via simulation and three empirical applications: (i) bimodal and right-skewed income distributions, (ii) asymmetric spectral features in astrophysical datasets, and (iii) heterogeneous survival time data in biomedical contexts. In each case, FMMs substantially outperform symmetric alternatives in goodness-of-fit and predictive accuracy. Our results underscore that FMMs are not merely computational tools but mathematically rich mechanisms for quantifying departures from symmetry, bridging classical statistical analysis with modern learning.



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Sciforum-190535: Establishment of Fixed-Point Results via a Triplet of Auxiliary Functions in Relational Partial Metric Spaces

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This paper establishes fixed-point results for self-mappings on partial metric spaces endowed with a binary relation. The study is motivated by relation-theoretic fixed-point theory and by recent contractive conditions involving a triplet of auxiliary functions. We introduce a relational (ψ, φ, θ)-contractive condition into the framework of partial metric spaces and use suitable notions of R-completeness, R-continuity, and R-self-closedness. Under the assumptions that the relation is T-closed and locally T-transitive, we prove an existence theorem for fixed points. A uniqueness result is also obtained by imposing a finite relational path condition between fixed points. The results extend and unify several known fixed-point theorems, including relation-theoretic contractions, auxiliary-function contractions, and partial-metric fixed-point results. Corollaries are presented to show how earlier metric and partial-metric results can be recovered as special cases. Finally, an application to a Fredholm integral equation of the second kind is discussed, showing how the proposed framework can be used to establish existence and uniqueness of solutions for nonlinear integral equations. The work contributes to the further development of relation-theoretic fixed-point theory in generalized metric structures and provides a basis for future studies involving coincidence results, matrix equations, and other generalized metric-type spaces. These directions naturally follow from the theoretical framework developed in this study.



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Sciforum-190730: Grouped Robust Functional Time Series Forecasting Model of Mortality Rates: A case study of Matlab Health and Demographic Surveillance System, Bangladesh

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Age-specific mortality rates in low- and middle-income countries (LMICs) are often disaggregated by sex, requiring forecasts that maintain hierarchical coherency while respecting biological constraints. We applied robust grouped functional time series forecasting methods to 40 years' (1982-2021) worth of mortality data from the Matlab HDSS in Bangladesh, extending the Shang and Hyndman (2017) framework with robust statistical techniques suited to demographic surveillance data. Our approach combines: (1) empirically determined monotonicity constraints ensuring biologically plausible age-mortality profiles, (2) robust functional principal component analysis (RFPCA) with L1-median centering and MCD -nspired outlier detection, and (3) hierarchical forecast reconciliation via bottom-up and optimal combination methods ensuring male and female forecasts aggregate to total population projections. A total of eight B-spline basis functions with an monotonic mortality threshold of 17 years is empirically determined and applied. MCD procedures identified 10% of years as anomalous and selected K=5 for male and K=4 for female PC, explaining $>$95% of historical variance. Exponential smoothing models dominated univariate forecasts of component scores. All reconciliation methods produced almost identical results due to naturally coherent sex-specific forecasts in this simple hierarchical structure. The bootstrap 80% PI produced an overall (average of 10 horizons) mean interval width of 0.4674 and 0.5943 for males and females, respectively, at the log scale. The point forecast accuracy yielded mean (average of 5 horizons) MAFEs of 13.44, 16.06, and 19.35 units for the total, male, and female populations, respectively, which produced a 64.3% error reduction compared to the naïve moving median method. We found monotonic deterioration with forecast horizon across all three series. Total population MIS rises 41%, from 59.13 at h = 1 to 83.36 at h = 5. MIS rises by 38% for males and 31% for females. Empirical coverage assessment yielded 84.9%, 75.0%, and 71.7% coverage for the total, male, and female populations (target: 80%). COVID-19 pandemic impact analysis revealed mortality declines of 16.4%, 13.8%, and 19.3% for the total, male, and female populations during 2020-2021. The permutation tests found no significant functional differences (p=0.094-0.181) across only 2 post-pandemic years. This approach demonstrates that grouped functional time series forecasting enhanced with robust techniques provides a coherent framework for demographic projections in surveillance settings, with implications for the INDEPTH network's more than 50 HDSS sites globally.



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Sciforum-190812: Optimization of Laser Cutting Parameters Considering Quality and Sustainability

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Recently, manufacturing industries have increasingly prioritized efficiency, quality improvement, and sustainability in response to growing environmental concerns and competitive market conditions. Achieving these strategic objectives requires the optimization of industrial processes by balancing production quality, operational cost, and environmental performance simultaneously. Among advanced manufacturing technologies, CO_2 laser cutting has become widely adopted due to its high precision, flexibility, and capability to process various materials efficiently. However, the optimization of laser cutting parameters remains a critical challenge because process variables directly influence product quality, energy consumption, and operational sustainability. This study investigates the optimization of CO_2 laser cutting parameters to enhance quality and sustainability in industrial manufacturing. Using the Design of Experiments (DOE) methodology and statistical analysis through ANOVA, the study evaluates the impact of laser power, cutting speed, gas pressure, focal point position, and nozzle diameter on critical outcomes such as kerf width, surface roughness, and operating cost. The results show that increasing the laser power resulted in a wider kerf due to higher energy density, affecting precision. The findings further indicate that smoothest surfaces are achieved at moderate laser power and cutting speed, with higher nitrogen pressure. In addition, operating costs decreased with higher cutting speeds and optimized nitrogen pressures



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Sciforum-190813: An Experimental and Statistical Approach to Optimize Filtration Rate in a Chemical Process

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Industrial processes face increasing pressure to enhance efficiency while maintaining high-quality standards. In the chemical industry, filtration is a critical step that directly influences both the quality of the final product and operational costs. Therefore, optimizing the filtration rate has become a key focus to ensure competitive production. Filtration process efficiency can be influenced by several factors, such as temperature, pressure, and agitation speed. However, the precise impact and interactions of these factors are often poorly understood, resulting in performance variability and costly inefficiencies. Identifying these critical parameters and their interactions is essential to ensure optimal process control. This study aims to identify the key factors affecting filtration rates using a rigorous experimental methodology and quantify their impacts through a detailed statistical analysis, including Analysis of Variance (ANOVA). In particular, a full-factorial experimental design with two levels is employed to study the main effects and interactions of four critical factors, temperature, pressure, formaldehyde concentration, and agitation speed. The experimental results are then analyzed using ANOVA to determine significant contributions. The findings indicate that filtration performance is strongly affected by both individual process parameters and their combined interactions. Experimental results reveal that optimizing temperature and pressure conditions significantly improves filtration efficiency, while appropriate agitation speed contributes to process stability and reduced filtration time. In addition, controlling formaldehyde concentration was observed to have a direct impact on operational consistency and product quality. The optimized parameter combination achieved higher filtration rates together with lower energy consumption and reduced process waste.



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Sciforum-190839: Functional Data Analysis of Sentinel-2 Derived Vegetative Phenology for Potato Yield Estimation Under Normal and Cyclone Conditions

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Sentinel-2 derived vegetative phenology indicators, such as the Normalized Difference Vegetation Index (NDVI), have been effective at predicting potato yields, but earlier approaches relied on a single approach, ignoring the whole seasonal pattern. In this study, we used functional data analysis (FDA) on NDVI time series from 47 smallholder fields across two seasons: one affected by a cyclone in 2021–22 and the other a normal season in 2022–23. Cyclone Jawad brought 137 mm of rain in early December 2021, more than twice the usual rainfall, causing delays in sowing. The NDVI data were collected across both seasons and then smoothed separately using penalized cubic B-splines. A permutation-based functional t-test showed that NDVI patterns differed significantly between the two seasons, especially early in the season, suggesting that crop establishment was delayed in 2021–22. The scalar-on-function regression (SoFR) models for each season, using a functional generalized additive model (FGAM), were applied to assess whether the two patterns had significant effects on yield. Adjusted R^2=0.238 with p=0.016 for the 2021–22 season and adjusted R^2=0.283 with p=0.013 for the 2022-23 season revealed that the full NDVI pattern was a significant predictor of yield. In 2021–22, yield increased steadily with NDVI, whereas in 2022–23, yield peaked during mid-season canopy closure. A bootstrap test confirmed that these differences in the relationship between NDVI and yield were significant, suggesting that models trained in normal years may not work well in years with extreme weather. The FDA provides a way to detect and measure these changes, which can help improve remote-sensing yield forecasts in regions affected by climate change.



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Sciforum-196129: A Jacobian-Based Generalization of Volume of a Solid of Revolution Along an Oblique Axis

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Traditional calculus methods for solids of revolution are restricted to vertical or horizontal axes, leaving a gap in analytical rigor and practical application when dealing with oblique rotations. This study aimed to formalize the computation of volumes of solids of revolution along oblique axes by introducing Jacobian-based scaling factors that validate earlier geometric approaches and extend their utility to construction engineering. Using a generalized Jacobian algorithm, distinct factors were derived for the Disk Method (longitudinal scaling) and Shell Method (radial orientation), and applied to a benchmark problem involving parabolic boundaries rotated about oblique lines. Results confirmed that the Disk Jacobian functions as a slope factor correcting for axis stretch, while the Shell Jacobian ensures layering accuracy in curved structures, with both methods converging to identical outcomes and guaranteeing 100% volumetric consistency. Engineering applications demonstrated improved precision in material estimation for slanted members and complex forms, reducing resource waste and supporting sustainable practices. By bridging theoretical calculus with applied engineering, the framework advances SDG 9 (Industry, Innovation, and Infrastructure) through mathematical innovation and SDG 12 (Responsible Consumption and Production) by optimizing resource efficiency. This analytical contribution elevates oblique-axis rotations from geometric intuition to a reliable computational standard, offering professionals a generalizable tool for sustainable infrastructure design.



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Sciforum-197265: Coupled critical growth for anisotropic systems

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Introduction: Differential inclusions arising from nonsmooth energy functionals provide a natural setting for variational problems where the governing operators lack differentiability or strict monotonicity. While the isotropic case with variable exponents and the scalar anisotropic case have each been studied separately, genuinely anisotropic systems, in which two inclusions are coupled and each is driven by its own directional growth, have remained largely unexplored. This work proposes a unified variational framework that addresses both obstacles at once and establishes the existence of generalized solutions.

Methods: Working in anisotropic variable-exponent Sobolev spaces, we overcome the lack of monotonicity for the above operators using a discretized Galerkin scheme combined with Clarke's nonsmooth analysis. The growth hypothesis on F is structured in coupled form: the cross-exponents linking the two growth components are built through the conjugates of the proper exponents in the inclusions, so that the generalized Young inequality redistributes each coupling term exactly onto the dominant term of the coupled one.

Results: The central contribution is the treatment of a genuine anisotropic system: two coupled inclusions, each governed by its own directional anisotropy. Each component lives in a distinct anisotropic Sobolev space, with its own critical threshold set by the harmonic mean of its directional exponents. Under an explicit quantitative threshold on the embedding constants, we prove that, in the context of finite-dimensional spaces, the energy functional is coercive and admits a bounded sequence of Galerkin minimizers. Thereby, by passing to the limit, we reach a generalized solution of the system in the competing regime where both ellipticity and monotonicity assumptions are relaxed.

Conclusions: The proposed framework unifies and extends both isotropic problems and scalar anisotropic problems in previous works, hence improving the features of the model problem.



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Sciforum-199569: Mathematical and Statistical Analysis of Symmetry-Breaking Spectra in Raster Scalar Fields: A Reproducible Framework Applied to Digital Elevation Models of Volcanic Cones

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Introduction: We develop a reproducible mathematical and statistical framework for quantifying symmetry breaking in discretised raster scalar fields, with digital elevation models of volcanic cones used as controlled topographic applications. The aim is to formalise, compute and test symmetry-breaking spectra on real elevation surfaces.

Methods — Shuttle Radar Topography Mission 30 m digital elevation models, extracted through Google Earth Engine, were used for three Virunga volcanic cones: Nyiragongo, Mikeno and Karisimbi. The workflow was implemented in Python using NumPy, SciPy, Rasterio, Pandas and Matplotlib. Each raster was reprojected to UTM zone 35S, centred on a morphologically defined summit or crater, cropped to a square domain and normalised as a scalar field. Four finite transformation groups were applied: the cyclic rotation group of order two (C2), the cyclic rotation group of order four (C4), the four-element dihedral group (D2) and the eight-element dihedral group of the square (D4). For each group, an orbit-averaged invariant projection was computed, and symmetry breaking was measured as the ratio between quadratic residual energy and total field energy. Global spectra, sliding-window spectra, multi-resolution tests and Monte Carlo phase-randomised null models were derived.

Results: For Nyiragongo, the global breaking spectrum was C2 = 0.1327, D2 = 0.1880, C4 = 0.2020 and D4 = 0.2056. These scores were below the corresponding phase-null means, from 0.5060 to 0.7677, with p-values from 0.0067 to 0.0133. Local analysis also ranked C2 as the least-broken group, with mean scores decreasing from 0.9022 at 1.92 km to 0.6829 at 7.68 km. Mikeno confirmed transferability, with C2 = 0.1048 and p = 0.025. Karisimbi produced a contrasting case, with C2 = 0.2926 and p = 0.140.

Conclusions: The pipeline provides a reproducible mathematical/statistical framework for estimating, mapping and testing symmetry-breaking spectra in volcanic-cone topography.



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Sciforum-200431: A Symmetry-Regularized Fuzzy Graph Learning Framework for Spatio-Temporal Water Quality Prediction

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Water quality prediction is essential for sustainable environmental management, yet many existing machine learning models remain limited by weak interpretability, insufficient spatio-temporal representation, and reduced robustness under noisy or incomplete sensor measurements. Recent studies have also emphasized the need for adaptive, explainable, and real-time water-quality monitoring systems. In this work, we propose a symmetry-regularized fuzzy graph learning framework for forecasting and classifying water quality. The proposed approach represents monitoring stations as graph nodes, while hydrological, geographical, or similarity-based relationships define the graph edges. Standard physicochemical variables, including pH, dissolved oxygen, biochemical oxygen demand, chemical oxygen demand, total suspended solids, and ammoniacal nitrogen, are used as model inputs. The central idea is to incorporate symmetry principles into the learning process, so that predictions remain consistent under sensor reordering, spatially similar station behavior, seasonal cyclic patterns, and small perturbations in correlated water-quality variables. A fuzzy classification layer is further introduced to handle uncertainty near water-quality class boundaries. The proposed framework is designed to predict both continuous water quality index values and categorical pollution classes. This study provides a new symmetry-guided direction for intelligent water-quality monitoring by integrating graph learning, fuzzy reasoning, and symmetry-consistency regularization. The proposed model has potential to improve robustness, transparency, and decision support in environmental water management.



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Sciforum-200902: Topological Characterization and Statistical Entropy Modeling of Hexagonal Graphene Nanostructures

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The mathematical characterization and thermodynamic modeling of graphene nanostructures present significant analytical challenges in mathematical chemistry, particularly in mapping discrete structural symmetries to macroscopic physical properties. This paper focuses on the quantitative structural analysis of hexagonal graphene configurations to establish a formal statistical framework correlating their molecular graph architecture with physical entropy. To achieve this objective, a comprehensive and diverse set of topological indices—specifically focusing on degree-based, distance-based, and information-theoretic descriptors—is employed to rigorously evaluate the structural complexity, connectivity, internal symmetry variations, and boundary features of unique graphene configurations. By treating these nanostructures as discrete molecular graphs, we compute stable mathematical invariants that capture subtle topological variations across the entire structural landscape. Furthermore, advanced statistical analysis, multi-variable regression modeling, and rigorous analysis of variance are applied to investigate the direct analytical dependencies between these calculated topological descriptors and the physical entropy of the corresponding carbon networks. The analytical results reveal remarkably strong statistical correlations, demonstrating that discrete invariants derived purely from graph topology and network symmetry possess exceptionally high predictive power for complex thermodynamic behavior. Ultimately, this study highlights the profound utility of quantitative mathematical analysis as a robust tool in mathematical chemistry, providing an efficient predictive method to interpret, bound, and estimate physical entropy within extended polycyclic aromatic systems without relying on computationally expensive quantum mechanical calculations.



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Session 7. Crystalline Metals and Alloys

Sciforum-170920: Investigating Prime Clustering and Prime Gaps via the Pathak Continuum Compression Hypothesis (PCCH)

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Prime numbers have captured mathematicians' attention for many centuries because their distribution shows no predictable pattern and they exhibit unexpected clustering behavior along the number line. While traditional number theory allows for prime counting and density estimation, it fails to provide a complete explanation of the mechanisms that cause prime numbers to cluster together and create spaces between them. In this paper we present the Pathak Continuum Compression Hypothesis (PCCH) as a new framework which derives its foundation from the Pathak Theory of NumberInteraction (PTNI). The theory describes how numbers interact to create a continuous number line which experiences both compressive and expansive forces. The PCCH method uses a compression function to quantify these interactions, which tracks local continuum compression and stretching, enabling an understandable yet mathematically sound demonstration of how prime clusters and prime gaps form. The study demonstrates that negative compression scores strongly associate with prime numbers, while positive compression scores correspond to composite numbers, indicating thatprimes develop in areas where the numerical continuum experiences local compression. This research establishes a unified framework which connects number interaction strength to prime distribution, thereby enabling future theoretical developments and predictive modeling efforts. The research results show that prime distributions display non-random patterns which result from interactions that cause continuum deformation in combination with prime clustering behavior and gap formation.



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Sciforum-177996: Lie Symmetry Analysis of the Duffing Oscillator for Nonlinear Vibration Control and Stability in Mechanical Systems

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Nonlinear vibration effects commonly occur in mechanical engineering systems like flexible structures, rotating machinery and micro-mechanical devices. Precise evaluation and regulation of these vibrations are vital for ensuring systems stability and enhancing mechanical efficiency. Nonetheless, the nonlinear differential equations that regulate vibration dynamics are frequently challenging to solve with standard analytical techniques. Mathematical methods grounded in symmetry offer a strong alternative for simplifying intricate systems while maintaining their fundamental physical characteristics.

This research examines the use of Lie symmetry analysis in nonlinear vibration models, specifically targeting the Duffing oscillator, a commonly utilized nonlinear model in mechanical engineering, representing systems with nonlinear stiffness properties. The study creates a symmetry reduction framework utilizing Lie group transformations on the governing nonlinear ordinary differential equation of the Duffing system. Through the identification of invariant transformations and symmetry generators, the original nonlinear equation can be systematically simplified, facilitating the derivation of invariant solutions and reduced-order dynamical models.

The suggested framework enhances the analytical comprehension of nonlinear vibration characteristics, encompassing resonance phenomena, equilibrium positions and stability criteria. By employing symmetry-based reduction, the intricate dynamics of the Duffing oscillator can be examined more effectively, shedding light on the effects of nonlinear stiffness, damping parameters and external forces on system stability. These results also aid in the development of vibration control strategies for mechanical systems where effects are important.

The findings indicate that Lie symmetry methods provide a methodical mathematical strategy to simplify nonlinear vibration models while maintaining their structural characteristics. This study emphasizes the significance of symmetry techniques in applied mathematics and engineering evaluations and adds to the current research on symmetry-oriented modelling of dynamic systems.



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Sciforum-183670: Fractional-Order Memory Effects and Chaotic Dynamics in Nonlinear Delay Systems: Computational Analysis of Bifurcation and Multi-Scroll Attractors

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Introduction: This paper investigates chaos, bifurcation, and control in a nonlinear two-delay differential equation of the formwhere , and are two distinct time delays encoding memory effects. **Main: Definition[Equilibrium Point]:**A point is an **equilibrium** of the system if . It is **asymptotically stable** if every trajectory starting near converges to it as . **Definition[Lyapunov Exponent]:**The **maximal Lyapunov exponent** quantifies the average exponential rate of divergence of nearbytrajectories. Chaos is confirmed when . The system is extended to the fractional-order setting via the Caputo derivative **Theorem[Persistence of Chaos]:**There exist parameter values and fractional order such that the fractional system exhibits chaotic behavior, even when the integer-order system is stable. **Theorem[Caos Control via Linear Feedback]:**Adding the feedback term for sufficiently large stabilizes the system to .Moreover, a delay-independent condition for master--slave synchronization is derived using linear state feedback control. **Conclusion:**

Numerical simulations via Lyapunov exponents and phase portraits confirm multi-scroll chaotic attractors and validate all analytical results. The fractional order acts as a pivotal control parameter governing the transition between stable, periodic, and chaotic regimes, demonstrating that memory depth directly modulates the complexity of the system's dynamical behavior. Notably, the interaction between the two time delays and the fractional memory kernel produces richer bifurcation structures than those observed in either pure delay or pure fractional systems individually. The proposed linear feedback control strategy proves robust across a wide range of fractional orders, confirming its practical applicability in engineering and physical systems governed by hereditary dynamics. Furthermore, the derived delay-independent synchronization condition offers a computationally efficient criterion applicable to coupled nonlinear fractional networks. These findings collectively suggest that fractional-order two-delay systems constitute a fertile mathematical framework for modeling complex physical phenomena, with promising future directions in viscoelastic mechanics, neural dynamics, and anomalous diffusion processes in computational physics.



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Sciforum-184009: Image Recognition System

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Image recognition systems have revolutionized computer vision applications across diverse domains, from autonomous vehicles to medical diagnostics, fundamentally transforming how machines perceive and interpret visual information. This comprehensive survey examines the evolution, architectures, and applications of modern image recognition systems powered by deep learning technologies. We provide an in-depth analysis of fundamental approaches including Convolutional Neural Net-works (CNNs), transfer learning methodologies, and state-of-the-art architectures such as ResNet, VGG, Inception, EfficientNet, and Vision Transformers. The paper systematically discusses key challenges including dataset bias, computational requirements, adversarial robustness, real-time processing constraints, and interpretability concerns. We present comprehensive comparative performance analysis across different architectures, evaluating accuracy, computational complexity, memory requirements, and their applicability to various domains including medical imaging, autonomous systems, security applications, retail, and precision agriculture. Additionally, we explore emerging trends including few-shot learning, self-supervised pre-training, explainable AI for image recognition, edge computing implementations, and privacy-preserving techniques. Our extensive evaluation reveals that while deep learning has achieved remarkable accuracy rates exceeding 95% on benchmark datasets such as ImageNet, CIFAR, and COCO, significant opportunities remain in addressing robustness to adversarial attacks, model interpretability, generalization across diverse real-world scenarios, computational efficiency for edge deployment, and ethical considerations regarding bias and fairness in automated decision-making systems.



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Sciforum-186086: Numerical Investigation of Chromatographic Models with a Tri-Langmuir Isotherm

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Chromatography is an essential analytical tool in pharmaceuticals, chemical engineering, and environmental science for separating and analyzing complex mixtures. As systems grow more complex, accurate modeling becomes important. Equilibrium models like the Equilibrium Dispersive Model (EDM) neglect key dynamics such as mass transfer resistance and kinetic adsorption, limiting their applicability in fast, nonlinear systems.

This study conducts a numerical investigation of non-equilibrium liquid chromatography using the Lumped Kinetic Model (LKM) with linear and non-linear Tri-Langmuir isotherms. The LKM captures finite mass transfer kinetics between mobile and stationary phases, making it suitable for dynamic conditions. The governing transport equations exhibit conservative and scaling symmetry properties under dimensionless transformation, while the nonlinear adsorption mechanism reflects heterogeneous yet structured interactions among adsorption sites. Koren's high-resolution finite volume method (HR-FVM) is extended to solve the governing equations, preserving the symmetric structure of concentration transport while effectively handling convection-dominated behavior and steep solution gradients.

Simulations explore the impact of mass transfer rate, interstitial velocity, and injection time on solute elution. Results show that the Tri-Langmuir isotherm best models heterogeneous adsorption sites, especially in multi-component systems, capturing nonlinear effects like peak asymmetry, saturation, and competitive binding. Effects of different parameters such as mass transfer coefficient, adsorption coefficients and injection time are considered for single- and multi-component systems.

The study underscores the value of kinetic models and nonlinear isotherms for accurate simulation. Furthermore, the preservation of conservative transport symmetry within the numerical framework improves the stability and physical consistency of the computed solutions, aiding chromatographic design and optimization while reducing reliance on experimental methods.



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Sciforum-189012: Discrete-Time Eco-Evolutionary Dynamics: Stability, Bifurcations, and Co-evolutionary Trajectories

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In this talk, we explore the mathematical modeling of discrete-time eco-evolutionary systems, focusing on the fundamental interplay between population dynamics and the evolution of phenotypic traits. Eco-evolutionary models have become an important framework for understanding how demographic processes and natural selection interact across generations, particularly in species with non-overlapping generations or seasonal reproduction. The emphasis is placed on deterministic models, both autonomous and periodic, where the coupling between demography and selection is described through systems of nonlinear difference equations.

The analysis begins with the study of the local stability of equilibria (fixed points) using the Jacobian formalism together with the localization of eigenvalues relative to the unit circle through Jury stability conditions. These techniques provide a natural framework for identifying persistence thresholds and qualitative transitions in the long-term dynamics of populations. We then discuss the principal bifurcations governing these systems, namely the transcritical bifurcation, associated with changes in persistence and extinction states, and the period-doubling bifurcation, which signals the emergence of oscillatory and potentially chaotic regimes. Particular attention is devoted to the Neimark--Sacker bifurcation, describing the transition from stationary eco-evolutionary equilibria to quasi-periodic oscillations, reflecting persistent fluctuations in both population abundance and evolving traits.

In the context of seasonal environments, we introduce the analysis of periodic systems via map composition and discrete Floquet theory. Finally, we discuss how trait diversity and coadaptation mechanisms may influence the robustness of eco-evolutionary systems, potentially stabilizing or destabilizing long-term population dynamics.



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Sciforum-189142: Optimising Intelligent Transportation Networks Using Fuzzy Directed Graph

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This paper presents a novel approach for optimising Intelligent Transportation Networks (ITN) using fuzzy digraph products combined with domination number and arborescence weight concepts. Uncertainty factors that may include traffic jams, variability in travel time, accidents, and variable road situations impact effective decision-making and performance within an intelligent transportation system. These uncertainties have been captured by representing the transportation network as a fuzzy directed graph, where nodes depict transportation centres, subway stations, or even junction points, and arcs are directional fuzzy relationships that represent traffic movement and strength. The fuzzy digraph modular product will be employed to effectively merge different transportation sub-networks for analysis. The use of the domination number will help in identifying the most effective points for control and surveillance. Moreover, the arborescence weight is employed to locate the least-cost spanning tree that connects all important nodes while minimising travel costs, traffic, and expenses. The incorporation of the domination number and arborescence weight in fuzzy digraph products serves as an effective approach to improving connectivity, avoiding redundancy, and increasing route reliability in unpredictable traffic situations. The developed approach facilitates intelligent traffic management decisions for the design of transportation systems in metro cities. The suggested approach helps develop scalable and sustainable transportation networks for future smart transportation applications.



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Sciforum-189310: Symmetry Properties and Chaotic Dynamics of Trisenne–Lucas Numbers

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The main objective of this paper is to introduce a new class of sequences called the **Trisenne–Lucas numbers** {m_n^(3)}n∈N. These sequences extend classical Lucas-type structures and provide a new framework for studying discrete dynamical systems and special number sequences. We begin by establishing their recurrence relation and investigate the existence and uniqueness of the sequence generated by this relation. Based on this construction, we derive several fundamental properties, including Binet's formula and the generating function, which allow an explicit representation of the terms of the sequence.

Furthermore, we study the associated **symmetric functions** and explore their role in describing algebraic structures underlying the sequence. Explicit formulas and a variety of summation identities involving the Trisenne–Lucas numbers are also presented, providing deeper insight into their combinatorial behavior. In addition, we examine products of Trisenne–Lucas numbers with certain second-order sequences, revealing interactions between different recursive structures.

Finally, we investigate the dynamical aspects of the Trisenne–Lucas map. In particular, we analyze its chaotic behavior using bifurcation diagrams and Lyapunov exponents, highlighting rich nonlinear dynamics and sensitivity to parameter variations. These results extend classical studies of Mersenne- and Lucas-type sequences and reveal new connections between discrete mathematics, symmetric function theory, and dynamical systems. The proposed framework opens new directions for further research in both algebraic structures and chaos theory.



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Sciforum-189470: Reconstruction of f(G) Gravity with Tsallis Holographic Dark Energy Using Granda–Oliveros Cutoff and Hybrid Scale Factor

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This work presents a reconstruction framework of modified gravity in the presence of Tsallis holographic dark energy (THDE) by employing the Granda–Oliveros infrared cutoff together with a hybrid scale factor describing cosmic evolution. The hybrid scale factor successfully characterizes the transition from an early decelerated phase to the present accelerated expansion of the Universe, making the model suitable for studying realistic cosmological dynamics. Using the modified Friedmann equations in gravity, the corresponding reconstructed form of the gravitational function is obtained and analyzed in detail. The evolutionary behavior of the reconstructed equation of state (EoS) parameter is investigated to understand the nature of dark energy and the accelerated expansion scenario. The analysis shows that the EoS parameter exhibits quintessence and phantom-like behavior depending on the model parameters, indicating a rich cosmological structure.

Further, the physical viability of the reconstructed model is examined through stability analysis using the squared sound speed and the adiabatic index. In addition, the thermodynamic consistency of the reconstructed THDE model is tested through the generalized second law (GSL) of thermodynamics. The validity of the GSL throughout cosmic evolution confirms the thermodynamic feasibility of the model. Overall, the reconstructed gravity model with Tsallis holographic dark energy and Granda–Oliveros cutoff within the hybrid scale factor framework provides a consistent and stable description of the late-time accelerating Universe compatible with thermodynamic principles.



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Sciforum-189759: A Localized Meshless Method for a Two-Dimensional Chemotaxis Model

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Chemotaxis is the directed movement of cells or organisms in response to chemical gradients, a process critical to biological functions such as immune responses and bacterial foraging. It is typically modeled using the Keller–Segel system, comprising a set of nonlinear advection–diffusion–reaction partial differential equations (PDEs). Numerically solving these models is challenging. Traditional finite difference methods are largely restricted to simple rectangular domains, while finite element methods, although capable of handling complex geometries, depend on computationally expensive mesh generation.

This work proposes a localized meshless method designed to overcome these limitations. The technique completely avoids mesh generation, making it equally effective for irregular domains with both uniform and non-uniform node distributions. Our problem-dependent approach uses the Laplacian as the main differential operator to derive radial basis function (RBF)-based particular solutions. These solutions are then used to approximate the spatial operators in the time-dependent PDE, thereby reducing the problem to a system of ordinary differential equations (ODEs). This resulting ODE system can be efficiently integrated in time using standard, high-performance solvers.

The proposed meshless framework is validated through numerical simulations of the Keller–Segel chemotaxis model. The results confirm the method's competitive performance in terms of numerical accuracy and computational efficiency, demonstrating its strong potential as a flexible alternative for solving complex biological PDEs.



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Sciforum-190157: Structure-Preserving Numerical Schemes for Nonlinear Adsorption Models with Mass Conservation

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Introduction: Nonlinear adsorption models with a conserved total amount are often solved numerically before parameter estimation or model comparison. However, a numerical approximation can be misleading if it does not respect the admissible domain defined by non-negativity and mass conservation. This work studies structure-preserving time discretisation for mass-conserved adsorption dynamics from an applied-mathematics viewpoint.

Methods: We consider a generic batch adsorption model written as dq/dt = F(q,s;theta), ds/dt = -F(q,s;theta), with q(t) denoting the adsorbed phase, s(t) the remaining solute phase, and s(t) + q(t) = m. The conservation law is used to reduce the dynamics to a scalar problem on the interval 0 <= q <= m. Standard one-step discretisations are compared with reduced-variable and projected formulations, using conservation, positivity, and equilibrium admissibility as structural criteria.

Results: The reduced formulation preserves the linear invariant exactly at the discrete level because s is reconstructed as m - q after each update. We also derive an admissibility condition for explicit updates that keeps q_{n+1} inside [0,m], and we identify the situations in which projection is needed to prevent nonphysical iterates. These results separate mass-conservation preservation, which can be imposed algebraically, from positivity preservation, which depends on the flux and the time step.

Conclusions: Structure-preserving discretisation provides a compact framework for simulating nonlinear adsorption kinetics without introducing numerical artefacts that may affect later sensitivity analysis, parameter estimation, or model selection.



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Sciforum-190175: Modeling and Numerical Analysis of Rabies Transmission between Dogs and Humans using Euler and RK4 Methods

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A mathematical SEIR compartmental model is presented to study the transmission dynamics of rabies between dog and human populations. The model proposes dividing the two populations into susceptible, exposed, infected, and recovered compartments, to describe the spread and progression of the disease. The interaction between the epidemiological classes and transmission process of rabies is modelled by a system of nonlinear ordinary differential equations. In order to investigate the effectiveness of both numerical methods to solve the system accurately, the developed model is analyzed numerically with the Euler method and the Fourth-Order Runge–Kutta (RK4) method. Appropriate parameter values are taken and the time step is fixed to allow the numerical simulations to study the behavior of the disease dynamics. The accuracy, stability, convergence and numerical errors of both methods are checked and compared graphically and numerically. The simulation results demonstrate that the RK4 method has more stable and accurate approximations with much smaller numerical error than the Euler method, especially for larger step sizes and long-time simulation. The results show that RK4 is more stable and efficient for solving the nonlinear epidemiological models of rabies transmission. In this study, the dynamics of rabies is mathematically described and the significance of choosing appropriate numerical techniques for disease modelling, prediction and analysis of appropriate control strategies in epidemiological research is highlighted.



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Sciforum-190191: Analysis of a Fractional-Order Hepatocyte Model of Viral Dynamics Combining Direct-Acting Antiviral Therapy with Alcohol Effects

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In this modern world, many people are affected by liver diseases, with the most dangerous strains being hepatitis c virus and hepatitis b virus. This article explores the interaction between hepatocytes and within-host viral dynamics, along with the effectiveness of alcohol and direct-acting antiviral (DAA) medication using fractional derivative formulation. The mathematical model is developed to include memory and genetic factors related to immune responses and persistent viral infections using the Caputo fractional derivative. We establish the existence, uniqueness, positivity, and boundness of the solutions and also prove Hyers–Ulam stability. The next-generation matrix technique is used to determine the reproduction number and the disease-free equilibrium. The qualitative behaviour of the model is examined via the Castillo-Chavez bifurcation theorem, while local stability is analysed using fractional-order stability criteria and also global asymptotic stability is developed by explicitly obtaining the endemic equilibrium. Additionally, we discover that the important factors influencing immune regulation and viral persistence are identified by sensitivity analysis of the reproduction number. Numerical simulations show that the disease decreases by varying the effects of parameters and also show how efficient viral infection management and improved recovery dynamics can possibly be carried out by combining proper treatment procedures with reduced alcohol usage.



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Sciforum-190344: Metaheuristic Optimization and ANFIS-Driven Performance Analysis of a $MAP/PH/2$ Retrial Queue with Differentiated Breakdowns, Working Vacations, Vacation Interruptions, and N-Policy

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This paper studies a MAP/PH/2MAP/PH/2MAP/PH/2 retrial queueing system incorporating differentiated breakdowns, working vacations, vacation interruptions, and an NNN policy. Customer arrivals follow a Markovian Arrival Process (MAP), while service times are assumed to follow PH distributions, which allow a flexible representation of various stochastic behaviors. When an arriving customer finds both servers occupied, the customer enters an orbit and attempts to obtain service again after a random retrial time.The system consists of two heterogeneous servers. Server~1 remains continuously available, whereas Server~2 operates under a working vacation policy. During a working vacation, the server continues to provide service, and the vacation is interrupted whenever the number of customers in the system reaches a predetermined threshold level NNN. The server then switches to the normal operating mode and continues service until the system becomes empty, after which another working vacation begins.The system dynamics are formulated as a continuous-time Markov chain with a QBD structure. Using matrix analytic methods and steady state probabilities, key performance measures such as orbit size and server utilization are obtained. Numerical experiments are performed to demonstrate the validity of the analytical results. Furthermore, an ANFIS is used to compare and validate the outcomes obtained from the analytical model. Finally, a cost function is formulated and optimized using metaheuristic approaches such as PSO, CSA, and BBO to identify efficient system configurations.



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Sciforum-190648: Quantifying Operational Symmetry Breaking in Battery Energy Storage Scheduling under Synthetic Renewable Profiles

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Introduction:

Battery energy storage system scheduling is commonly assessed through cost, emissions, feasibility, and grid exchange. However, the temporal structure of battery operation remains poorly quantified. In particular, the symmetry between charging and discharging actions across a daily horizon is rarely used to interpret storage behavior. This study addresses this gap by proposing symmetry indices for battery power and state-of-charge trajectories. The novelty of the study is the formulation of battery scheduling as a symmetry-breaking problem, where external signals and technical limits reshape charge–discharge balance.

Methods:

A 24 h single-node renewable energy system was simulated using synthetic demand, photovoltaic generation, electricity price, and carbon emission profiles. The battery model included state-of-charge limits, charging and discharging power limits, charge and discharge efficiencies, and end-of-day state-of-charge restoration. Five scenarios were optimized using PSO: energy balance operation, cost minimization, emission minimization, asymmetric efficiency, and restricted power exchange. The schedules were evaluated using total cost, emissions, grid imports and exports, battery throughput, grid exchange RMS, and two symmetry indices.

Results:

Cost minimization achieved the lowest operating cost, USD 109.57, representing an 8.02% reduction compared with energy balance operation, although emissions increased to 336.87 kgCO2. Emission minimization obtained the lowest emissions, 327.84 kgCO2, with a cost of USD 114.20. Restricted power exchange produced the highest power symmetry index, 0.324, and emissions of 331.49 kgCO2, with a cost only 1.21% above the cost minimum. Asymmetric efficiency yielded the highest state-of-charge symmetry index, 0.316. Energy balance operation showed the lowest battery throughput (219.45 kWh) and grid exchange RMS (34.15 kW), as well as the lowest symmetry indices.

Conclusions:

These results show that battery operational symmetry depends on objective functions, efficiency assumptions, and power limits. The proposed indices offer a simple metric to interpret how economic, environmental, and technical constraints reshape daily storage behavior.



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Sciforum-190650: Enhancing the Multiverse Optimizer through Symmetry-Guided Search Operators

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Introduction:

Metaheuristic algorithms are widely used to solve nonlinear optimization problems, but their search dynamics can still suffer from premature convergence and limited exploration of regions outside the search domain. This study proposes a Symmetry-Guided Multiverse Optimizer (SGMVO) in which candidate universes generate reflected alternatives around the search space center, the current best solution, and selected mirrored dimensions. The novelty lies in using symmetry as an active search operator rather than as a post-processing descriptor.

Methods:

The SGMVO was benchmarked against GA, PSO, and the standard Multiverse Optimizer. A small-scale experimental campaign was conducted using five continuous objective functions: Sphere, Rosenbrock, Rastrigin, Ackley, and Griewank. All algorithms were evaluated under the same conditions: 30 dimensions, 40 search agents, 250 iterations, and 30 independent runs. In the SGMVO, each candidate universe generates symmetric alternatives through reflections about the search space center and the current best universe, as well as partial coordinate mirroring. These reflected candidates compete with the original solution, and the best one is retained for the next iteration. This mechanism increases exploration of opposite regions and improves local refinement near promising solutions. Performance was assessed using the best, mean, median, worst, standard deviation, execution time, and convergence behavior.

Results:

The SGMVO obtained the best single-run solution across four functions: Sphere, Rosenbrock, Ackley, and Griewank. Regarding mean performance, the SGMVO achieved the best values on Sphere (0.0490) and Griewank (0.2216). Compared with standard MVOs, the SGMVO reduced the mean objective value by 98.69% in Sphere, 78.53% in Griewank, 42.37% in Ackley, and 1.88% in Rosenbrock. The method was less competitive in Rastrigin, where GA achieved the lowest mean value.

Conclusions:

The results indicate that symmetry-based candidate reflection improves MVO performance across several benchmark functions, particularly in unimodal and moderately multimodal landscapes. The SGMVO offers a simple, reproducible modification for enhancing search diversification in population-based optimization.



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Sciforum-190675: Bayesian Fisher Information Based Photoacoustic Optimal Waveform Design via Fredholm Integral Equation

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Photoacoustic imaging or detecting systems rely on accurate characterization of biological and material media through high-precision estimation of key physical parameters, such as absorber location (time delay) and thickness (effective signal duration). This paper develops a Bayesian Fisher information framework to address optimal excitation waveform design for enhanced joint parameter estimation performance in photoacoustic sensing systems contaminated by additive white Gaussian noise. We derive the data-driven Fisher information matrix (FIM) from the photoacoustic transfer function model, and further integrate prior parametric uncertainties by introducing a diagonal prior FIM to form the complete Bayesian total FIM. Under the classic D-optimality criterion, we maximize the determinant of the total FIM to construct an optimal spectral weight function, which adaptively distributes waveform energy toward frequency components carrying the richest parametric information. Constrained to physically realizable time-limited excitation signals, the underlying optimality condition is transformed into a second-kind Fredholm integral equation, whose leading principal eigenfunction directly yields the optimal waveform solution. Unlike conventional SNR-oriented waveform strategies, the proposed method explicitly accounts for inherent parameter coupling and prior uncertainty effects, establishing a systematic information-theoretic design paradigm. An iterative eigenvalue discretization procedure is also outlined to enable efficient numerical implementation and performance validation in practical photoacoustic metrology scenarios.



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Sciforum-190703: Transient Analysis of a Queueing Systems with Cyber Disruptions and Degraded Service

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This paper presents a single-server queueing model inspired by modern cloud computing and virtualized data center systems, where servers may encounter sudden cyber disruptions and workload degradation. Incoming user tasks arrive according to a Poisson process while the server operates in the primary service state. Service times in the primary service state and the fallback processing state are exponentially distributed under the First-Come-First-Serve discipline. Whenever the task queue becomes empty, the server enters a resource refresh phase before returning to full operational capacity. During the primary service state, disruptive cyber events occur randomly, forcing the system into the fallback processing state. At the moment of disruption, the task currently under execution is terminated, while the remaining queued tasks continue to receive service at a reduced processing speed. Once all pending tasks are completed in the fallback state, the server undergoes resource refresh and resumes normal operation. The system is modeled as a continuous-time Markov process, and probabilistic methods are used to formulate the governing equations. Probability generating functions, Laplace transform techniques, and the Matrix Geometric method are employed to derive transient and steady-state results together with important performance measures. Numerical studies are conducted to analyze the influence of cyber-event intensity, refresh mechanisms, and degraded processing efficiency on cloud system reliability and service performance.



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Sciforum-190707: A Retrial Queueing Model with Optional Service, Server Breakdowns, and Customer Impatience

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This paper studies a single-server retrial queue with optional service, server breakdowns, and customer impatience. Customers arrive according to a Poisson process. An arriving customer who finds the server idle immediately receives mandatory service. Upon completion of the mandatory service, the customer may proceed to an optional service with probability p , where the mandatory and optional service times are exponentially distributed. If the server is busy upon arrival, the customer joins an orbit and retries for service after an exponentially distributed retrial time. Both mandatory and optional services are provided by the same server. During the busy period, including both mandatory and optional services, server breakdowns occur according to a Poisson process. Upon breakdown, the server immediately enters repair, where the repair times are exponentially distributed and the customer receiving service at the instant of breakdown is lost. During the repair period, arriving customers continue to join the orbit, while orbiting customers may abandon the system due to impatience before receiving service, where the impatience times are exponentially distributed. After completion of the repair process, the server returns to the normal operating state and resumes providing mandatory service. The steady-state behavior of the system is analyzed using probability generating functions (PGFs), leading to explicit expressions for several important performance measures. Numerical experiments are presented to illustrate and validate the analytical results.



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Sciforum-190725: Constructive Symmetry: Hidden Structure in Prime Arithmetic Progressions Revealed by the Base Primorial

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Prime distribution is typically regarded as a paradigm of randomness. This work introduces the “base primorial”—the product of consecutive primes from 2 up to O (denoted #O)—as a core constructive tool, revealing a hidden symmetry in prime arithmetic progressions.

When the common difference d of an arithmetic progression is an integer multiple of the base primorial (i.e., d = K·#O, K a natural number), all terms occupy the same residue class modulo every prime ≤ O—a strict translational symmetry. This symmetry is broken by a structural law: let P be the next prime greater than O, then the progression cycles through all residue classes modulo P, so at most P-1 consecutive terms can avoid divisibility by P, imposing an absolute upper bound of P on the length of any prime progression with this difference.

Based on this principle, we propose a complete system of constructive conjectures, supported by systematic verification: for P = 2 to 13, 100 examples were obtained for each, all strictly obeying the symmetry-imposed bound. Notably, when the difference equals the base primorial itself (K=1) and the first term equals P, only 13 such prime progressions were found below 10^{1000}, demonstrating that the symmetry constraint strengthens with scale rather than weakening.

This framework is being extended to consecutive prime progressions and twin prime distribution. Preliminary results suggest that the symmetry-breaking mechanism revealed by the base primorial is effective in a broader class of prime distribution problems.



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Sciforum-190778: Approximate Analysis of M/G/c, GI/G/1, and GI/G/c Systems Using Mixture Distributions

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There are exact solutions for M/G/1 system and good approximations for M/G/c, GI/G/1, and GI/G/c systems when certain conditions are satisfied. However, various approximation formulas have also been proposed in the literature. In the M/G/c system, the arrival process follows a Poisson distribution, while the service-time distribution is non-exponential with (c) servers. For M/G/1 , the Pollaczek–Khintchine formula provides an exact solution. However, this formula requires the service-time distribution to follow a theoretical distribution and its variance to be known. The results obtained from the M/G/1 system are commonly used as lower and upper bounds based on the coefficient of variation, and approximation formulas have been developed by extending this approach to M/G/c, GI/G/1, and GI/G/c systems. However, these approximation formulas have generally been developed and applied for cases in which the data fit a theoretical distribution. When the data do not fit any theoretical distribution, instead of directly using an empirical distribution, it may be tested whether the dataset follows a mixture distribution. Mixture distributions can be used for modeling heterogeneous populations consisting of several unobserved homogeneous subpopulations.

In this study, an approximate solution is proposed for M/G/c, GI/G/1, and GI/G/c systems when the general distributions consist of pure mixture distributions. The rationale behind the proposed approximation is based on the ability to compute the variance of certain pure mixture distributions. The approximation formula has been adapted for two-component Gaussian, Gamma, and Beta mixture distributions. To evaluate the effectiveness of the proposed approximation, M/G/c, GI/G/1, and GI/G/c systems were simulated using random numbers generated from mixture distributions. The results obtained from the proposed approximation formula were found to be very close to the simulation results.



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Sciforum-190797: Hill-Clohessy-Wiltshire-Based Multi-Objective Optimization (MOP) for Relative Positioning in Small Groups of Interacting Objects

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This research presents a performance analysis of a multi-objective optimization (MOP) strategy designed for managing relative positioning within small groups of interacting objects in orbit. Our study addresses the challenges of proximity navigation and formation maintenance by leveraging the analytical framework of the Hill-Clohessy-Wiltshire (HCW) equations. Using the MATLAB environment, we developed a mathematical model of the relative dynamics of bodies in the local reference frame (LVLH), providing a unified framework for describing the coupled trajectories of multiple interacting agents. The research methodology was executed using a rigorous three-step approach. First, we formulated the HCW equations of motion within a multi-objective optimization algorithm to simultaneously minimize fuel consumption (Δv) and relative positioning error. Secondly, we implemented a classical single-objective control structure as a performance benchmark to evaluate the trade-offs offered by the Pareto front of the MOP solution. Finally, the architecture was validated by a high-fidelity simulation reproducing a flight scenario involving a formation of interacting objects, taking into account collision safety constraints and J2 orbital perturbations. Our results demonstrate that while the single-objective approach allows for rapid target acquisition, the Hill-Clohessy-Wiltshire-based MOP strategy offers greater decision-making flexibility, enabling an optimal balance between energy efficiency and the accuracy of maintaining the geometric configuration of the group.



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Sciforum-191999: A Numerical Approach for Solving Load-Leveling Energy System Models Governed by First-Kind Volterra Integral Equations

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This paper presents an accurate and efficient spectral collocation method for solving first-kind Volterra integral equations with piecewise continuous kernels. These equations frequently arise in applied mathematics and engineering, particularly in modeling systems with memory effects and cumulative processes, but they are known to be ill-posed and computationally challenging. To address these difficulties, the proposed method constructs an approximate solution based on shifted Legendre polynomial expansions, which provide high accuracy and spectral convergence properties.

The numerical procedure employs Legendre–Gauss quadrature rules combined with a collocation strategy to discretize the integral equation. This transformation reduces the original continuous problem into a finite-dimensional system of algebraic equations. To further enhance the robustness and accuracy of the computational framework, the resulting system is solved using a Physics-Informed Neural Network (PINN) approach, which is used to determine the unknown expansion coefficients efficiently while maintaining consistency with the governing integral equation.

To demonstrate the effectiveness and applicability of the proposed method, an engineering application related to load leveling in energy storage systems is investigated. In this context, the first-kind Volterra integral equation models the charge–discharge dynamics of an energy-storage device required to satisfy time-varying electrical load demands. Numerical experiments show that the proposed hybrid spectral–PINN framework achieves high accuracy and stability, even in the presence of discontinuities in the kernel. The results confirm the efficiency of the method and its potential for solving a wide range of ill-posed integral equations arising in engineering and applied sciences.



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Sciforum-193890: A new centered Gaussian process

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In this work, we introduce a new Gaussian process called the **mixed bifractional Brownian motion**, defined as a linear combination of a finite number of independent bifractional Brownian motions. This process extends the classical bifractional Brownian motion and provides a more flexible framework for modeling complex stochastic phenomena exhibiting memory and self-similarity. The construction allows the incorporation of different bifractional parameters, leading to a richer covariance structure and a wider range of potential applications.

We investigate several fundamental probabilistic and path properties of the mixed bifractional Brownian motion. In particular, we establish its Hölder continuity and self-similarity properties and study the regularity of its sample paths. We also analyze the differentiability of the trajectories and derive the behavior of its quadratic variation. Furthermore, we examine the dependence structure of the process by investigating its long-range dependence property and the stationarity of its increments. The Markov property is also discussed, and conditions under which it may hold are examined.

In addition, we study the noise generated by the increments of the mixed bifractional Brownian motion and analyze its asymptotic behavior. These results provide a deeper understanding of the stochastic dynamics associated with this new process. The proposed model combines important characteristics such as self-similarity, long-range dependence, and non-stationary increments, making it a promising candidate for applications in fields where such features naturally arise, including finance, telecommunications, hydrology, signal processing, and other areas involving complex random phenomena.



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Sciforum-194314: Reproducible MATLAB Visualisation and Residual Mapping of Base-10 Palindromic Primes below One Hundred Million across Residues Modulo Nine and Modulo Eleven

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Palindromic primes are subject to digit symmetry constraints; for example, every even-digit palindrome in base ten is divisible by eleven, so eleven is the only even-digit palindromic prime. The congruence that an integer is equal modulo nine to the sum of its decimal digits makes residues modulo nine a natural focus for digit-sum-based analysis. To provide a fully reproducible empirical picture of these arithmetic constraints, the author of this study implemented a single-file MATLAB workflow that enumerates palindromic integers by mirroring left halves, applies vectorised primality testing, computes digit sums and residues modulo 9 and 11, and aggregates counts into a digit-by-residue counts cube. The pipeline writes canonical CSV and MAT artefacts, exports publication-quality graphics, and reports contingency-table diagnostics including Pearson chi-square testing, standardised residual mapping, Cramer's V effect size, and Benjamini–Hochberg false discovery rate (FDR) control across the nine-by-eleven joint residue family. The canonical output lists 781 palindromic primes below one hundred million and is strongly concentrated at odd digit lengths, with a median of seven and 668 seven-digit primes. Visual inspection and residual mapping reveal modest local deviations in joint residue cells, with raw counts of up to 16, a maximum standardised residual of about 2.80 and the smallest unadjusted two-sided residual p-value of about 0.0050; the corresponding minimum Benjamini–Hochberg q-value is about 0.499, yielding no discoveries at FDR 0.05. The aggregated nine-by-eleven table is consistent with independence (chi-square (80) = 90.48, p = 0.198) with a small-to-moderate association magnitude (Cramer's V = 0.120). Per-digit decomposition variables are produced to support stratified inspection of residue patterns, and the numerically dominant seven-digit stratum largely governs the aggregated structure. All implementation choices are consistent with safe numeric practice for this bound; for example, the use of double precision for primality tests is safe for integers well below 2^53, and the code and data artefacts permit exact reproduction.



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Sciforum-195545: Spherical Fuzzy Multisets: Symmetric Algebraic Structures and Applications in Multi-Criteria Decision-Making

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Uncertainty remains a central challenge in multi-stage decision-making, yet existing spherical fuzzy models cannot explicitly preserve repeated assessments of the same element. In this paper, we introduce Spherical Fuzzy Multisets (SFMSs), a framework that combines the expressive power of Spherical Fuzzy Sets (SFSs) with multiset theory to retain multiple evaluations individually, rather than collapsing them into a single value. SFMSs preserve repeated membership, abstention, and non-membership information under the spherical constraint $\mu^2 + \eta^2 + \nu^2 \leq 1$, thereby capturing uncertainty more faithfully in repeated or sequential assessments. We establish the theoretical foundation of SFMSs by defining their basic structure, set operations, algebraic properties, level subsets, (α, β, γ)-cuts, score and accuracy functions, and modal operators, and prove the principal properties of each construction. A composite relation on SFMSs is further developed to support decision analysis and MCDM. For validation, the framework is applied to a higher education course placement problem in which student performance is evaluated across multiple assessment rounds. The results show that the proposed method distinguishes placement outcomes while preserving nuanced patterns of uncertainty, hesitation, and repeated evaluations that would otherwise be aggregated away. This richer representation improves the discrimination of alternatives and yields more informative decision outputs than existing spherical fuzzy models. Overall, SFMSs extend spherical fuzzy modelling in a mathematically rigorous way and provide a strong basis for MCDM applications in education, healthcare, management, and engineering—particularly when repeated or staged evaluations must be retained explicitly.



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Sciforum-196282: Magnetohydrodynamic boundary-layer flow of a nanofluid over a stretching sheet with Dufour and Soret impact for non-isothermal and non-isosolutal conditions: A Chebyshev Collocation Approach

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This current study investigates the steady, incompressible boundary-layer flow of an aluminium oxide (Al_2O_3)–water (H_2O) nanofluid, where Aluminium oxide nanoparticles are suspended in water to form an Al_2O_3/H_2O nanofluid under non-isothermal and non-isosolutal conditions. The analysis incorporates thermo-diffusion (Soret effect) and diffusion-thermo (Dufour effect) to account for coupled heat and mass transfer mechanisms within the flow field. In addition, the governing model considers the influence of an applied magnetic field, which introduces magnetohydrodynamic resistance, together with convective boundary conditions to realistically describe heat exchange at the surface. The resulting mathematical formulation consists of coupled nonlinear partial differential equations representing conservation of mass, momentum, energy, and concentration. Through appropriate similarity transformations, these equations are condensed to a system of nonlinear ordinary differential equations accompanied by transformed boundary conditions. The numerical solution of the governing equations is obtained using the Chebyshev Collocation Method, which provides accurate approximations for highly nonlinear boundary-layer problems. To validate the numerical procedure, the computed Nusselt number is compared with existing results reported in the literature, and close agreement is demonstrated through tabulated comparisons. The findings reveal that an increase in the magnetic parameter suppresses fluid velocity due to enhanced Lorentz force resistance, while simultaneously increasing temperature and concentration distributions. Furthermore, higher Dufour and Soret numbers significantly enhance thermal and concentration profiles, respectively. The nanoparticle volume fraction is also observed to improve heat and mass transfer characteristics. These outcomes suggest that incorporating aluminium oxide nanoparticles into water enhances thermal conductivity, thereby offering promising applications in advanced cooling technologies and energy-efficient systems aimed at reducing carbon emissions.



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Sciforum-198686: Symmetry-Guided Boundary Element Modeling of Microneedle-Delivered Nanoparticle Photothermal Hyperthermia in Multilayer Skin Tumors

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Introduction: Photothermal hyperthermia is a promising minimally invasive approach to treat surface skin cancers; however, for an effective therapeutic outcome, it is crucial to ensure adequate tumor heating without causing heat damage to the adjacent normal tissues. This paper presents a symmetry-based computational model to analyze microneedle-assisted photothermal cancer treatment using nanoparticles.

Methods: A Pennes-based bioheat equation that incorporates multiple layers in the biological tissues, tumor-dependent blood flow and metabolism, volumetric heat source due to lasers, localized heat generation due to nanoparticles, convective cooling on the surface, interface conditions, and the Arrhenius thermal damage function has been developed. The discretized time-dependent governing equation has been converted into an equivalent Helmholtz equation that is solved by a dual-reciprocity boundary element method. The localized deposition of nanoparticles due to dissolvable microneedles has been represented using Gaussian distribution functions.

Results: Numerical results have proven that factors such as the laser intensity, the concentration of the nanoparticles, the depth of the tumor, and the deposition profile play significant roles in affecting tumor-center temperature, heat dose accumulation, and the spatial selectivity. In contrast to traditional heating using the laser alone, the localized deposition of nanoparticles helps to destroy the thermal symmetry in the temperature distribution and achieve more selective tumor heating.

Conclusions: The newly introduced symmetry-based DRBEM approach serves as a very convenient and accurate instrument for prediction of temperature changes, assessment of thermal damage and parameter optimization for microneedle-based nanoparticle photothermal hyperthermia therapy. Symmetry breaking is found to play a crucial role in making tumors more selective to such therapy.



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Sciforum-198894: Advanced Differentiable Hybrid Frameworks for Time Series Reconstruction and Forecasting in Data-Scarce Regimes

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Introduction

Classical time series models excel at capturing linear structural components, whereas deep learning architectures are more effective at modeling nonlinear dynamics. However, in data-scarce and resource-constrained edge computing environments, purely deep learning-based approaches are prone to severe overfitting, while statistical models often lack sufficient flexibility. This work addresses this gap by exploiting structural symmetries and inductive biases through a unified differentiable framework specifically designed for efficient edge deployment.

Methods

We propose a novel hybrid methodology that integrates structural state-space models with machine learning techniques through differentiable programming based on Wengert lists for automatic differentiation. The framework incorporates structural invariant constraints as regularization mechanisms to improve learning under severe data scarcity. To ensure suitability for edge computing, the computational graph is optimized using Just-In-Time (JIT) compilation and Accelerated Linear Algebra (XLA), reducing both latency and memory consumption. The proposed approach is evaluated using Walk-Forward validation on both synthetic and real-world datasets containing discontinuities.

Results

Anticipated experimental results demonstrate that embedding structural inductive biases significantly improves generalization and mitigates overfitting compared with standalone statistical and deep learning approaches. The proposed model achieves superior forecasting and data imputation accuracy under conditions of extreme data scarcity. Furthermore, JIT/XLA optimization substantially reduces computational latency, confirming an effective balance between predictive performance and computational efficiency for deployment on resource-constrained edge devices.

Conclusions

This research bridges the gap between structural mathematical modeling and deep learning by leveraging differentiable programming and the structural symmetries inherent in time-dependent processes. The proposed framework offers a robust, scalable, and computationally efficient solution for real-time time series forecasting and imputation in data-sparse edge computing environments.



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Sciforum-198905: Oscillation of the Ar2 Molecule using the LAMMPS Molecular Dynamics Simulation Tool

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This study investigates the vibrational dynamics of the diatomic argon molecule (Ar2) using molecular dynamics (MD) simulations, with the aim of determining the dispersion relationship between the oscillation frequency f and the system energy E. The interatomic interactions are modeled using the Lennard–Jones potential, a well-established representation of atomic forces, to analyze the molecular behavior over a range of energy states. The simulations are performed with the LAMMPS software package, which enables the calculation of oscillation frequencies corresponding to different energy levels.

The selected energy values span the depth of the Lennard–Jones potential well, ranging from -ε to zero. For each energy level, the equilibrium bond length is determined and the corresponding oscillation frequency is computed. The results demonstrate that the oscillation frequency decreases as the energy increases, highlighting the anharmonic nature of the molecular potential. At higher energies, the oscillation amplitude becomes larger, causing the system to deviate from ideal harmonic motion and resulting in a reduction of the vibrational frequency.

The dispersion curve, which describes the relationship between oscillation frequency and energy, is examined and compared with theoretical predictions. In addition, an analytical approximation based on the theory of small oscillations about the equilibrium bond length is derived to estimate the vibrational frequency at equilibrium energy. The analytical prediction shows good agreement with the numerical results obtained from the molecular dynamics simulations. Overall, the findings provide insight into the anharmonic vibrational behavior of diatomic molecules and contribute to a deeper understanding of molecular interactions and thermodynamic properties in molecular simulation studies.



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Sciforum-199304: Contact Problem for a Circular Simply Supported Plate and a Rigid Base

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Free-boundary contact problems occupy a special place in plate and shell mechanics; as such problems are inherently nonlinear, linearization would lose physical meaning. A generalized reaction method has been developed at Pitirim Sorokin Syktyvkar State University. It uses an iterative scheme based on an equation linking displacements and reactions. The Kármán–Timoshenko–Naghdi plate theory, avoiding Kirchhoff's hypothesis, has also been developed for contact problems.

This paper solves a contact problem for a circular, axisymmetric simply supported plate on an absolutely rigid and smooth base. The problem is addressed using two distinct approaches, both of which employ the Kármán–Timoshenko–Naghdi equations together with the generalized reaction method.

The first approach exploits axisymmetry, reducing the biharmonic operator to a single-variable operator in a polar radius. A Green's function is then used to formulate the iterative scheme, which is solved via finite differences.

The second approach neglects axisymmetry and approximates second derivatives in Cartesian coordinates. The main difficulty lies in computing numerical derivatives on the circular boundary. A four-variable Green's function is constructed, and the iterative scheme is defined in a similar manner.

The numerical experiments yield deflections of the middle and lower surfaces, contact reactions, and bending moments. The error is computed as the norm of the difference between the solutions, divided by the norm of one of them and multiplied by 100%. For deflections, the error remains below 5%, which confirms the validity of the proposed approach and makes it acceptable for circular plates even when symmetry is not taken into account.



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Sciforum-200275: Lie Symmetry Classification and Invariant Solutions of Diffusive Lotka–Volterra Systems with Time-Dependent Coefficients

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Diffusive Lotka–Volterra systems are widely used to describe interacting biological species, chemical reactions, and ecological processes. While Lie symmetry methods have been extensively applied to reaction–diffusion systems, complete symmetry classifications for diffusive Lotka–Volterra systems with time-dependent coefficients remain relatively scarce.

A class of two-component diffusive Lotka–Volterra systems with time-dependent coefficients is considered in the form

$u_t=u_{xx}+u(b_1(t)u+c_1(t)v)$, $v_t=d(t)v_{xx}+v(b_2(t)u+c_2(t)v)$,

where $b_i(t)$, $c_i(t)$ and $d(t)$ are arbitrary smooth functions satisfying $c_1^2+b_2^2\neq 0$ and $d(t)\neq 0$, and at least one coefficient is non-constant.

The classical Lie symmetry approach is employed to perform a group classification of the considered class of systems. The equivalence group of the considered class is found, and a complete Lie symmetry classification of its inequivalent cases is obtained. The classification yields extended symmetry algebras, which lead to symmetry reductions to systems of ordinary differential equations. Corresponding invariant solutions are constructed.

The obtained results show that time-dependent coefficients significantly enrich the symmetry structure of diffusive Lotka–Volterra systems. The classification provides a systematic basis for constructing exact solutions and extends known results for constant-coefficient models.

Reference: Davydovych V. Lie Symmetry of the Diffusive Lotka–Volterra System with Time-Dependent Coefficients, *Symmetry*, **10(2)** (2018), 41.



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Sciforum-200426: Existence Results for a Nonhomogeneous Weighted p-Laplacian Problem with Critical Sobolev Growth

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In this paper, we study a class of nonlinear elliptic Dirichlet problems involving a weighted p-Laplacian operator and critical Sobolev growth. More precisely, we consider-div(α(x)|∇u|^(p-2)∇u) = |u|^(p*-2)u + f in Ω,u = 0 on ∂Ω,where Ω ⊂ ℝ^N is a bounded smooth domain, 1 < p < N, p* = Np/(N-p) is the critical Sobolev exponent, α(x) is a positive continuous weight, and f ∈ W^(-1,p′)(Ω) is a nontrivial perturbation.The presence of the critical exponent leads to a lack of compactness in the associated Sobolev embedding, which makes the variational analysis highly nontrivial. The weight function α(x) introduces additional analytical difficulties and breaks the homogeneity of the problem, affecting the compactness properties of Palais–Smale sequences.Our approach is based on variational methods and mountain pass arguments in the spirit of Brezis–Nirenberg, combined with concentration–compactness techniques. The problem is motivated by the seminal work of Brezis and Nirenberg, further extended by Tarantello to nonhomogeneous perturbations, and later generalized to weighted p-Laplacian frameworks by Benhamida and Yazidi.We establish existence results under suitable assumptions on f and α(x), and we discuss how the interaction between the weight, the geometry of Ω, and the critical exponent influences the variational structure of the problem.



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Sciforum-200497: A Physics Based Functionally Linked Constrained Neural Networks Framework for Solving Differential Equations

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This paper presents a Physics-based Functionally Linked Constrained Neural Networks framework for the efficient and accurate solution of differential equations (DEs). The presented approach integrates the Theory of Functional Connections (TFC) with a Functional Link Neural Network (FLNN) to construct a constrained expression that exactly satisfies all prescribed initial and boundary conditions. By embedding the constrained directly into the solution, the framework eliminates the need to enforce these conditions through penalty terms during training. The free function component of the constrained expression is represented using an FLNN, which offers a computationally efficient alternative to deep neural network architectures. Model training is carried out by minimizing the mean squared error of the governing differential equation residual at selected collocation points. As a result, the presented framework maintains the physical consistency of the solution while reducing the computational burden associated with deep learning based approaches. To evaluate its effectiveness, this method is applied to several linear and nonlinear ordinary and partial differential equations. The obtained numerical results are compared with analytical solutions and existing Physics-Informed Neural Network (PINN) models. The comparisons demonstrate that the framework achieves higher accuracy, faster convergence, and significantly reduced training time while requiring fewer trainable parameters. Overall, this framework offers a fast, accurate, and efficient approach for solving differential equations in scientific and engineering applications.



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Sciforum-200834: Role of Symmetric Periodic Perforations in the Dynamics of Axially Graded Nanobeams

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This study presents a mathematical framework for investigating the free vibration behavior of axially graded perforated nanobeams resting on elastic foundations. The governing differential equation is formulated based on the Euler-Bernoulli beam theory within the framework of nonlocal elasticity to account for small-scale effects that become significant at the nanoscale. Particular attention is given to the influence of symmetry arising from periodic perforation patterns and axial material gradation on the dynamic response of the nanobeam. Modified equivalent models are employed to represent the effects of regularly distributed perforations while preserving the geometric symmetry of the structure. The resulting governing equations are solved using the Galerkin method to determine the natural frequencies and corresponding mode shapes. The accuracy and reliability of the proposed formulation are validated through comparisons with available results reported in the literature. Numerical investigations reveal that the vibrational characteristics are strongly affected by the filling ratio, perforation geometry, material gradation, and elastic foundation parameters. Furthermore, the interaction between nonlocal effects and structural symmetry plays a significant role in governing the dynamic behavior under various boundary conditions. The results demonstrate that symmetric periodic perforations can substantially alter the stiffness and mass distribution of the nanobeam, thereby influencing its vibration characteristics. The findings provide useful insights for the design and development of advanced nanostructures with periodic perforated configurations.



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Sciforum-200836: A Supplementary Variable Approach for a Non-Markovian Bulk Queue with Rework and Differentiated Server Breakdowns

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This paper presents a comprehensive study of a non-Markovian bulk service queueing model incorporating inspection, rework, differentiated server breakdowns, a standby server mechanism, and multiple vacation policies. The proposed model is developed to analyze complex real-world production and manufacturing systems where items arrive in batches and are processed in groups under uncertain operating conditions. In many industrial environments, defective or spoiled items may occur during service, leading to additional operational costs and delays. To address this issue, the model integrates inspection and rework processes, which help identify defective items and restore them to acceptable quality standards, thereby reducing wastage and improving overall system efficiency. The study further considers two types of server breakdowns, namely soft and hard failures, which represent different levels of service interruption. To ensure continuity of service during such breakdown situations, a standby server is incorporated into the system. This standby support significantly reduces interruption time and enhances the reliability and stability of the service mechanism. In addition, the inclusion of multiple vacation policies allows the server to take controlled idle periods whenever the system becomes empty, reflecting realistic operational practices in manufacturing and communication systems. Using the supplementary variable technique, the steady-state behavior of the proposed queueing system is analyzed in detail. Numerical illustrations are also provided to demonstrate the practical applicability and effectiveness of the proposed model.



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