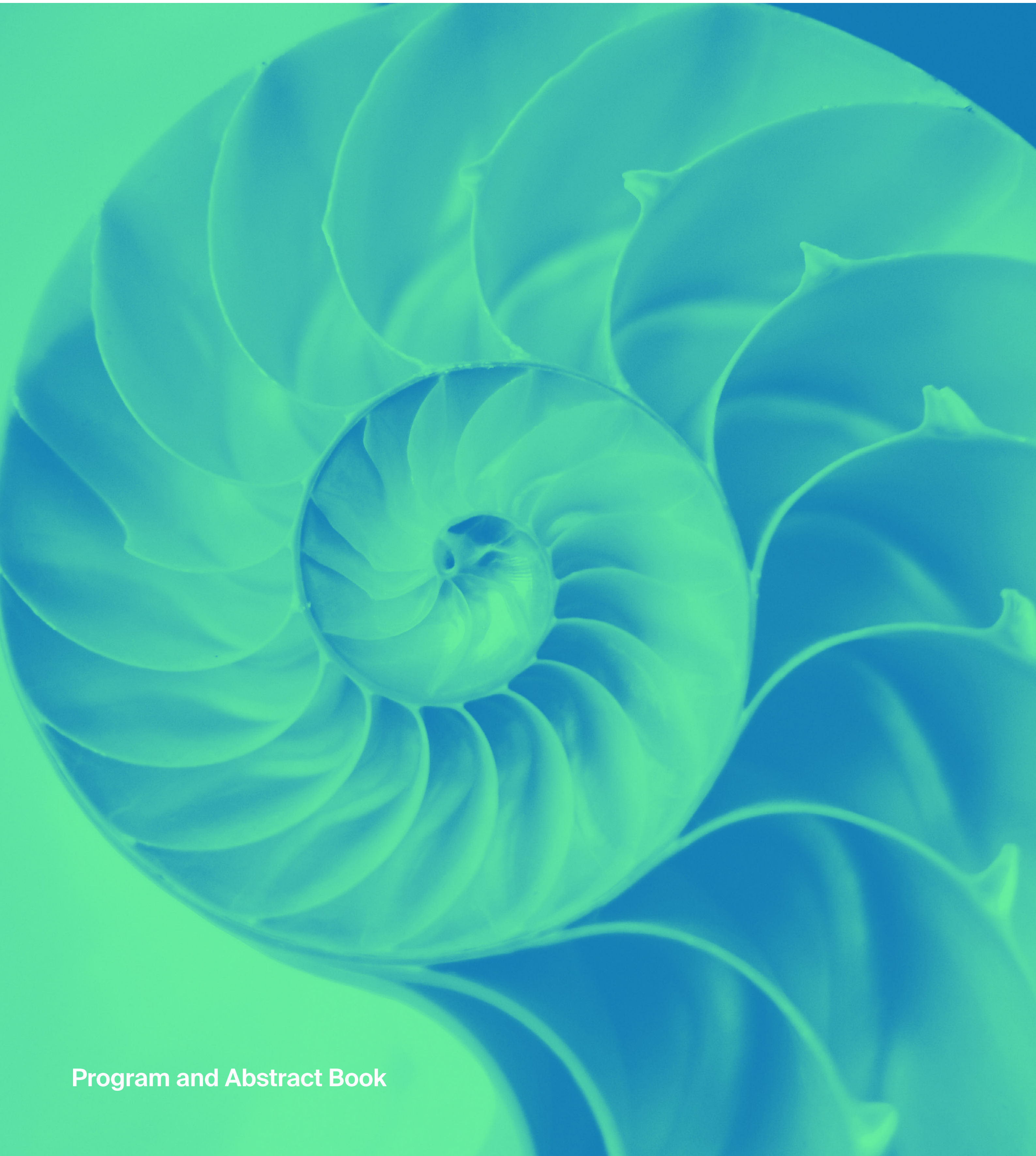
IOCFF 2026 Conference

The 1st International Online Conference on Fractal and Fractional

13–15 April 2026 | Online



Program and Abstract Book

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#IOCFF2026

The 1st International Online Conference on Fractal and Fractional

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MDPI

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Prof. Dr. Haci Mehmet Baskonus

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Conference Secretariat

Email: iocff2026@mdpi.com

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Welcome from the Chairs

Dear Colleagues,

It is our sincere pleasure and great honor to announce **the 1st International Online Conference on Fractal and Fractional: Theoretical Foundations and Interdisciplinary Applications (IOCFF 2026)**, which will be held online by the MDPI open access journal ***Fractal and Fractional*** (ISSN: 2504-3110; Impact Factor: 3.3) from **13 to 15 April 2026**.

Fractals and fractional calculus have changed the way we model and understand complexity across mathematics, physics, engineering, and beyond. Fractals offer a powerful framework for describing irregular, self-similar structures found in nature and science, while fractional calculus provides a flexible and accurate tool for modeling memory effects, anomalous diffusion, and nonlocal phenomena. Together, they have provided fundamental theories and inspired innovative applications in diverse fields, ranging from fluid dynamics and materials science to finance, biology, and data analysis. Their impact continues to expand in the study of complex systems in the modern scientific landscape.

IOCFF 2026 will focus on recent theoretical methods, advanced computational algorithms, and emerging applications of fractals and fractional calculus. Through analytical innovations, numerical methods, or interdisciplinary research, the conference aims to highlight the growing role of these fields in addressing complex problems across science and engineering. We invite you to join a dynamic exchange of ideas that reflects both the rich foundations and the evolving future of fractal and fractional research.

All scientists or engineers interested in fractals and fractional calculus are welcome to join this event and share their findings around the following general and related themes. However, the list is not exclusive.

S1. Recent Advances in Fractional-Order Differential and Integral Operators
S2. Fractional Calculus and Its Applications in Engineering Systems
S3. Numerical Methods for Fractional Calculus
S4. Fractional Calculus in Complex and Nonlinear Dynamical Systems
S5. Fractional Calculus in Machine Learning: Applications and Challenges
S6. Fractal Geometry: Mathematical Foundations and Real-World Applications

IOCFF 2026 will allow you to share and discuss your most recent research findings with a vibrant worldwide community of scientists and engineers in the field. Moreover, numerous internationally renowned speakers will share their current state-of-the-art research during the event through six live sessions, including comprehensive Q&A segments.

We look forward to having you join us at this exciting event.

Prof. Dr. Carlo Cattani
Conference Chair
University of Tuscia, Viterbo, Italy

Prof. Dr. Haci Mehmet Baskonus
Conference Chair
Harran University, Sanliurfa, Türkiye

fractal and fractional

an Open Access Journal by MDPI

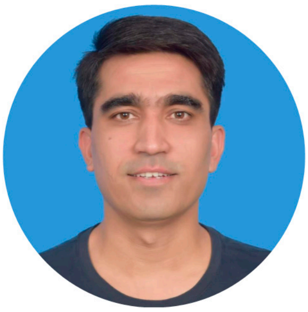
Fractal and Fractional (ISSN 2504-3110) is an online, peer-reviewed and open access journal, which provides an advanced forum for studies related to fractals and fractional calculus and their applications in different fields of science and engineering; publishing reviews, regular research papers and short notes.

Our aim is to encourage scientists to publish their experimental and theoretical results in as much detail as possible. There is no restriction on the maximum length of the papers as details of the experiment and any calculations must be provided so that the results can be reproduced. *Fractal and Fractional* additionally offers the following two unique features:

- Manuscripts regarding research proposals and research ideas are particularly welcome.
- Electronic files containing details of calculations and experimental procedures can be deposited as supplementary material.

Impact Factor: 3.3 (2024); 5-Year Impact Factor: 3.2 (2024)

Session Chairs



Dr. Anwarud Din

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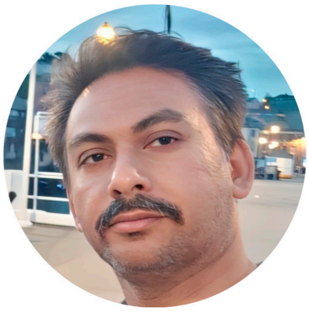
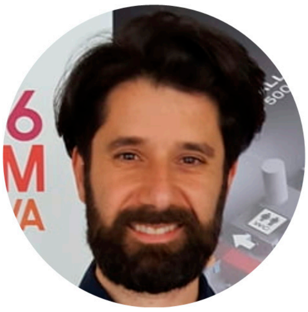


Dr. Saptarshi Das

Centre for Environmental Mathematics, University of Exeter, Penryn Campus, Cornwall, UK

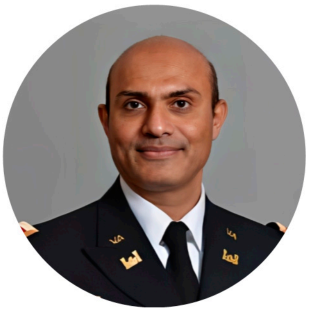
Prof. Dr. Alex Iosevich

Department of Mathematics, University of Rochester, Rochester, NY, USA



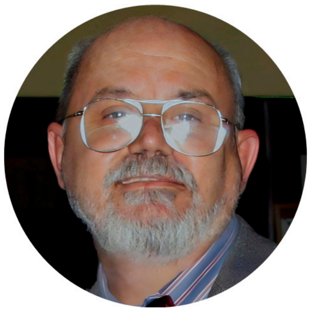
Prof. Dr. Camillo Porcaro

Department of Neuroscience and Padova Neuroscience Center, University of Padova, Padova, Italy



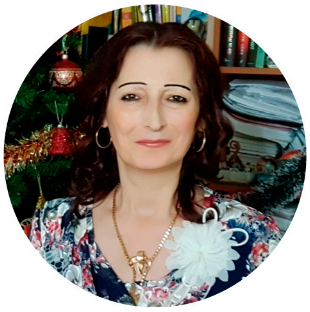
Prof. Dr. Dimplekumar N. Chalishajar

Department of Applied Mathematics, Virginia Military Institute (VMI), Lexington, VA, USA



Prof. Dr. Jordan Hristov

Department of Chemical Engineering, University of Chemical Technology and Metallurgy, Sofia, Bulgaria



Prof. Dr. Rodica Luca

Department of Mathematics, "Gheorghe Asachi" Technical University of Iasi, Iasi, Romania

Prof. Dr. Yangquan Chen

Department of Mechanical Engineering, University of California, Merced, CA, USA

Dr. Da-Yin Liu

INSA Centre Val de Loire, Bourges, France

Invited Speakers

Prof. Dr. Boris Malomed

Department of Physical Electronics, School of Electrical and Computer Engineering, Faculty of Engineering, Tel Aviv University, Tel Aviv, Israel

Prof. Dr. Chris Goodrich

School of Mathematics and Statistics, UNSW Sydney, Sydney, Australia

Prof. Dr. George Anastassiou

Department of Mathematical Sciences, University of Memphis, Memphis, TN, USA

Prof. Dr. Jesus Manuel Muñoz-Pacheco

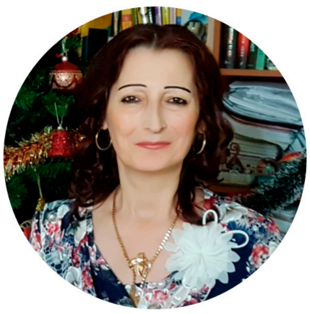
Faculty of Electronics Sciences, Autonomous University of Puebla, Puebla, Mexico

Prof. Dr. Jocelyn Sabatier

IMS (Integration from Material to System) laboratory, Bordeaux University, Talence, France

Prof. Dr. Katica (Stevanović) Hedrih

Mathematical Institute of Serbian Academy of Science and Arts, Belgrade and Faculty of Mechanical Engineering, University of Niš, Nis, Serbia



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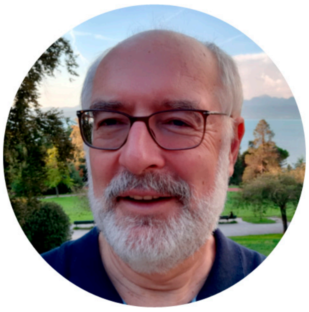
Prof. Dr. Stanisław Drożdż

Faculty of Computer Science and Mathematics, Cracow University of Technology, Kraków, Poland; Complex Systems Theory Department, Institute of Nuclear Physics, Polish Academy of Science, Kraków, Poland



Prof. Dr. Yuri Luchko

Department of Mathematics, Physics and Chemistry, Berlin University of Applied Sciences and Technology, Berlin, Germany

Program at a Glance

April 13th	April 14th Morning	April 15th Morning
Abstract Book & Poster Gallery	**09:00 AM CEST** **Session 2.** Fractional Calculus and Its Applications in Engineering Systems **Session 4.** Fractional Calculus in Complex and Nonlinear Dynamical Systems	**09:00 AM CEST** Session 1. Recent Advances in Fractional-Order Differential and Integral Operators
	April 14th Afternoon	**April 15th Afternoon**
	14:00 PM CEST **Session 6.** Fractal Geometry: Mathematical Foundations and Real-World Applications	**14:00 PM CEST** **Session 3.** Numerical Methods for Fractional Calculus **Session 5.** Fractional Calculus in Machine Learning: Applications and Challenges

IOCFF 2026 Program

14th April 2026 (Tuesday)

Opening Ceremony
S2. Fractional Calculus and Its Applications in Engineering Systems
S4. Fractional Calculus in Complex and Nonlinear Dynamical Systems

Time: 09:00 (CEST, Basel) | 03:00 (EST, New York) | 16:00 (CST Asia, Beijing)

CEST Time	Speaker	Title
9:00–9:10	*Prof. Dr. Carlo Cattani and Prof. Dr. Haci Mehmet Baskonus* *Welcome from the conference chairs*	
9:10–9:20	*Dr. Da-Yan Liu* *Welcome from the session chair*	
9:20–9:40	**Prof. Dr. Katica (Stevanović) Hedrih** Invited Speaker	Approximate Analytical Solution of the Motion Modes of Dynamics of Two Rheological Maxwell–Faraday Discrete Dynamic Systems, Creeper Type, and Fractional Type, with Piezoelectric Polarization Property of the Faraday Element
9:40–10:00	**Prof. Dr. Jocelyn Sabatier** Invited Speaker	Some Proposals for a Renewal in the Field of Fractional Behaviour Analysis and Modelling
10:00–10:15	**Fernando Olivar Romero** Selected Speaker	Fractional Time–Space Viscoelastic Wave Equation: Analysis and Solution Framework
10:15–10:30	**Romolo Marcelli** Selected Speaker	Spectral Analysis of Sierpinski Microwave Resonators
10:30–10:45	**Beibei Zhang** Selected Speaker	Investigation of Turbulent Heat Transfer in a Straight Tube Using Fractional Maxwell Nanofluid
10:45–10:55	*Dr. Da-Yan Liu* *Welcome from the session chair*	
10:55–11:15	**Prof. Dr. Boris Malomed** Invited Speaker	A Review: Basic Fractional Nonlinear-Wave Models and Solitons
11:15–11:30	**Paul-Erik Haacker** Selected Speaker	Fractional Differential Equations as Nonlocal Dynamical Systems: A New Perspective
11:30–11:45	**Ernestas Uzdila** Selected Speaker	Continual Impulse-Based Finite-Time Fractional Difference Logistic Map Stabilization Scheme
11:45–12:00	**Haiqa Ehsan** Selected Speaker	Influence of Fractional and Integer Derivatives on the (4+1)-Dimensional Fractional Davey–Stewartson–Kadomtsev–Petviashvili Equation
12:00–12:15	**Audrey Helen Moore** Selected Speaker	A Fractional Hodgkin-Huxley-Lotka-Volterra Model of Neuronal Synchronized Electromechanics
12:15–12:30	**Mark Edelman** Selected Speaker	On Fractional Generalizations of the Logistic Map and Their Applications

S6. Fractal Geometry: Mathematical Foundations and Real-World Applications

Time: 14:00 (CEST, Basel) | 08:00 (EST, New York) | 21:00 (CST Asia, Beijing)

CEST Time	Speaker	Title
14:00–14:10	*Prof. Dr. Camillo Porcaro* *Welcome from the Session Chair*	
14:10–14:30	**Prof. Dr. Stanisław Drożdż** Invited Speaker	Testing the Time Series for Multifractality
14:30–14:45	**Simone Cauzzo** Selected Speaker	Fractal Dimension Analysis: Unlocking Ageing-Related Changes in Brain Criticality
14:45–15:00	**Sadaf Moaveninejad** Selected Speaker	A Hybrid Fuzzy Logic System Leveraging Higuchi Fractal Dimension for Transparent and Predictive Control of Adaptive TMS Protocols
15:00–15:15	**Filippo Zappasodi** Selected Speaker	Brain Dynamics Fractality as a Biomarker for Drug-Responsive Epilepsy

15:15–15:30	**Arianna Caputo** Selected Speaker	Meditation-Related Increases in EEG Fractal Dimension Reveal Enhanced Brain Complexity
15:30–15:45	**François Berkmans** Selected Speaker	Fractal-Inspired Multiscale Surface Topography Characterization via ISO 25178 Sdr
15:45–16:00	**Jaume Anguera** Selected Speaker	Applications of Fractal Geometry for Antenna Design in Wireless Communications
16:00–16:15	**Lilian Aurora Ochoa Ontiveros** Selected Speaker	Construction of Fractal Stiffness and Mass Matrices for Two-Dimensional Self-Similar Frames

15th April 2026 (Wednesday)

S1. Recent Advances in Fractional-Order Differential and Integral Operators

Time: 09:00 (CEST, Basel) | 03:00 (EST, New York) | 16:00 (CST Asia, Beijing)

CEST Time	Speaker	Title
9:00–9:10	*Prof. Dr. Jordan Hristov and Prof. Dr. Rodica Luca* *Welcome from the session chairs*	
9:10–9:30	**Prof. Dr. Rodica Luca** Invited Speaker	Positive Solutions to a System of h-Riemann–Liouville Fractional Differential Equations with Coupled Boundary Conditions
9:30–9:50	**Prof. Dr. Yuri Luchko** Invited Speaker	The Generalized Convolution Taylor Formula Involving the General Fractional Integrals and Derivatives with the Sonin Kernels
9:50–10:05	**Maria Rosaria Lancia** Selected Speaker	On Non-Autonomous Fractional Semilinear Equations
10:05–10:20	**Pratibha Verma** Selected Speaker	A Darbo Fixed-Point Approach for Variable-Order Hybrid Caputo–Hadamard Sequential Differential Equations
10:20–10:35	**Pranay Goswami** Selected Speaker	Analytical Solution of a Generalized Fractional Diffusion–Advection Equation
10:35–10:50	**Yuri Drobotov** Selected Speaker	Riesz Fractional Calculus on Power-Weighted Variable Generalized Hölder Spaces over Metric Measure Spaces
10:50–11:05	**Asifa Tassaddiq** Selected Speaker	Gamma Transform
11:05–11:20	**Octavian Postavaru** Selected Speaker	A Hybrid Fibonacci-Based Approach for Inverse Problems in Nonlinear Fractional Differential Equations
11:20–11:35	**Jehad Alzabut** Selected Speaker	Long-Term Behavior of Non-Oscillating Solutions in High-Order Forced and Disturbed Fractional Difference Systems

S3. Numerical Methods for Fractional Calculus

S5. Fractional Calculus in Machine Learning: Applications and Challenges

Time: 14:00 (CEST, Basel) | 08:00 (EST, New York) | 21:00 (CST Asia, Beijing)

CEST Time	Speaker	Title
14:00–14:10	*Prof. Dr. Dimplekumar N. Chalishajar* *Welcome from the Session Chair*	
14:10–14:30	**Prof. Dr. Chris Goodrich** Invited Speaker	Qualitative Properties of Discrete Fractional Operators
14:30–14:45	**Reema Gupta** Selected Speaker	Operational Matrix Method for Variable-Order Fractional Stochastic Systems
14:45–15:00	**Mallanagoud Mulimani** Selected Speaker	Solution of Fractional-Order Brusselator Chemical Model by Homotopy Analysis Method
15:00–15:15	**Dipesh Dalal** Selected Speaker	Effects of Fractional Dynamics and Time Delay on Nutrient Transport in Blood Flow
15:15–15:25	*Prof. Dr. Yangquan Chen* *Welcome from the Session Chair*	

15:25–15:45	**Prof. Dr. George Anastassiou** Invited Speaker	Fractional Case of Composition of Activation Functions and the Reduction to Finite Domain
15:45–16:05	**Prof. Dr. Jesus Manuel Muñoz-Pacheco** Invited Speaker	Fractional-Order Memristive and Nonmemristive Hopfield Neural Networks: From Model to Circuit Implementation
16:05–16:20	**Yaroslav Olegovich Tsvira** Selected Speaker	Practical PINN Implementation for a Fractional-Order Damped Oscillator with CppAD-Computed Gradients
16:20–16:35	**Alexandra Maria Borș** Selected Speaker	Understanding the Black Box of Fractional Machine Learning Models
16:35–16:50	**Yuxuan Yang** Selected Speaker	Solving Fractional Squeeze-Film Flow in the Knee Joint with Physics-Informed Neural Networks
16:50–17:05	**Oluwaseun Olumide Okundalaye** Selected Speaker	Beyond Integer-Order Learning: The Role of Fractional Calculus in Next-Generation Machine Learning
17:05–17:20	**Mohammad Partohaghighi** Selected Speaker	Fractional-Order Differential Privacy

Session 1. Recent Advances in Fractional-Order Differential and Integral Operators

sciforum-165221: A Darbo Fixed-Point Approach for Variable-Order Hybrid Caputo–Hadamard Sequential Differential Equations

Pratibha Verma, Wojciech Sumelka

Institute of Structural Analysis, Poznan University of Technology, Piotrowo 5 Street, 60-965 Poznan, Poland

In this study, we discuss the boundary value problem involving a hybrid Caputo–Hadamard fractional sequential differential equation with variable order. The use of a variable fractional order is motivated by the fact that, in many applications, memory effects and system behavior change over time rather than remaining constant. Therefore, fractional models with constant order may be inadequate for describing the dynamics of complex systems. Our primary tool for constructing the results is the Banach space of continuous functions. Existence and uniqueness results are obtained by applying Darbo's fixed-point theorem in combination with Kuratowski's measure of noncompactness. This approach allows dealing with operators that are not compact while still ensuring the existence of solutions. The hybrid concept, in which Caputo and Hadamard fractional derivatives appear sequentially, offers a flexible formulation that can describe a wider class of problems than models involving a single fractional operator. A key idea in the approach is reformulating the considered boundary value problem as an equivalent operator equation. This representation enables the direct application of fixed-point theorems and facilitates a discussion of solution uniqueness under additional suitable assumptions that are easy to verify rationally. The stability of solutions is also examined. Using the concept of Ulam–Hyers stability, it is demonstrated that small perturbations in the problem data result in bounded variations in the corresponding solutions. To illustrate the theoretical results, an example supported by numerical computations is included. The results indicate that the proposed technique is suitable for problems in which system properties evolve with time.

sciforum-154383: Riesz Fractional Calculus on Power-Weighted Variable Generalized Hölder Spaces over Metric Measure Spaces

Yuri Drobotov

Institute of Mathematics, Mechanics and Computer Science named after I. I. Vorovich; Southern Federal University; Rostov-on-Don, 344058; Russia

Riesz fractional integro-differentiation is recognized as an analytical framework that defines fractional powers of the Laplace operator through potentials and hypersingular integrals. Furthermore, hypersingular integrals are established as natural extensions of partial differential operators to fractional orders, thereby enabling the development of fractional calculus for multivariate functions based on these operator classes. This study is devoted to the generalization of Riesz fractional calculus to abstract metric measure spaces, where classical results are subsumed as particular instances. The central objective is to enhance the theoretical framework with new fundamental results on weighted continuity characterized by specific generalizaitons of the Hölder condition. Three principal contributions are presented. First, the local modulus of continuity is refined through a generalized definition compatible with advanced theoretical constructs, extending its applicability to broader function classes on metric measure spaces. Second, Zygmund-type estimates are derived for power-weighted functions via a novel analytical approach, resolving technical challenges introduced by power-law weights that are absent in unweighted settings. Third, boundedness theorems are established to characterize the behavior of potential-type operators and hypersingular integrals on power-weighted variable generalized Hölder spaces, thereby completing the analytical foundation of Riesz fractional calculus in these generalized settings. The scientific significance of this work lies in its advancement of fractional calculus as a discipline within function analysis and operator theory. Specifically, the developed theory rigorously formalizes continuity properties of functions on abstract metric measure spaces, offering an approach that unifies classical and generalized perspectives. These contributions are anticipated to hold substantial implications for applications in integral equations and mathematical modeling, where the derived continuity estimates and operator boundedness properties may be leveraged to analyze complex systems under non-standard regularity assumptions.

sciforum-155882: The quasi-integral Read-Bajraktarević functional, symmetries and Hardy-Orlicz spaces

Waldo Arriagada

Department of Mathematics, New Uzbekistan University, Tashkent 100007, Uzbekistan

In this talk we prove the existence of complex local fractal functions of the Hardy-Orlicz class. A local fractal function is the fixed point of the Read-Bajraktarevic (RB) functional. The graph of the fixed point is the attractor of an appropriate iterated function system (IFS), whose construction is fairly standard. When the underlying function space is a Hardy-Orlicz space, the local fractal function of the Hardy-Orlicz class (fixed point of the RB operator) enjoys of peculiar properties stemming from the complex conjugation. For example, it is known that both the fixed point and its graph are intrinsically real. The latter reflects as an embedding of the realified attractor of the induced iterated function system into the product $[-1, 1] \times [-1, 1]$. The induced IFS in this case is denominated a Complex Iterated Function System (CIFS). In this talk we provide a characterization of this type of IFSs via a quasi-integral representation of the Read-Bajraktarevic (RB) functional. The realization of a contractive IFS in the (previously untreated) analytic case is obtained via a finite increments theorem for holomorphic functions, dating back to 1965. We prove that both the fractal function and the induced CIFS are actually the complex extension of their real counterparts. In a wide sense, our results generalize some of the outcomes obtained by Massopust *et al.* on Lebesgue and Sobolev spaces to higher orders, dimensions and function spaces (where the role of the norm is now played by a Young function). These results somewhat show that it would be natural to extend the Read-Bajraktarević operator to other function spaces on subdomains of differentiable and real analytic manifolds. Other questions, such as the existence of fixed points in higher-orders or more sophisticated function spaces, etc., remain open as well. Our generalizations may be useful in applications.

sciforum-163112: Uniform Weak Convergence Rates for CTRW Approximations of Time–Fractional Diffusions with Unbounded Coefficients

Artur Sidorenko [1,2,3,*], Vasiliy Kolokoltsov [2,3,4] and Anastasiya Kostrova [3,4]

[1] Faculty of Mechanics and Mathematics, Department of Probability Theory, Lomonosov Moscow State University, Moscow 119991, Russia
[2] Faculty of Economics, Laboratory of Stochastic Analysis and its Applications, National Research University Higher School of Economics, Moscow 109028, Russia
[3] Vega Institute Foundation, Moscow 119991, Russia
[4] Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, Moscow 119991, Russia

We study uniform weak convergence rates for probabilistic numerical schemes that approximate solutions of time-fractional diffusion equations with unbounded coefficients. The spatial part of the model is generated by a diffusion process with linearly growing drift and diffusion coefficients, covering in particular geometric Brownian motion. The time-fractional structure is described by the classical Caputo–Dzherbashian derivative. In probabilistic terms, the solution can be represented as a subordinated Markov process obtained by time-changing a diffusion with the inverse of a stable subordinator.

We investigate a solution method based on continuous-time random walk (CTRW) approximations. The construction combines discrete-time Markov chain schemes for the diffusion component with heavy-tailed random walk approximations of the stable subordinator.

Our analysis develops a high-order sensitivity framework for the associated diffusion semigroup, relying on Kunita’s stochastic flow theory and the chain rule for tensor fields. This allows us to control derivatives of the flow with respect to the initial condition as random tensor fields, obtain uniform (in space) and exponentially growing (in time) bounds for all orders of sensitivities, and derive corresponding estimates for the derivatives of the Markov semigroup. We also establish a quasi-contraction property of the semigroup in suitable weighted function spaces.

Using these ingredients, we prove explicit uniform weak convergence rates for the CTRW approximation applied to smooth test functions of linear growth. Depending on the spatial part of the equation, the convergence rate is either logarithmic in the time step or follows a power-law. Our results extend previous CTRW convergence theory from bounded to linearly growing coefficients, including applications to fractional Black–Scholes models.

sciforum-167377: A delay decomposition approach to stability of fractional-order neutral-type systems

Yener Altun

Faculty of Economics and Administrative Sciences, Department of Business Administration, Yuzuncu Yil University, Van 65100, Turkey

Fractional calculus, which enables the modeling of dynamic systems by generalizing the concepts of derivatives and integrals to fractional orders, is an important research topic. Fractional calculus is widely used in many scientific fields because it models real-world events more accurately than integer-order systems. This research aims to present new stability criteria for fractional-order neutral systems using a delay decomposition approach. One of the most important indicators of the qualitative behavior of fractional order differential equation systems is system stability. The Lyapunov method and the linear matrix inequality technique we used in this study are the most preferred methods. The stability criteria are obtained by constructing appropriate Lyapunov-Krasovskii functionals and using linear matrix inequalities. The difficulty of obtaining a fractional-order Lyapunov functional lies in how to design a positively defined functional V and easily determine whether the fractional derivative of V is less than zero. The Lyapunov method used in this study offers the advantage of being able to directly calculate the integer-order derivatives of the system under consideration. In conclusion, some theoretical results have been obtained for the fractional-order neutral type systems considered. A few simple examples are presented using MATLAB and Simulink to demonstrate the effectiveness and applicability of these theoretical results.

sciforum-164492: A Hybrid Fibonacci-Based Approach for Inverse Problems in Nonlinear Fractional Differential Equations

Octavian Postavaru

CITI (Research and Training Center in Innovative Techniques of Applied Mathematics in Engineering “Traian Lalescu”), Politehnica University of Bucharest, 060042 Bucharest, Romania

In this paper, we present a numerical method for solving inverse problems associated with nonlinear fractional-order differential equations, with the goal of identifying an unknown right-hand side function from over-measured data. The proposed approach is based on a newly introduced hybrid basis, referred to as the *fractional-order hybrid Fibonacci function*, constructed by combining block-pulse functions with Fibonacci polynomials. This hybrid representation exploits the strong approximation capabilities of Fibonacci polynomials together with the effectiveness of block-pulse functions in modeling discontinuous behavior, allowing for accurate approximation of both continuous and discontinuous solutions. The fractional-order feature is incorporated through the transformation $x \text{->} x^{\alpha}$ applied to the Fibonacci polynomials, where α is a real parameter. To the best of our knowledge, this hybrid basis is employed for the first time in the context of inverse problems for fractional differential equations. An exact Riemann-Liouville fractional integral operator is derived in closed form using the regularized beta function. By expanding the solution in terms of the proposed hybrid functions, the inverse problem is reduced to a system of algebraic equations with unknown coefficients corresponding to the right-hand side function. Discretization using Newton-Cotes quadrature nodes leads to a homogeneous system of algebraic equations, from which the coefficients of the hybrid expansion are determined. Substituting these coefficients into the solution representation allows for the recovery of the unknown nonlinear term. Several numerical examples are presented to demonstrate the accuracy and effectiveness of the proposed method. The results indicate that the use of an exact fractional integral operator significantly improves the quality of the approximation when compared with existing numerical approaches.

sciforum-153124: A Novel Approach to the Inverse Problem of Identifying Fractional Derivative Orders in Partial Differential Equations

Ravshan Radjabovich Ashurov

Laboratory of Differential Equations and their Applications, V.I. Romanovskiy Institute of Mathematics, Uzbekistan Academy of Science, Tashkent 100174, Uzbekistan

Determining the order of fractional derivatives in partial differential equations (PDEs) presents a significant challenge in applied mathematics. Fractional derivatives play a crucial role in accurately modeling complex physical phenomena, including anomalous diffusion, wave propagation, viscoelasticity, and various fluid dynamic processes. Unlike classical derivatives, fractional-order models account for memory and nonlocal effects, making them highly relevant for real-world systems. However, the direct measurement of the fractional order is typically impractical due to the lack of appropriate instruments or experimental techniques. As a result, this leads to an inverse problem—estimating the unknown fractional order from available, often indirect, observational data of the solution to the governing PDE.

Recent studies have made substantial progress in analyzing such inverse problems, particularly for time-fractional subdiffusion equations, where uniqueness results have been rigorously established. Nevertheless, many physical processes cannot be fully captured by subdiffusion models alone. This presentation explores a newly developed inverse problem formulation that extends the analysis to a broader class of equations, including fractional-wave equations, Rayleigh-Stokes-type models, and mixed-type fractional PDEs.

Our approach not only ensures the uniqueness of the estimated fractional order but also proves the existence of solutions under suitable conditions. This provides a more complete and versatile theoretical framework with strong potential for applications in engineering, physics, and beyond.

sciforum-162165: Analysis of Fractional Pantograph Equations with Riemann–Liouville Derivatives and Nonlocal Conditions

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This work develops a rigorous analytical framework for the study of nonlinear fractional pantograph equations involving sequential Riemann--Liouville derivatives of different fractional orders together with nonlocal integral boundary conditions. These models arise naturally in systems that exhibit memory, scaling effects, and nonlocal interactions, making their qualitative analysis both challenging and essential. The methodological approach relies on a combination of Schauder's fixed-point theorem and the Banach contraction principle to obtain fundamental results on the solvability of the problem. Within the Banach space $L^{1}(J)$, we establish the existence of at least one solution by demonstrating the compactness of the associated integral operator through the Kolmogorov compactness criterion. Under suitable Lipschitz-type assumptions, uniqueness is further guaranteed by showing that the corresponding operator is a contraction.

In addition, we investigate the continuous dependence of solutions on initial data and nonlinear terms, confirming the stability and robustness of the proposed model under small perturbations. An illustrative example is included to verify the theoretical framework and to demonstrate how the analytical results can be applied in practice. Overall, the findings of this study contribute to the qualitative theory of fractional pantograph systems and provide a unified basis for modeling complex nonlocal phenomena with memory effects in applied mathematics, physics, and engineering.

sciforum-164929: Analytical Solution of a Generalized Fractional Diffusion-Advection Equation

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This study investigates a generalised nonlinear fractional diffusion–advection equation incorporating concentration-dependent diffusion and nonlinear advection terms. The model $\mathcal{D}_t^\beta u(x,t) = \frac{\partial}{\partial x} \left(f(u) \frac{\partial u}{\partial x} \right) - \frac{dK}{du} \frac{\partial u}{\partial x}$, where, \(\mathcal{D}_t^\beta \) represents the Caputo derivative of the fractional order \(\beta \in (0,1] \), and \(f(u), \frac{dK}{du} \) are nonlinear functions. This formulation incorporates a concentration-dependent diffusion coefficient, allowing diffusive behaviour to vary with the state variable, together with a nonlinear advection term that accounts for convective effects more realistically. The mathematical model is expressed using the Caputo fractional derivative, which is particularly suitable for problems with physically meaningful initial conditions and memory effects. To establish mathematical well-posedness, sufficient conditions for the existence and uniqueness of solutions are derived by applying the Banach fixed-point theorem. This analysis guarantees that the problem admits a unique solution within an appropriate functional framework, providing a solid theoretical foundation for further investigation. For constructing approximate analytical solutions, the homotopy perturbation method (HPM) is employed. This technique yields solutions as rapidly convergent series without requiring small parameters or linearization of the governing equation. Several illustrative examples demonstrate the effectiveness, accuracy, and simplicity of the proposed approach. The obtained results confirm the applicability of HPM to nonlinear fractional diffusion–advection problems and generalise earlier studies on fractional diffusion equations and Burgers-type equations, offering a unified framework for analysing a wide range of nonlinear fractional models.

sciforum-153164: Application of a Generalized Fractional Fourier Transform with Memory to Nonlocal Fractional Differential Equations

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Application of a Fractional Fourier Transform with Memory to nonlocal fractional differential equations is proposed in this work. The new transform is defined as

$$
\mathcal{F}_{\alpha,\beta}[u](\omega) :=
\int_{-\infty}^{+\infty}
u(t)\,
\exp\!\Bigg(
i\, I^{\beta}\, g(\omega)\,
\int_{0}^{t} h(s)\,ds
\Bigg)\, dt ,
$$

where $g(\omega)$ and $h(s)$ characterize memory effects, while I^{β} provides an additional fractional degree of freedom. This formulation allows us to capture nonlocal and hereditary behaviors commonly observed in fractional-order systems, and reduces to the classical Fourier transform as a special case when $g(\omega)$ is constant, $h(s)=1$, and $\beta=0$.

Beyond its formal definition, the proposed framework offers a more flexible mathematical structure that is particularly well-suited to problems where memory effects play a dominant role. Many physical and engineering systems—such as anomalous diffusion, viscoelasticity, and biological processes—cannot be accurately described without explicitly incorporating hereditary dynamics. The memory-based kernel of the transform provides a natural mechanism for integrating such effects, thereby extending the applicability of Fourier analysis to a much broader class of problems.

In addition to its theoretical relevance, the transform establishes new operational relations with Caputo-type derivatives and fractional integrals. These relations enable the development of efficient analytical and semi-analytical methods for solving nonlocal fractional differential equations. Numerical illustrations further confirm that the proposed transform achieves better adaptability to varying memory characteristics compared with the classical Fourier and even standard fractional Fourier frameworks.

Overall, this study highlights the potential of the Fractional Fourier Transform with Memory as a versatile and powerful tool, bridging the gap between traditional Fourier analysis and the growing need for models that faithfully represent memory-dependent, nonlocal, and fractional-order dynamics.

sciforum-153128: Certain sandwich-type results obtained for Atangana-Baleanu fractional integral applied to hypergeometric function

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Atangana-Baleanu fractional integral operator presents particular importance due to its nonsingular Mittag-Leffler kernel which allows for this operator to be used in many branches of applied mathematics for development and study of mathematical models which involve it. The interest related to the study of hypergeometric functions in their connection to the theory of univalent functions has reappeared when L. de Branges used hypergeometric functions in the proof of the famous Bieberbach conjecture. Confluent (Kummer) hypergeometric function was studied lately from many points of view. Applying the differential subordinations and differential superodinations theories introduced by Miller and Mocanu, interesting inequalities can be established for the complex-valued functions, which generate sandwich results. In this paper we introduce a new operator defined by the Atangana-Baleanu fractional integral applied to confluent hypergeometric function which takes the following form and we establish several subordination and superordination results, which give considering together sandwich-type theorems. Interesting corollaries are stated using particular functions as best subordinant and best dominant of the subordinations and superordinations studied in theorems stated in this paper.

sciforum-162369: Estimate the non-Newtonian fractional Weddle's inequalities involving h-convex functions

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The definition and properties of the multiplicative derivative for the function $f: I^{\circ}\subseteq \mathbb{R}\to \mathbb{R^+}$ are given at the beginning of this abstract. Additionally, it gives a comprehensive description of the multiplicative integral and clarifies the characteristics that differentiate it from other types of integrals. In the realm of non-Newtonian calculus, the technique of multiplicative integration by parts is considered an important instrument. Secondly, this constitutes the first definition of h-convexity that is multiplicative by the use of the B-function, in addition to a number of cases in which convexity is multiplicative. This research provides evidence of a new form that utilizes the multiplicative absolute value, which is represented as $| \cdot |^{*}$, as well as other distinctive traits that have not been seen before. A presentation is offered that discusses the multiplicative Riemann-Liouville fractional integral operators of order $\alpha > 0$. These operators are formed based on a function $f : [a,b] \to \mathbb{R^{+}}$, where $\mathbb{R^{+}}$ represents the set of positive real numbers. This strategy framework proposes a new technique for dealing with the idea of multiplicative Weddle inequalities, and it is outlined in this publication. This is based on the ideas of fractional integrals as described by Riemann-Liouville. We put forward evidence that suggests the existence of yet another level of inequality. This conclusion is reached on the premise that the function in issue is multiplicative h-convex and that a multiplicative absolute value with unique properties exists. The findings that are provided are calculated by using multiplicatively P-functions in conjunction with multiplicatively s-convex functions.

sciforum-167464: Existence and Nonexistence for a Nonlinear Viscoelastic Fourth Order Equation with a Fractional Damping

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This study investigates the existence and nonexistence of solutions for a nonlinear fourth-order viscoelastic equation with a fractional damping in a bounded domain. The model at hand describes the dynamic behavior of elastic plates, taking into account both memory effects and nonlocal dissipation. In this framework, the viscoelastic term is represented by a convolution with a suitable relaxation kernel, while the damping mechanism is defined through a fractional power of the associated linear operator.

To establish the local existence, we reformulate the governing equation as an abstract evolution system within an appropriate Hilbert space. By demonstrating that the linear operator generates a strongly continuous semigroup of contractions and ensuring the local Lipschitz continuity of the nonlinearities, we apply semigroup theory to prove the local existence and uniqueness of solutions. Furthermore, we provide conditions under which these solutions attain the regularity of weak solutions based on the initial data.

Regarding long-term behavior, we derive energy identities and a priori estimates to show that solutions exist globally for sufficiently small initial data. Here, the interplay between viscoelastic memory and fractional damping enhances the dissipative structure, which is vital for the stabilization of the system. Conversely, we explore the potential for finite-time blow-up when the system is driven by large initial energy. Using a Lyapunov functional approach and the concavity method, we establish that solutions fail to exist globally under specific conditions related to the nonlinear source. Our findings also highlight the role of the fractional damping term, noting that while it may delay the blow-up, it cannot entirely prevent it in the presence of strong nonlinearities. This work extends the existing literature by integrating fractional calculus into the analysis of fourth-order evolution equations, offering new insights into the balance between memory effects and nonlocal dissipation.

sciforum-166488: Fractional power of resolvent operators and applications in fractional neutral integro-differential equations with delay

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In this talk, we discuss recent advances in the theory of resolvent operators for abstract fractional differential equations. Our focus is on the construction and analysis of resolvent operators associated with abstract evolutionary fractional integro–differential equations involving the Caputo derivative of order $\alpha \in (0,1)$. Using tools from perturbation theory, we establish sufficient conditions for the existence and well-posedness of the corresponding fractional resolvent operators in Banach spaces.

By employing the concept of the fractional potential of a strongly continuous semigroup of linear operators, we derive fractional power estimates for the associated fractional resolvent operator. These estimates play a crucial role in the qualitative analysis of solutions and allow us to relax spatial regularity assumptions on the forcing terms appearing in the equations. As an application of these results, we prove the existence of mild solutions for a class of fractional neutral integro–differential equations with infinite delay, formulated in suitable phase spaces.

Finally, we illustrate the applicability of the developed theoretical framework by studying a coupled fractional partial integro–differential system involving the Caputo derivative. For this system, we establish the existence of mild solutions by combining the obtained resolvent estimates with fixed point arguments. The results presented in this talk provide a unified and robust approach to the analysis of fractional evolution equations with memory and delay effects, contributing to the ongoing development of the theory of fractional dynamical systems.

sciforum-163523: Gamma Transform

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The Gamma function $\Gamma(z)$ is fundamental in analysis and fractional calculus; however, its poles at the negative integers prevent its direct use as an integral transform kernel. In this paper, we introduce the **Gamma transform**, a novel analytic operator derived from a distributional decomposition of $\Gamma(z)$. By representing the Gamma function as a convergent series of complex delta functionals, we obtain a regularized formulation that is free of singularities and well defined for real, fractional, and complex orders. The proposed transform exhibits structural properties analogous to classical integral transforms, including the Laplace and Mellin transforms, while incorporating discrete factorial-type weighting inherent to the Gamma function. Fundamental properties of the Gamma transform are established, including linearity, boundedness, inversion, and convolution theorems, ensuring consistency with standard operational calculus. This framework naturally extends classical transform theory to fractional settings without loss of analytic rigor. Applications to ordinary and fractional differential equations are presented to demonstrate the effectiveness of the Gamma transform in solving initial- and boundary-value problems, where it provides compact representations and inherent regularization. The results illustrate how the transform unifies classical analytic methods with modern fractional analysis. The Gamma transform offers a new, singularity-free tool for fractional calculus and related fields, contributing to the development of analytic techniques within the context of fractal and fractional systems.

sciforum-167446: Generalized Caputo-Type Fractional-Order Differential Equations with the Mittag-Leffler Function and Their Exact Solutions via the Laplace Integral Transform Operator

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In this study, within the framework of recent advances in fractional-order differential and integral operators, a fractional differential equation defined by means of a generalized Caputo-type fractional derivative is investigated. The proposed approach extends the classical Caputo fractional derivative by introducing a generalized kernel structure, thereby allowing a more flexible and comprehensive description of memory and hereditary effects. From this perspective, the study contributes to the theoretical development of fractional-order operators and their analytical properties.

The considered fractional differential equation involves the Mittag-Leffler function, which plays a fundamental role in the analysis of fractional-order dynamical systems. This special function naturally arises in the solution structure of fractional differential equations and is essential for characterizing the qualitative and quantitative behavior of fractional-order models. To derive an explicit analytical solution, the Laplace transform method is employed as a powerful and systematic tool for handling generalized Caputo-type operators. By applying the Laplace transform, the fractional differential equation is transformed into an algebraic equation in the Laplace domain, significantly simplifying the analytical treatment of the problem.

Subsequently, the inverse Laplace transform is applied to obtain the exact solution in closed form, expressed in terms of the Mittag-Leffler function. This result clearly demonstrates the intrinsic connection between generalized fractional derivatives and special functions commonly used in fractional calculus. Moreover, it is shown that, under suitable parameter selections, the proposed model reduces to the classical Caputo fractional differential equation, and several well-known results available in the literature are recovered as special cases.

The methodology and results presented in this work provide a unified and effective framework for the analysis of a broader class of fractional differential equations involving generalized kernels. The proposed model and solution technique offer valuable insights into the behavior of fractional-order systems and have potential applications in mathematical physics and various applied sciences where complex memory-dependent processes are encountered.

sciforum-162050: Long-Term Behavior of Non-Oscillating Solutions in High-Order Forced and Disturbed Fractional Difference Systems

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This paper develops a comprehensive asymptotic framework for analyzing the long-term behavior of non-oscillatory solutions in high-order forced and disturbed fractional difference systems of Caputo type. We consider a general nonlinear model in which memory effects, external forcing, and nonlinear disturbance terms interact within a higher-order nabla fractional operator. Despite the growing interest in discrete fractional calculus, existing results primarily address first-order or unforced systems, leaving a significant gap in understanding the asymptotic dynamics of complex high-order models with multiple nonlinearities. To address this gap, we derive new sufficient conditions ensuring that all eventually non-oscillatory solutions remain bounded within an explicit asymptotic envelope of the form

$$|\Psi(\iota)| = O\!\left((\iota^{n-1})^{1/\upsilon} R(\iota,c)\right),$$

where ($R(\iota,c)$) is a summable fractional kernel depending on the system's coefficients. The established criteria incorporate delicate growth restrictions on the forcing term, the nonlinear damping functions, and their relative exponents, thereby generalizing earlier theorems and offering sharper bounds.

The analysis further yields refined exponential-type bounds under additional summability conditions, highlighting how fractional memory and nonlinear perturbations shape the qualitative behavior of solutions. Two detailed numerical examples validate the theoretical findings and illustrate the precision of the derived envelopes. The results significantly extend the current theory of fractional difference equations, providing new analytical tools for models arising in discrete population dynamics, engineering, and other applications where memory and external disturbances play essential roles.

sciforum-162002: On a Boundary–Initial Value Problem for a Multi-Term Sequential Caputo Fractional Equation

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Fractional differential equations with sequential derivatives have become increasingly important because they can capture memory effects and complex behaviours that classical derivatives fail to describe. These tools are widely used today to model real processes in areas such as biology, chemistry, and physics, where systems often evolve in ways that depend not only on their current state but also on their history. Motivated by these applications, we investigates a new multi-term fractional differential equation involving sequential Caputo derivatives, motivated by recent research on fractional models with multiple memory effects. Unlike classical single-term models, the presence of sequential derivatives introduces additional analytical challenges, particularly in constructing explicit solution formulas.

Using techniques inspired by earlier works on sequential fractional operators, we derive the exact analytic solution of the problem in terms of the bivariate Mittag-Leffler function. The obtained representation provides a clear description of how the interaction of the two fractional orders influences the temporal dynamics of the system. Additionally, several useful properties of the bivariate Mittag-Leffler function are formulated to support the solution construction.

The results demonstrate that multi-term sequential fractional equations admit explicit closed-form solutions under suitable boundary and initial conditions. These findings contribute to the theoretical development of sequential fractional models and offer a practical framework for analysing complex diffusion-type processes with layered memory effects.

sciforum-167250: On non autonomous fractional semilinear equations

maria rosaria lancia, Simone Creo

Sapienza

We study non-autonomous semilinear evolution equations
$\partial_t^\alpha u=A(t)u(t)+J(u(t))$ $t\in (0,T)$
$u(0)=u_0$
with fractional-in-time derivatives, governed by sectorial operators A(t) that satisfy the classical Acquistapace–Terreni conditions.

These conditions ensure the well-posedness of the associated linear evolution families despite the lack of time invariance. Our analysis introduces the fractional solution operators S_alpha(t, τ) and P_alpha(t, τ), for which we establish ultracontractivity estimates that generalize classical heat-kernel bounds to the fractional and non-autonomous setting. These estimates provide a crucial tool for controlling nonlinearities.

Building on this linear foundation, we address the semilinear equation through fixed-point arguments formulated in weighted function spaces adapted to fractional temporal behavior. We prove local well-posedness for a broad class of nonlinearities, requiring only localized Lipschitz continuity and suitable growth conditions. Furthermore, under additional smallness assumptions on the initial data, we obtain global-in-time existence results. These findings extend and refine existing theories for both autonomous fractional equations and classical parabolic problems.

To illustrate the applicability of our abstract theory, we discuss a fractional heat equation with time-dependent, uniformly elliptic operators in non-divergence form. This example highlights how the developed framework accommodates PDEs with variable coefficients, nonlinear effects, and fractional temporal dynamics.

Reference: S. Creo and M. R. Lancia, Non-autonomous semilinear fractional evolution equations: well-posedness and ultracontractivity results, 2025.

sciforum-162035: Positive solutions to a system of h-Riemann-Liouville fractional differential equations with coupled boundary conditions

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Fractional-order differential and integral operators, as well as fractional differential equations, play a significant role in modeling real-world phenomena across a wide range of scientific and engineering fields. Compared to classical integer-order differential equations, fractional differential equations provide a more effective framework for capturing hereditary and memory effects inherent in many materials and processes. In this talk we investigate the existence of positive solutions for a system of two h-Riemann-Liouville fractional differential equations (S) in the unknown functions u and v, involving singular sign-changing nonlinearities and two positive parameters. This system is supplemented with general coupled boundary conditions (BC) that incorporate various h-Riemann-Liouville fractional derivatives together with Riemann-Stieltjes integrals. The h-Riemann-Liouville fractional derivative extends both the classical Riemann-Liouville derivative (when $h(t)=t$) and the Hadamard derivative (when $h(t)=\ln t$). We first derive the Green functions associated to problem (S)–(BC) and establish several upper and lower bounds for them. Then we make a change of the unknown functions (u,v) to (x,y), and the new problem (P) is written equivalently as a system of two integral equations (I). We then construct a suitable Banach space X and define an operator A in this space corresponding to system (I), noting that (x,y) is a solution of (I) (or problem (P)) if and only if (x,y) is a fixed point of operator A. Under appropriate assumptions on the nonlinearities of system (S), we identify intervals of the parameters for which the operator A admits at least one fixed point or at least two fixed points, which correspond to positive solutions of problem (P). The proofs of our main results rely on the Leray–Schauder nonlinear alternative and the Guo–Krasnosel'skii fixed point theorem. Finally, we provide two examples that illustrate the applicability of the obtained results.

sciforum-155890: Read-Bajraktarević functional on higher order Orlicz-Sobolev space

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In this talk, we consider a class of local Iterated Function Systems (IFSs) whose attractors are the graphs of local fractal functions both in a real Orlicz space and in a real Orlicz-Sobolev space over a domain of $\mathbb{R}^N$. We prove that these maps are in correspondence with the fixed points of the Read-Bajraktarević operator and justify a step in the proof of a theorem recently published in the literature. This result states that local fractal function of an Orlicz-Sobolev class of order $m>1$ (m is an integer) appear naturally as fixed points of the restriction of the Read-Bajraktarević functional. This was well-known since the works of Massopust et al. in the context of Lebesgue and Sobolev spaces. Our method extends a number of known theorems on the existence of local fractal functions to more general function spaces (where the role of the norm is now played by a Young function, also known as an N function), and to higher orders and dimensions. In this talk, we will relax a condition (inequality) that ensures contractivity of the Read-Bajraktarević operator, and we prove that an asymptotic quantity appearing in the proof is not required to be greater than 1. The existence of local fractal functions of the Orlicz and of the Orlicz-Sobolev classes is demonstrated through an intermediary result. The realization of a contractive IFS in the (previously untreated) multidimensional case is obtained via a stronger version of the mean-value theorem. Our results demonstrate that it is natural to extend the Read-Bajraktarević operator to other function spaces on subdomains of differentiable and real analytic manifolds. Other questions, such as the existence of fixed points in higher-orders etc., remain open as well.

sciforum-156256: Some families of complex and quaternionic regular-fractional Dirichlet-type spaces

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Theory of complex Dirichlet spaces is a very important topic in complex analysis. These function spaces consist of holomorphic functions whose derivatives are Lebesgue square integrable in norm. Similarly, the theory of quaternionic Dirichlet modules of slice regular functions is widely known as a natural extension of the theory of complex Dirichlet spaces via quaternionic analysis.

On the other hand, fractional calculus is a theory that allows us to consider integrals and derivatives of any real or complex order, where the fundamental theorem of usual calculus is also extended. Then, extending the concept of derivative through the fractional proportional derivative with respect to the truncated exponential function in the complex Dirichlet spaces and quaternionic Dirichlet modules we obtain many families of function spaces.

Some families of complex Banach spaces and quaternionic Banach modules of functions asociated to generalized fractional derivatives with respect to a truncated exponential function are presented. The aim of this talk is to present an interesting extension of the usual function theory of complex Dirichlet spaces and quaternionic Dirichlet modules in terms of the fractal-fractional calculus finding a large variety of families of complex Banach spaces and quaternionic Banach modules that contain the already known spaces.

sciforum-144912: Studying variable-order caputo fractional differential equations through non-compactness and fixed point theorem

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Intoduction: Fractional calculus is a branch of mathematical analysis that studies the possibility of extending the order of the differentiation and integration operators to a noninteger order,In this study we istablish new results of existence, using advanced mathematical tools such as the Kuratowski measure of noncompactness and fixed point theorems.

Main Results: in this research we are studying the the boundary value problem

where is a continuous function, and is the caputo fractional derivative of variable order.

be a partition of the interval and let be a PWCF with respect to , i.e., where are constants,(with). Using (H1) the BVP(1) becomes, ,

Lemma: Assuming that ,is continuous function and there exists a number such that

The solution of the integral equation, given by where

solves(2)

Theorem: Let the conditions of the Lemma be satisfied and there exist a constants, such that and the inequality holds. Then, (2) possesses at least one solution in Π_{ϑ}. The proof is established using the measure of noncompactness and Darbo's fixed point theorem, which leads to the existence results for BVP(1), and the uniquness is proved using the Banach fixed point theorem.

Conclusion:This Study is a valuable contribution to the expanding field of fractional calculus, in which we skillfully employed the Darbo's Fixed Point Theorem in conjunction with the Kuratowski Measure of Non Compactness, which is a valuable.

sciforum-171114: Synchronization of chaotic systems via hybrid adaptive backstepping for the comparison of different derivatives with novel fractional MABC operator

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The hybrid control law incorporating adaptive backstepping is suggested for the synchronization of two identical modified piecewise Rossler models. The suggested approach gives rapid, high rigorous chaos synchronization in presence of external disturbances and unknown system parameters. The adaptive backstepping control approach is established to stabilize the dynamics of synchronization error occurring from the conflict (mismatch) between the response and drive systems. The unknown external disturbance and unknown system parameters are determined by the adaptive backstepping law in such a way that whole system with the controller stabilizes. Performance of the suggested law is examined for a numerical solution and compared against frequently utilised chaos synchronization approach including active control, optimal control, and adaptive backstepping control approach. It was investigated for the aspect of chaos synchronization and synchronization errors. The synchronization of models via hybrid adaptive backstepping for the comparison of different derivatives with novel MABC fractional operator is exhibited. The MABC fractional operator is a nonsingular operator obtained by integration by parts of the standard fractional operator with Mittag-Leffler kernel. Effective approach is suggested for stabilizing chaos. Computer simulation results demonstrate the superior performance of the hybrid control approach over the state of art for the synchronization of modified Rossler models.

sciforum-162402: The generalized convolution Taylor formula involving the general fractional integrals and derivatives with the Sonin kernels

Yuri Luchko

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In this talk, contributions by Abel and Sonin to the origins of the general fractional integrals and derivatives with the Sonin kernels are first discussed. Whereas Abel introduced and employed the integral and integro-differential operators with the power law kernels that are nowadays referred to as the Riemann-Liouville fractional integral and the Caputo fractional derivative, Sonin extended his method to the case of pairs of arbitrary kernels whose Laplace convolutions are identically equal to one.

Then we mention some recent results regarding properties of the general fractional integrals and derivatives with the Sonin kernels. In particular, the first and the second fundamental theorems of Fractional Calculus for the general fractional derivatives, the regularized general fractional derivatives, and the sequential general fractional derivatives are formulated in the appropriate spaces of functions.

As an application of this theory, we discuss the convolution series that are a far-reaching generalization of the power law series as well as a representation of functions in form of the generalized convolution Taylor series. Another application is the generalized convolution Taylor formula that contains convolution polynomials and remainders given in terms of the general fractional integrals and the general fractional sequential derivatives. A known particular case of this formula is the fractional Taylor formula with the polynomials involving the power law functions with the fractional exponents and a remainder in terms of the Riemann-Liouville fractional integrals and the sequential Riemann-Liouville fractional derivatives.

Session 2. Fractional Calculus and Its Applications in Engineering Systems

sciforum-162909: Fractional Time–Space Viscoelastic Wave Equation: Analysis and Solution Framework

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We investigate a fractional generalization of the one-dimensional scalar wave equation with a singular memory kernel, a model originally introduced to describe single-mode propagation in viscoelastic media exhibiting weakly singular hereditary effects. The fractional formulation is constructed by replacing the standard second-order time derivative with the Caputo fractional derivative of order α, where the order ranges from one to two, and by replacing the spatial Laplacian with the Riesz fractional pseudo-differential operator of order β, with the order likewise ranging from one to two.

An explicit solution is obtained in the form of a series involving Fox H-functions, which arise naturally from the combined influence of temporal and spatial fractional operators. This series representation provides a mathematically tractable structure that facilitates asymptotic analysis and numerical approximation. Furthermore, in the limiting cases $\alpha=2$ and $\beta=2$, the formulation reduces to the classical viscoelastic wave equation, thereby illustrating the consistency of the fractional model with established theories of wave propagation in media with memory.

The results demonstrate that the fractional extension captures a broadened spectrum of propagation behaviors characterized by smoother wave fronts, delayed response, and modified dispersion patterns that depend sensitively on the fractional orders. These findings underscore the capacity of fractional operators to extend classical viscoelastic models and to describe complex dynamical effects associated with memory and spatial nonlocality in a unified mathematical setting.

sciforum-160890: Modeling and Analysis of Fractional Maxwell Fluid in Intra-Articular Drug Injection Flow

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Intra-articular drug injection is a core therapeutic approach for knee osteoarthritis (KOA), and its efficacy is closely related to the spatial distribution of the injected fluid. Supported by fractional-order derivative theory, this study first constructs an anatomically accurate model of the human knee joint cavity as a foundation for flow simulation. Given that the injected drug exhibits time-dependent viscoelastic relaxation with pronounced memory effects, the fractional Maxwell constitutive equation is adopted to more accurately describe its mechanical behavior. This constitutive model is then embedded into the fluid dynamic governing equations, which are discretized and solved using the finite volume method combined with the L1 algorithm. Through numerical simulations, we systematically investigate how key parameters—such as the characteristic relaxation time and the fractional derivative order—affect the intra-articular distribution of the injected drug. The results indicate that increasing the fractional derivative order enhances the effective distribution volume of the drug within the joint cavity and increases its retention in peripheral regions, while reducing distribution uniformity in the central region. By introducing fractional-order derivatives, this study enables a more precise representation of drug flow behavior during injection. The results elucidate the regulatory roles of key parameters in shaping drug distribution, thereby offering valuable scientific guidance for the development of personalized KOA injection-based treatment strategies.

sciforum-163039: Numerical Simulation of Caputo Fractional Oldroyd-B Fluid in a Top-Covered Square Cavity

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With the rapid advancement of materials science and chemical engineering, engineering applications involving complex viscoelastic fluids—such as blood flow, asphalt paving, and colloidal processing—are increasingly prevalent. The traditional Oldroyd-B model remains one of the most classical constitutive models for viscoelastic fluids, yet it exhibits limitations when characterizing fluids with intricate microstructures. Caputo fractional derivatives demonstrate significant advantages in describing the complex rheological properties and long-time memory effects of viscoelastic fluids. Therefore, this study aims to investigate the flow behavior of Caputo fractional Oldroyd-B fluids in a top-driven square cavity through numerical simulation. Based on ANSYS Fluent software and employing the finite volume method, this study coupled the fractional Oldroyd-B fluid constitutive equations with the Navier-Stokes equations through user-defined functions and user-defined scalars. The effects of different fractional derivative orders, Weissenberg numbers, and Reynolds numbers on flow field structures, vorticity distributions, and vortex center locations were investigated. To overcome numerical instability at high Weissenberg numbers, an artificial viscosity term was introduced into the transport equation, drawing on strategies from integer order models. Comparison with benchmark data from existing literature demonstrates good agreement in key metrics such as velocity distribution and vortex center location, validating the accuracy and effectiveness of the numerical model.

sciforum-167229: Parameter Space-Based Fractional-Order PD Controller Design and Analysis for Cooperative Adaptive Cruise Control Systems

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Cooperative adaptive cruise control (CACC), an extension of adaptive cruise control (ACC), is an important intelligent transportation solution for future mobility. It leverages vehicle-to-vehicle information to improve longitudinal tracking, traffic throughput, energy consumption and string stability. However, controller tuning remains sensitive to model uncertainties and communication delay. This paper presents an analytical parameter space based fractional-order PD (FOPD) controller tuning framework for the CACC problem. For the constant time headway spacing policy, the fractional-order controller parameters are explored over the (kp, kd, μ) parameter space considering the plant uncertainties. To enable a tractable stability assessment for the commensurate fractional-order characteristic equation, a variable transformation is used to obtain an equivalent polynomial form, and stability is then verified using the Hurwitz criterion. The resulting parameter space maps provide a transparent graphical approach during the controller gain and fractional-order selection. Moreover, the CACC design is completed by a feedforward controller that uses preceding vehicle acceleration information within the predecessor vehicle following communication topology. The proposed method is tested in a vehicle platoon simulation environment in terms of string stability and the time-domain responses of position, velocity, acceleration and headway time. The proposed controller is compared with both the integer-order ACC and the integer-order CACC. Simulation studies under representative CACC maneuvers demonstrate that the proposed parameter space approach enables systematic FOPD tuning and achieves improved performance requirements compared to integer-order PD control.

sciforum-163203: Analysis of Multivariable Control Loops in Distillation Columns Using Variable-Order Fractional Calculus

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This paper presents an advanced analysis of multivariable control loops applied to industrial processes using variable-order fractional calculus. The study focuses on a Multi-Input Multi-Output (MIMO) system represented by the classical Wood & Berry model of a binary distillation column. This benchmark model is characterized by cross-coupling and time delays, relating the distillate (X_D) and bottoms (X_B) compositions to the manipulated variables: reflux flow rate and reboiler heat duty.

To address the challenges of these coupled dynamics, the proposed control strategy employs Fractional Order PI controllers (Gc_1 and Gc_2) based on the Scarpi variable-order fractional integral. The fractional order od the integral parte of the controller, $\alpha(t)$, is modeled as a time-varying exponential function rather than a fixed value, allowing the controller to adapt its aggressiveness over time. To obtain the time-domain response, the closed-loop equations formulated in the Laplace domain were numerically inverted. While various inversion methods are reported in the literature, this work used the method proposed by de Hoog et al., implemented via the invertlaplace function from the Python MpMath package (Version 1.2.1).

The controller parameters were estimated by minimizing the Integral of Squared Error (ISE), resulting in a total objective function value of 4.04. The transient response analysis indicates a settling time of approximately 60 time units. The variable-order strategy successfully decoupled the system, suppressing the interaction peak on the bottoms composition (X_B) and returning it to the set-point, demonstrating robust disturbance rejection capabilities.

sciforum-162673: DRT, diffusive representation and infinite state description: application to fractional behaviours analysis and modelling

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The study of complex dynamical systems exhibiting memory effects, anomalous diffusion, or multi-scale relaxation has grown significantly across physics, electrochemistry, porous media, and biology. Experimental evidence shows that many real-world systems do not follow classical exponential laws or standard integer-order diffusion models. This has motivated the development of alternative mathematical frameworks capable of capturing long-range temporal correlations and broad relaxation spectra.

Fractional-order models have become popular because they naturally produce power-law memory kernels and can reproduce several experimentally observed behaviors. However, their use now appears increasingly problematic. Major limitations include: lack of clear physical interpretation of fractional parameters, structural non-uniqueness, numerical difficulties, multiplicity of definitions of fractional derivatives, physical inconsistence of initial conditions and the risk of misinterpreting fractional behavior.

These issues call for caution and for systematic criteria to determine whether a system truly requires fractional-calculus-based modeling. In many cases found in the literature, such justification is absent.

In this context, two alternative approaches, the Distribution of Relaxation Times (DRT) and diffusive representations, provide physically interpretable and mathematically transparent modeling tools. Both rely on expressing the system response as a continuous superposition of exponentially decaying modes, and they can be seen as equivalent formulations of the same underlying spectral framework.

The article therefore aims to: (1) clarify the theoretical links between DRT, diffusive representation, and infinite-state models; (2) show on several examples from the literature that DRT can rigorously distinguish genuinely fractional behaviors from behaviors that are not fractional (in order to avoid using fractional models unjustifiably); (3) demonstrate how DRT enables direct time-domain modeling from experimental data produced by a gas sensor with comparable accuracy and as few parameters as a fractional model ; and (4) illustrate how DRT-derived kernels yield accurate and physically meaningful models. Overall, the goal is to promote DRT and diffusive representations as robust alternatives to fractional-order models.

sciforum-163036: Fractional Approach to Dynamic Analysis of Coupled Circular Plates with Hereditary Effects

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This study investigates the dynamic behaviour of coupled circular plates connected by viscoelastic layers exhibiting creep effects. Such multilayer systems are widely used in aerospace, civil engineering, and microsystems, where time-dependent material properties and long-term loading significantly affect structural stability and performance.

The proposed analytical framework applies fractional calculus to capture hereditary effects in viscoelastic materials, offering a more accurate representation of time-dependent behaviour than classical models. Starting from D'Alembert's principle and the theory of hereditary viscoelasticity, we derive a system of partial integro-differential equations describing transverse oscillations of the plates. These equations are systematically reduced to ordinary integro-differential form using Bernoulli's method and solved via Laplace transforms, enabling closed-form solutions for key dynamic characteristics.

The results provide explicit expressions for natural frequencies, mode shapes, and time-dependent response functions. Analysis reveals the emergence of multi-frequency vibration regimes, with modal families remaining temporally uncoupled. This property facilitates precise identification of resonance conditions and dynamic absorption phenomena. The fractional parameter acts as a tunable damping factor: lower values allow prolonged oscillations, while higher values induce rapid decay. Additionally, increasing the kinetic stiffness of coupling layers raises vibration frequencies and amplifies hereditary effects.

These findings deliver a robust tool for designing multilayered structures with tailored dynamic responses. By adjusting fractional order and stiffness ratios, engineers can optimise vibration control, enhance stability, and mitigate long-term deformation risks under complex loading conditions.

sciforum-165616: Fractional Calculus for Space-Fractional Thermoelasticity

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Classical continuum theories cannot accurately describe thermoelastic behavior at the micro- and nano-scales due to scale-dependent effects. At these scales, long-range interactions significantly influence the thermomechanical response of structural systems. Fractional calculus has emerged as an effective framework for representing nonlocal interactions through space-fractional formulations within continuum mechanics. Although space-fractional structural models are capable of capturing scale-dependent mechanical behavior, thermoelastic effects in such formulations are not yet adequately addressed. This study develops a space-fractional thermoelastic framework to analyse thermal effects in micro- and nano-scale truss structures. Thermal influences are incorporated through temperature-dependent material properties, allowing the effect of temperature variation on structural response to be examined within a continuum-based formulation. A parametric investigation is carried out to study the influence of Young's modulus variation, material grain size, and material order, represented by the fractional order, under different temperature levels and boundary conditions. The analysis is performed within a thermal continuum-based space-fractional formulation, in which nonlocal interactions are described through a fractional-order representation embedded in the governing equations. The proposed framework advances thermomechanical modelling at small scales and it supports the design and optimisation of micro- and nano-scale devices that account for the thermal effects, including MEMS and NEMS structural components.

sciforum-163012: Investigation of Turbulent Heat Transfer in a Straight Tube Using Fractional Maxwell Nanofluid

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To address the issue of insufficient cooling efficiency in traditional thermal management solutions for high-heat-flux electronic devices, this study proposes a coupled model integrating a straight circular turbulent flow channel with a high-heat-flux chip. A carboxymethyl cellulose (CMC)-based Al_2O_3 nanofluid is selected as the working medium. The CMC base fluid exhibits typical linear viscoelastic behavior with notable viscoelastic memory decay characteristics. The addition of Al_2O_3 nanoparticles enhances the thermal conductivity of the fluid while preserving the viscoelastic properties of the base fluid, making it highly compatible with the fractional-order Maxwell constitutive model. Therefore, the Caputo fractional-order Maxwell fluid model is adopted to accurately characterize the viscoelastic memory effects of the CMC base fluid. Through user-defined functions (UDFs) in Fluent, this model is coupled with the k-ω SST turbulence model for numerical simulation. After validating the turbulence model, the influence of fractional-order parameters—namely the relaxation exponent (α) and relaxation time (λ)—on the turbulent flow structure, heat transfer performance, and temperature reduction of the electronic device is systematically investigated. The results demonstrate that the viscoelastic behavior of the fluid and turbulent fluctuations work synergistically to enhance heat transfer. Under optimized fractional-order parameters, a significant cooling effect on the electronic device is achieved. This research provides a novel cooling strategy for high-power electronic devices, and the established fractional-order modeling framework offers a valuable reference for similar thermal design and fluid dynamics studies.

sciforum-163014: Numerical Simulation of Fluid-Flexible Fin Fluid-Structure Interaction Based on a Fractional Model

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Existing studies have confirmed that fluid-structure interaction (FSI) between flexible structures and fluids can enhance heat transfer in enclosed and ventilated cavities, with relevant characteristics affected by structural elasticity, flow parameters, and other factors. However, current integer-order models fail to account for the viscoelastic memory effect of time-fractional Maxwell fluids and lack the application of fractional derivatives, making it difficult to accurately characterize the dynamic flow-deformation interaction. Taking the square cavity flow with flexible fins as the research object, and based on the advantages of Caputo fractional derivatives—including compatibility with physical initial conditions, simplicity of numerical discretization, and superior ability to fit long-term memory characteristics—this paper introduces Caputo fractional Maxwell shear stress into the constitutive relation and incorporates time-fractional derivatives into the momentum and energy equations respectively to establish the governing equations, thereby constructing an FSI model of the fluid and flexible fins. Numerical simulations comparing the coupling results of integer-order and fractional-order fluids show that the fractional order (α) directly regulates the intensity of viscous memory; analysis of differences in velocity distribution, vortex structure, and wall shear stress reveals that the fractional-order model exhibits a more lagging flow response. The research indicates that fractional derivatives are a key tool for characterizing the long-term memory properties of Maxwell fluids, which can effectively improve the physical authenticity of FSI models and provide reliable theoretical support for engineering applications such as micro flexible fluid devices and fluid transportation.

sciforum-161407: On the Analytical Treatment of Fractional Elliptic Equations: Applications to Steady-State Heat Conduction

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Modeling fractional behaviors is fundamental for characterizing complex phenomena governed by power-law dynamics, long-range temporal and spatial correlations, fractal structures, and memory effects, which are commonly observed in diverse scientific and engineering applications. In this work, we provide an analytical derivation of a fractional formulation to steady-state heat-conduction modeling. The boundary-value problem is defined on an annular domain in which the boundary conditions are prescribed as functions of the angular coordinate. The proposed approach integrates two analytical techniques, namely separation of variables and fractional power-series expansion, within a framework based on the Caputo fractional derivative. Beyond heat conduction, the methodology presented here is also directly applicable to a broader class of boundary-value problems governed by elliptic fractional differential equations, supporting advancements in modeling heterogeneous materials, porous media, biological tissues, and systems with multiscale or memory-driven behavior. A distinguishing aspect of this study is its emphasis on analytical transparency and didactic clarity, with each mathematical step introduced and developed systematically. The resulting closed-form solution offers a useful reference for future investigations involving fractional differential equations, contributing both methodological guidance and conceptual insight. The results include the explicit solution in polar coordinates and its interpretation as a function of an arbitrarily selected fractional order, thereby illustrating how fractional operators modulate steady-state heat conduction under the specified boundary conditions.

sciforum-162801: On the Optimal Mechanism of Aerodynamic Efficiency in Near-Ground Unmanned Aerial Vehicle Flight Using Fractional-Order Entropy

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During near-ground flight, unmanned aerial vehicles (UAVs) experience enhanced aerodynamic efficiency due to the ground effect. However, the underlying flow mechanisms governing this optimal state—particularly the critical role of flow-structure orderliness—remain insufficiently clarified. To quantify flow orderliness and uncover its relationship with efficiency, this paper proposes a diagnostic method based on fractional-order entropy. The method utilizes unsteady lift time-series data obtained from computational fluid dynamics. Recognizing that this data embodies the historical memory and hereditary properties of the flow field, we introduce the Grünwald–Letnikov fractional-order derivative—well-suited for discrete sequences—to perform multi-scale fractional differentiation on the lift series. This process extracts embedded features reflecting the memory and scale-invariant characteristics of the flow structure. Permutation entropy is then calculated from the resulting fractional-order differential sequences and directly defined as the fractional-order entropy, which quantifies the dynamic orderliness of the flow field. Results indicate that at the flight altitude corresponding to peak aerodynamic efficiency, fractional-order entropy exhibits a distinct minimum. This extremum signifies that the flow field under this condition achieves the highest level of dynamic order, where ineffective turbulent dissipation is significantly suppressed and energy is efficiently converted into lift. Moreover, a strong monotonic correlation exists between fractional-order entropy and aerodynamic efficiency, confirming its effectiveness as a quantitative diagnostic indicator. From the novel perspective of flow-structure orderliness, this study systematically explains the formation mechanism of optimal efficiency in near-ground flight. The proposed fractional-order entropy framework not only provides a new theoretical tool for understanding complex ground effects but also lays a methodological foundation for the development of autonomous energy-saving flight control systems based on real-time flow-field perception.

sciforum-162734: Spectral Analysis of Sierpinski Microwave Resonators

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Triangular Sierpinski structures are fractal geometries that have recently been studied for microwave frequency applications and implemented in microstrip and coplanar waveguide configurations. Antennas and planar components can be proposed using the Sierpinski geometry. The main advantage of the above solution is the definition of a building block with increased internal complexity to modulate the frequency of operation within a frequency range defined by the triangle's edge length, set as a fixed starting parameter. Increasing the complexity of the internal geometry by the number of sub-triangles inside the original figure helps define a shifted resonance frequency, thereby enabling frequency tuning by geometry and, through coupled structures, enhancing the electrical performance of the single triangle via edge coupling. A wider bandwidth is achieved by coupling Sierpinski triangles with increased internal complexity. The single building block must be appropriately excited to obtain the primary resonance frequency and the higher-order modes expected for the entire, non-subdivided triangle. Then, a spectral analysis is mandatory to get a working frequency close to the natural one and minimally affected by the feeding system. In this work, a systematic theoretical approach to reconstructing the resonator spectrum will be presented based on the edge length, the internal complexity, and the feeding system for microwave band-stop triangular Sierpinski resonators in the X-Band (8-12 GHz), using the equilateral triangle, and comparing the response of an electromagnetically simulated structure with the analytical results. Planar electromagnetic 2D simulations will be presented for silicon-based devices, i.e., metallized structures on high-resistivity silicon substrates with Sierpinski geometries, fed by microstrip lines. A perturbative approach will be used to predict the resonance frequencies of an internally subdivided triangle. Internally stretched triangles will also be presented, to propose a method to modulate the resonator's working frequency as an alternative to those characterised by increased internal complexity.

sciforum-159365: The use of generalised fractional models to elucidate the rheological behaviour of nanofluids

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Introduction: Recent research has demonstrated that nanofluids exhibit a nonlinear relationship between their shear stress and shear rates due to the differing particle characteristics. Some complex classical mechanical models have been employed to describe the rheological properties of nanofluids. Given the heterogeneity of nanofluids, the effects of anomalous diffusion, Brownian motion-induced nano-convection, clustering of nanoparticles, and thermophoresis often occur and cause a large disparity between the simulation results obtained by classical mathematical models and measured experimental data. The traditional rheological constitutive equations may not be suitable to describe these anomalous phenomena; however, they can be effectively addressed by using fractional calculus.

Methods: Justification for the use of fractional calculus in nanofluid modelling is discussed from the viscoelastic mechanical viewpoint. The well-established models, including Newtonian, Maxwell, Kelvin-Voigt, and anti-Zener models, are summarised and generalised to the fractional setting. Several classes of convolution kernel functions are derived, which are suitable for describing the rheological behaviour of viscoelastic nanofluids. The effects of using different fractional operators in the constitutive equation of nanofluids are also investigated.

Results: It is found that the Maxwell constitutive equation with the Caputo-Fabrizio or Atangana-Baleanu-Caputo derivative is a special case of the generalised constitutive model with the fractional Riemann-Liouville derivative. Validation of the fractional Maxwell constitutive model against experimental data of nanofluids demonstrates that the proposed fractional constitutive formulation can effectively capture the rheology of non-Newtonian nanofluids exhibiting shear-thinning type curves. Based on a novel development of the nonlocal constitutive relationship, a unifying generalised nanofluid model incorporating rheological and heat relaxation is proposed.

Conclusions: This study provides useful insights into considering the nanofluids as a kind of viscoelastic fluid using fractional calculus and highlights the potential of using nonlocal mathematical models to simulate the nanofluid flow in real applications.

sciforum-166017: Time-Dependent Electroosmotic Transport of a Fractional Second-Grade Tetra-Hybrid Nanofluid in a Compliant Multi-Stenosed Bifurcated Artery

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This study presents a theoretical investigation of time-dependent electroosmotic hemodynamics of a fractional second-grade fluid laden with tetra-hybrid nanoparticles (Au, Al_2O_3, TiO_2, and SWCNTs) flowing through a compliant bifurcated artery affected by multiple atherosclerotic stenoses. The mathematical model incorporates the coupled effects of time-varying electroosmotic forcing, arterial wall elasticity, and complex multi-stenosed bifurcation geometry to realistically capture the dynamics of blood-based nanofluid transport and targeted drug delivery. A fractional constitutive framework is employed to account for the hereditary and memory-dependent viscoelastic behavior of blood, while the inclusion of tetra-hybrid nanoparticles significantly enhances the effective thermal and electrical conductivities, thereby improving flow regulation and nanoparticle dispersion. The resulting nonlinear fractional governing equations are solved numerically using *Mathematica software*. The parametric analysis demonstrates that an intensification of the time-dependent electroosmotic field markedly suppresses axial velocity in both the parent and daughter arteries, while an increase in the fractional-order parameter further attenuates the velocity due to enhanced memory effects. The temperature field is observed to decrease with increasing nanoparticle volume fraction, whereas the absence of a nanolayer around nanoparticles leads to a pronounced enhancement in temperature, accompanied by dominant velocity modulation. The present model elucidates the synergistic interplay between electroosmotic actuation, wall compliance, and fractional rheology in optimizing nanofluid-based drug transport, offering novel insights for the design of advanced electrohydrodynamic therapeutic strategies in cardiovascular disease management.

Session 3. Numerical Methods for Fractional Calculus

sciforum-158885: Spectrally Accurate Collocation Methods for Fractional Differential Equations

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Over the past twenty years, the mathematical modeling of Fractional Differential Equations (FDEs) has attracted growing attention and expanded significantly across various scientific fields, including bioengineering, mathematical biology, physics, chemistry and computational medicine. In this context, finding analytical solutions of FDEs is often more challenging than for classical ordinary differential equations, while the derivation of accurate and reliable numerical methods suffers from the possible non-smoothness of the solution and/or the vector field at the starting time, not to mention that the efficient treatment of the persistent memory term can make long-time simulations computationally demanding, due to the non-locality of the operator. To mitigate the aforementioned issues, the class of Runge-Kutta type methods, named Fractional HBVMs (FHBVMs) is presented, covering its design, development and analysis. In particular, a novel extension is proposed, allowing for a mixed graded/uniform stepsize strategy, that gives rise to an updated version of the pre-existing fhbvm Matlab code. As confirmed by the numerical tests, the novel approach is mainly tailored for problems with non-smooth vector field at the starting time, whose solution is both non-smooth at the origin and oscillatory, to concurrently gain accuracy in reproducing the initial non-smooth behaviour of the solution, while maintaining efficiency over extended time periods.

sciforum-167506: Comparative Numerical Analysis of Bessel Integrals Arising in Space-Fractional Diffusion

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Diffusion within porous biological media, such as brain tissue often exhibits anomalous behaviour that deviates from classical Fickian laws, motivating the use of space-fractional diffusion models. Analytically tractable space fractional diffusion problems are typically resolved with the help of an integral transformation. Bessel integrals arise in the scope of radial Fourier transform on the plane, which is equivalent to the Hankel transform, and suffer from slow convergence.

This study focuses on the numerical evaluation of a space-fractional reaction-diffusion system with cylindrical symmetry that models the foreign body reaction around an implanted neural electrode. The system incorporates a space-fractional Riesz Laplacian to capture microscopic tissue heterogeneity, and its steady-state solution is derived via the Hankel transform, yielding expressions involving oscillatory Bessel integrals.

Numerical evaluation of these integrals is essential for practical application and parameter estimation. The foundational work implemented a Double-Exponential (DE) quadrature method, enhanced by a sine hyperbolic transformation for oscillatory kernels and accelerated through Wynn's epsilon algorithm applied over intervals partitioned at Bessel function zeros. In this extended analysis, we conduct a comprehensive comparison of the initial DE approach with two additional advanced quadrature methods: the Ogata quadrature and the sinc integration rule. Each method is systematically assessed for accuracy, convergence rate, and computational efficiency across a range of fractional orders and spatial ranges.

Our results demonstrate that fractional exponents produce heavy-tailed concentration profiles distinct from the exponential decay of integer-order solutions, and they provide clear performance benchmarks for these quadrature techniques. This comparative study not only validates the robustness of the numerical framework but also offers practical guidance for selecting efficient integration strategies when calibrating transport parameters from experimental data, thereby enhancing the reliability of anomalous diffusion models in biomedical applications.

sciforum-165039: Effects of Fractional Dynamics and Time Delay on Nutrient Transport in Blood Flow

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In this paper, the fractional-order delay differential equation (FDDE) model is used to explore the dynamics of nutrient transport of a vascular environment to delayed nutrient uptake, and pulsatile blood flow. The external nutrient input and nonlinear interaction term, the product of nutrient concentration and blood flow over time are also studied. The delay is introduced to consider the physiological lags which are causes to metabolic processing and absorption of nutrients via Caputo fractional derivative. The stability of the system is examined at a steady and periodically variable blood flow conditions. To get the constant flow, analytical expressions of the equilibrium points and the critical values of delay are examined. When the blood flow is pulsatile, and considered to be a sinusoidal function, the system has complex oscillatory dynamics and numerical simulations are used to analyze when the system becomes unstable. A sensitivity analysis is examined to analysis the impact of main model parameters. Findings show that longer delay times and larger amplitudes of pulsation enhance instability and steady oscillation of nutrient concentration. These results lend some understanding to the physiological connotation of the delayed interactions in the nutrient transport and could be used to guide future studies on metabolic control and circulatory mechanisms.

sciforum-162454: Modified Cubic Spline Approach for the Numerical Solution of the Fractional Order Generalized Wave Type Equation

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The present study examines the numerical simulation of the generalized fractional-order wave-type Regularized Long Wave (RLW) equation, an advanced model for characterizing nonlinear dispersive wave phenomena in shallow water dynamics, plasma physics, and other fluid-like media. The incorporation of fractional-order derivatives enriches the RLW framework by capturing memory effects and nonlocal interactions features that classical integer-order formulations cannot adequately represent. The proposed model is of great significance in simulating the behavior of complex wave phenomenon arising in real world systems such as anomalous diffusion and nonlocal elasticity.

For the numerical solution of the wave-type RLW model, the spatial domain is discretized using modified cubic B-spline basis functions to obtain smooth and accurate approximations. At the same time, the temporal fractional derivative is handled through a finite difference scheme. The method is assessed under various mesh sizes and time-step combinations to analyze its convergence properties, error behavior, and computational efficiency. The study further demonstrates how variations in the fractional order affect both the qualitative and quantitative characteristics of the resulting wave profiles. The obtained numerical results will provide deep insight into the underlying mechanisms that control wave propagation in nonlocal environments, which classical integer-order models cannot reveal. Overall, this combined spline-difference framework shows strong promise for efficiently solving fractional nonlinear dispersive wave equations.

sciforum-159783: Numerical simulation of flow and heat transfer of variable viscosity fractional order Maxwell fluid on stretched plate

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This paper investigates the unsteady flow and heat transfer behaviors of viscoelastic fluids over a stretched plate. A novel fractional-order Maxwell constitutive relationship is proposed, which forms a coupled velocity-temperature-stress system and innovatively incorporates the temperature dependence of relaxation time. The governing partial differential equations, characterized by high nonlinearity and coupling effects, are numerically solved via a hybrid method combining the spectral method (for spatial discretization, ensuring high accuracy) and the finite difference method (for temporal discretization, improving stability). Analytical solutions under simplified conditions are derived to validate the reliability and accuracy of the numerical results. Comparative analyses of velocity, temperature, and shear stress fields are conducted across different constitutive, focusing on the influence of the Improved Variable Viscosity Model on flow and heat transfer characteristics. This work provides a reliable theoretical framework and numerical tool for optimizing fluid manipulation and process design in engineering applications involving stretched plate configurations. It enriches the theoretical system of fractional viscoelastic fluid mechanics and offers critical guidance for enhancing the performance of related engineering systems such as polymer processing and heat exchanger design. Moreover, the proposed hybrid numerical method and constitutive model can serve as a reference for subsequent studies on complex fluid flow and heat transfer problems in other similar configurations.

sciforum-166750: Operational Matrix Method for Variable-Order Fractional Stochastic Systems

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Introduction: Variable-order fractional stochastic integro-differential equations (VO-FSIDEs) have attracted increasing attention due to their capability to model complex dynamical systems involving memory effects, hereditary properties, and stochastic perturbations. Such equations arise naturally in a wide range of applications, including anomalous diffusion, viscoelastic materials, population dynamics, and financial mathematics, where uncertainty and time-dependent fractional behavior play a significant role.

Method: In this study, an efficient numerical framework based on the operational matrix method is presented for solving VO-FSIDE . The unknown stochastic solution is approximated using Chelyshkov, Chebyshev, and Euler polynomial expansions. Operational matrices corresponding to the variable-order Caputo fractional differential operator are constructed for each polynomial basis. By employing Newton–Cotes nodes as collocation points, the proposed scheme transforms the original variable-order fractional stochastic integro-differential equations into a system of algebraic equations, which can be solved efficiently using standard numerical techniques such as the Gauss elimination method and Newton's iterative method.

Results: A comparative numerical study is conducted to evaluate the performance of the proposed polynomial-based methods in terms of accuracy, convergence rate, and computational efficiency. Several numerical examples, including problems with known exact solutions, are presented to validate the proposed methodology. The numerical results demonstrate excellent agreement with the exact solutions and reveal that the Chelyshkov polynomial-based operational matrix method achieves higher accuracy with fewer basis functions compared to the Chebyshev and Euler polynomial approaches.

Conclusions: The proposed operational matrix framework provides a reliable, accurate, and computationally efficient tool for solving VO-FSIDEs. The comparative results highlight the effectiveness of the Chelyshkov polynomial approximation for capturing variable-order fractional dynamics, making the proposed approach well suited for practical applications in science and engineering.

sciforum-160884: Solution of fractional order Brusselator chemical model by homotopy analysis method

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The Brusselator chemical model is a well-known theoretical framework for describing autocatalytic reactions, oscillatory behavior, and pattern formation in chemical and biological systems. It has been widely employed to investigate nonlinear phenomena such as morphogen distribution, biological pattern selection, and periodic dynamics arising from reaction–diffusion mechanisms. Owing to its ability to capture complex temporal and spatial behaviors, the Brusselator model also finds applications in diverse scientific areas, including plasma physics and laser dynamics.

In this study, a fractional-order Brusselator chemical model is considered to incorporate memory and hereditary effects inherent in many real-world processes. The governing fractional differential equations are analyzed using two distinct approaches: the Homotopy Analysis Method (HAM) and Euler's Modified Method (EMM). HAM is employed as a semi-analytical technique to construct approximate solutions in the form of convergent polynomial series, facilitated by an appropriate selection of the convergence-control parameter. In parallel, EMM is used as a numerical scheme to obtain approximate solutions of the model for comparison purposes.

The obtained semi-analytical and numerical solutions are validated by comparing them with the solutions generated using the NDSolve routine in Mathematica. A detailed investigation of the model behavior is carried out for different values of the system parameters and fractional orders. The comparative analysis demonstrates excellent agreement among the results obtained by HAM, EMM, and NDSolve, confirming the accuracy and reliability of the proposed approaches. The comparative study highlights the rapid convergence and flexibility of HAM, compared to the numerical method EMM.

Finally, this work provides new insights into the fractional Brusselator model and establishes an effective framework for analyzing fractional nonlinear dynamical systems. The findings contribute to the numerical analysis of fractional chemical models and are relevant to broader applications in physics, biology, and engineering.

sciforum-139501: The Caputo-like discrete fractional 4D hyperchaotic Rossler system

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Chaotic behavior is a common feature of nonlinear dynamics, as well as The subject of our discussion today is to study the stability of hyperchaos in high-dimensional systems. This study serves as an introductory guide to a discrete fractional four-dimensional hyperchaotic Rossler system with a Caputo-like operator is a complex system that can be used to study chaos of discrete fractional nonlinear dynamics. Our results demonstrate the existence of a hyperchaotic invariant set in these systems, which leads to extended hyperchaotic transient behaviors. The coexistence of chaos and hyperchaos is evident in the numerical results, which are presented in phase plots and bifurcation diagrams for various fractional orders and different parameters and specific initial conditions . These diagrams provide a comprehensive explanation of the dynamics of the proposed discrete system. This research substantiates the presence of chaos in discrete fractional hyperchaotic Rössler systems that are reminiscent of Caputo like discrete systems. Control low is offered to display synchronization of coupled Caputo-like discrete fractional hyperchaotic Rossler systems and to force the states of the proposed system to converge asymptotically to zero. The findings of the study are demonstrated through the following implementation of numerical simulations has been a significant development in our research.

Session 4. Fractional Calculus in Complex and Nonlinear Dynamical Systems

sciforum-163280: A Fractional Hodgkin-Huxley-Lotka-Volterra Model of Neuronal Synchronized Electromechanics

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Introduction. Neuronal synchronization is critical to various brain processes such as memory formation, motion control, and some brain disorders like epilepsy. Neuronal functions and synchronization are achieved through the dynamics of action potentials and chemical neurotransmitters that are intricately coupled with the mechanical responses of neurons that depend on neuronal structure and interactions of neurons with their shared environment. Mathematically, the propagation of action potentials can be described by the Hodgkin-Huxley model, and neuronal synchronization may be modelled using Hodgkin-Huxley models coupled by prescribed controller-based synchronizers. These mathematical models depict transport across neuronal membranes of specific ions but ignore neuronal mechanics. In particular, various studies have shown that neuronal functions are highly sensitive to changes in neuronal volume. The homeostatic mechanisms that conserve neuronal volume are facilitated by the mechanical interactions of neurons with the cerebrospinal fluid surrounding them. The aim of this work is to develop a mathematical model linking functions and volumes of synchronized neurons.

Methods. A mathematical model of two adjacent neurons that are functionally synchronized and volumetrically coupled is proposed. Neuronal functions are described by a fractional Hodgkin-Huxley model where the ion channels are assumed to behave mechanically as variable-order fractional Maxwell linear viscoelastic materials. Neuronal synchronization is described by a synchronization time, and the coupled volumetric dynamics is depicted by a modified coupled Lotka-Volterra model where the rates depend on action potentials and regulatory volumetric processes are represented as Michaelis–Menten-like terms.

Results. Computer simulations in MATLAB show oscillatory volumetric dynamics like the ones of action potentials, and the return of volumes to their initial values when regulatory mechanisms are present.

Conclusion. The proposed model allows the study of neuronal function-volume dynamics of brain health and disease that may help with the design of better therapies for various disorders.

sciforum-157335: A Study on Fractional Operators and Their Modelling via Generalized Integral Transform

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This study presents a generalized integral transform (GIT) approach for analyzing and modeling a wide class of fractional-order mathematical systems arising in physical, engineering, and biological sciences. The investigated models include Newton's law of cooling, the logistic population growth equation, and the blood alcohol concentration model, each formulated using distinct fractional derivatives such as the Caputo, Caputo–Fabrizio (CF), modified Atangana–Baleanu–Caputo (mABC), and constant proportional Caputo (CPC) derivatives. These fractional operators effectively describe memory-dependent and non-local characteristics inherent in many natural and engineered processes. Analytical solutions of the proposed models are derived through the generalized transform method, and graphical illustrations are provided to demonstrate the influence of various fractional orders on system dynamics. The results reveal that the GIT technique offers a unified, powerful, and efficient framework for solving a broad spectrum of fractional differential equations with diverse kernels. Furthermore, it integrates several classical and modern transforms—including the Laplace, Sumudu, Elzaki, and Formable transforms—as special cases, thereby simplifying computation and enhancing both generality and adaptability. This unified formulation provides researchers with a flexible analytical tool capable of addressing diverse problems without redefining operators for each model. The comparative analysis validates the stability, accuracy, and consistency of the proposed technique. Overall, this work highlights the efficacy, robustness, and broad applicability of the generalized integral transform, establishing a firm foundation for future explorations of hybrid fractional models and their interdisciplinary applications.

sciforum-166456: Bifurcation control and parametric influence analysis in a fractional bi-rhythm Van der pol-Rayleigh system driven by Gaussian colored noises

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The stochastic P-bifurcation characteristics of a bistable Van der Pol-Rayleigh system incorporating fractional-order inertia and damping are examined under the joint effects of additive and multiplicative Gaussian colored noises. By invoking the minimum mean square error criterion, the fractional-order inertial and damping components are reformulated as the combinations of integer-order stiffness, damping, and inertial terms, allowing the original fractional system to be transformed into an isovalent integer-order dynamical system.

On this basis, the stationary probability density function of the system amplitude is obtained through the stochastic averaging technique. The parametric conditions governing the emergence of stochastic P-bifurcation are then identified using singularity theory. Furthermore, the qualitative topology evolution of the steady-state probability density is investigated by analyzing representative parameters located in distinct regions delineated by the transition curves.

To verify the analytical process, the numerical results carried out by Monte Carlo simulations and Radial Basis Function Neural Network (RBFNN) are compared with the theoretical solutions obtained. The high agreement among these results confirms the validity of the analytical framework and the accuracy of the transition set acquired. The findings provide theoretical insights into the control of bifurcation and vibration suppression in nonlinear fractional systems under random excitation, with potential applications in engineering design, aeroelastic stability and vibration control of structures with memory-dependent properties.

sciforum-162172: Continual Impulse-Based Finite-Time Fractional Difference Logistic Map Stabilization Scheme

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This research is about an impulse-based control scheme capable of stabilizing the unstable period-1 orbit of the fractional difference logistic map. In 1876, a biologist named Robert May popularized the first known simple mathematical model with chaotic solutions. This system utilizes ordinary differences and has a memory horizon of one step. A few decades later, the fractional difference logistic map was created, exhibiting a memory horizon reaching the initial condition through the utilization of Caputo fractional differences. Due to its valuable properties, the fractional difference logistic map has already been adapted in economics, steganography, epidemiology, and engineering. In recent years, we have explored some impulse-based control schemes to stabilize this map's unstable period-1 orbit. In our recent research, it was shown that a naive control scheme does not achieve finite-time stabilization of the unstable period-1 orbit. Instead, an H-rank-based approach is used to achieve finite-time stabilization. However, this version of the control scheme exhibited violent oscillatory transient processes following the control impulse. To improve the stabilization scheme, several findings were made. Firstly, at the cost of some stabilization duration, it is possible to stabilize the unstable period-1 orbit by finding a suitable initial condition within the tolerance corridor δ. Secondly, the coordinate for the unstable period-1 orbit drifts each time a control impulse is applied. And finally, this drift is caused not only by the perturbations caused by numerous stabilization impulses but also by computational effects. In conclusion, this proposed control scheme is minimally invasive compared to continuous feedback control because it preserves the system model and only requires a series of small, sparse, and instantaneous control impulses to achieve continuous adaptive stabilization for the unstable period-1 orbit of the fractional difference logistic map.

sciforum-166507: Control and synchronization of Julia sets of a discrete three-dimensional fractional HCV model

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This paper investigates the fractal dynamics of a discrete Caputo fractional-order hepatitis C virus (HCV) system. Due to the memory effect and the hereditary property, the discrete fractional model helps in the study of the fractal behavior of the HCV model. First, the three-dimensional integer-order HCV model is extended to the fractional-order one, and the corresponding Julia set is defined. Next, a fractional-order controller based on a coordinate transformation is designed to control the system's Julia set. The stability interval of the controlled system is determined by calculating the spectral radius of the Jacobian matrix at the system's fixed point, and numerical simulations are then presented to illustrate how the Julia set changes as the control parameters are increased within the stability interval. In addition, a nonlinear coupling controller is constructed and added to the three-dimensional model to achieve synchronization between two discrete fractional-order systems with different fractional orders and different parameters. Rigorous mathematical proofs are provided to establish the correctness and effectiveness of the proposed synchronization method. Moreover, numerical simulation figures are presented to illustrate the synchronization of the response system's Julia sets toward the target system's Julia sets as the synchronization parameters vary. These results contribute to a systematic characterization of fractal dynamical behavior of the fractional-order HCV system and provide useful insights for proposing HCV control strategies.

sciforum-163341: Fractal and Fractional Approaches to the Analysis of Heart Rate Variability

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Heart rate variability (HRV) is an essential physiological marker widely used to assess cardiovascular regulation and autonomic nervous system activity, reflecting complex, nonlinear dynamics in biological control mechanisms. HRV signals demonstrate long-range correlations, scale-invariant properties, and memory effects that challenge traditional integer-order modeling approaches. Fractal geometry and fractional calculus have emerged as powerful frameworks for analyzing such physiological signals, enabling a deeper understanding of temporal and structural complexity. This work explores the application of fractal measures and fractional-order modeling techniques to HRV analysis, emphasizing their ability to capture multiscale structures, self-similarity, and non-local temporal dynamics. Various fractal descriptors and fractional formulations reported in the literature are conceptually compared in terms of their biological interpretability, analytical performance, and potential application under different physiological or pathological conditions. Special attention is given to how non-integer order operators enhance the representation of physiological variability, anomalous fluctuations, and adaptive responses in cardiac rhythms. Furthermore, the discussion includes potential clinical and research implications, highlighting how these approaches may contribute to improved monitoring, diagnosis, and understanding of cardiovascular health. The synthesis highlights methodological trends, advantages, and current limitations of fractal and fractional approaches, providing a comprehensive perspective on the study of complex biological rhythms. Overall, this work supports the integration of advanced mathematical tools in quantitative biology and underscores their potential for improving understanding and monitoring of cardiovascular dynamics.

sciforum-162312: Fractional Differential Equations as Nonlocal Dynamical Systems: A new Perspective

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The study [Cong, N. D., & Tuan, H. T. (2017). {Generation of nonlocal fractional dynamical systems by fractional differential equations.}] examines nonlinear Caputo fractional-order differential equations $D^{\alpha} x = f(t,x)$ to initial conditions x_0 in the reals R^d of dimension d and the extent to which this differential equation generates a nonlocal dynamical system on R^d . Therein, a main conclusion is that this is generally not possible for dimensions d larger than 1, as different trajectories may meet in finite time.

On the other hand, linear time-periodic systems $D^{\alpha} x = A(t)\, x$ with $A(t) = A(t+T)$ and period time T are yet to be rigorously analyzed in the fractional-order case. In the case of functional differential equations in the sense of Hale, the respective generated dynamical system on the space of continuous functions C plays an important role in the development of Floquet theory for linear time-periodic systems. Both functional and fractional differential equations show nonlocal behavior.

We thus study a framework of Caputo-type differential equations to the lower bound - ∞, namely $D_{-\infty}^{\alpha}\, x = f(t,x)$, where an initial condition $\varphi \in S$ must be prescribed to formulate an initial value problem and S is a subset of the space of continuous functions $C(-\infty,0]$. We show, that this initial condition manifests as a time-dependent forcing term $|F\varphi\,(t)| \leq b\,(t+\eta)^{-\alpha}$ on the right-hand side of the differential equation, decaying algebraically, where $b, \eta > 0$. Further, we use this bound to analyze solutions of linear time-autonomous systems.

Finally, we investigate, how such a system may be posed to generate a nonlocal dynamical system on S. In particular, we provide nontrivial conditions under which a solution trajectory belongs to S.

sciforum-163208: Influence of Fractional and Integer Derivatives on the (4+1)-Dimensional Fractional Davey Stewartson Kadomtsev Petviashvili Equation

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Fractional-order models are capable of capturing memory, nonlocality, and anomalous dispersion, making them powerful tools for describing nonlinear wave propagation in complex media. This work investigates the influence of fractional and integer-order derivatives on the multidimensional fractional Davey–Stewartson–Kadomtsev–Petviashvili (FDSKP) equation, a higher-dimensional model widely used to characterize internal waves, optical pulses, and fluid–structure interactions. Here, the term fractional parameters refers explicitly to the orders of the fractional time derivatives appearing in the governing equations no spatial fractional derivatives are considered. Physically, fractional time derivatives introduce temporal nonlocality through memory effects, implying that the system evolution depends on its entire past history and can capture long-memory dynamics beyond the capability of classical integer-order models. Fractional beta, M-truncated, and classical integer-order derivatives are incorporated, and the resulting modifications in wave structures under different operator definitions are systematically analyzed. The FDSKP equation is reduced to an ordinary differential equation via appropriate traveling-wave transformations corresponding to each derivative type, and exact analytical solutions are obtained using the Modified Auxiliary Equation (MAE) and Jacobi Elliptic Function (JEF) methods. The proposed framework generates a rich family of wave structures, including bell-shaped, W-shaped, composite dark–bright, and periodic waves. The results demonstrate that the fractional parameters effectively regulate the amplitude, steepness, and overall wave dynamics with greater flexibility than classical integer-order derivatives. Furthermore, two and three dimensional visualizations reveal the dependence of dispersion characteristics and symmetry on the fractional order. Overall, the findings confirm that the choice of fractional operator significantly influences the physical interpretation of the model, highlighting the importance of fractional calculus for accurately representing complex wave dynamics in higher-dimensional nonlinear systems.

sciforum-165977: Modeling Air Pollution–Mortality Interactions Using Fractional Calculus and Stochastic Processes

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Air pollution is one of the most significant threats to environmental sustainability and public health, with particulate matter PM2.5PM_{2.5}PM2.5 and PM10PM_{10}PM10 recognized as among the most hazardous pollutants. Elevated concentrations of these fine particles are strongly associated with cardiovascular and respiratory diseases, leading to increased mortality worldwide. Understanding the dynamics of particulate matter and its health impacts is therefore essential for effective mitigation strategies and policy development. In this study, we propose a novel mathematical model to investigate air pollution dynamics by incorporating population, PM2.5, PM10, and pollution-induced mortality as key variables. The model is analyzed under deterministic, fractional-order, and stochastic frameworks. Fractional-order modeling is formulated using the Caputo derivative to capture memory and hereditary effects, while stochastic differential equations are employed to account for environmental randomness and uncertainty. The well-posedness of both the fractional and stochastic models is rigorously established. Stability and boundedness properties of the fractional-order system are examined, and the feasibility of equilibrium points is analyzed using isoclines and asymptotic behavior. For numerical simulations, the Adams–Bashforth–Moulton method is applied to the fractional-order model, whereas Milstein's scheme is utilized for the stochastic system. Sensitivity analysis is conducted to evaluate the influence of key parameters on system dynamics. To enhance predictive performance, machine learning techniques are integrated with the mathematical framework. Data-driven forecasting methods, including the ARIMA model and random forest regression, are employed to capture both short-term fluctuations and long-term trends in pollutant levels. By combining analytical modeling with data-driven approaches, the proposed framework improves forecasting accuracy and provides deeper insights into the complex interactions among air quality, particulate matter, and associated health risks.

sciforum-162159: Modulating Neuronal Entrainment via Distinct Fractional Derivative Orders in the FitzHugh-Nagumo Model

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Synchronization of neural activity to external rhythms is fundamental to information processing and network coordination. However, classical integer-order models often neglect the power-law adaptation and memory effects inherent in biological tissues, such as ion channel gating and membrane impedance. This study investigates the synchronization dynamics of a periodically forced FitzHugh-Nagumo system, incorporating fractional-order calculus to explicitly model these history-dependent behaviors. We also introduce an asymmetry in the model by assigning distinct fractional orders to the fast membrane potential and the slow recovery variable to isolate their specific biophysical contributions.

The system is numerically investigated across a broad two-dimensional parameter space of forcing amplitude and frequency. We employ rotation numbers and inter-spike interval statistics to systematically map phase-locking zones -- known as Arnold tongues -- and to classify complex dynamical regimes versus stable periodic firing.

The investigation reveals that the two fractional orders play fundamentally different, non-interchangeable roles in shaping the entrainment landscape. Introducing memory into the voltage dynamics acts primarily as a stabilizing damping mechanism, significantly suppressing chaos and higher-order locking in favor of robust 1:1 synchronization. In contrast, increasing memory in the recovery variable functions as a timescale modulator, systematically shifting the system's resonant frequency and entrainment windows toward lower bands. Furthermore, we observe that these effects can compete, where recovery memory can partially restore dynamical complexity to a system otherwise simplified by voltage memory.

We conclude that fractional order asymmetry serves as a flexible biophysical mechanism for tuning neuronal response. By independently adjusting stability and frequency selectivity, neurons can optimize their entrainment to rhythmic stimuli, offering new theoretical insights into the modulation of neural circuits and the potential for targeted neuromodulation strategies.

sciforum-144710: On fractional generalizations of the logistic map and their applications

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The regular logistic map was introduced in 1960s, served as an example of a complex system, and was used as an instrument to demonstrate and investigate the period doubling cascade of bifurcations scenario of transition to chaos. The first fractional generalization was introduced in 2002. The two generalizations which are based on the continuous and discrete Caputo fractional calculus are the fractional logistic map (FLM) and the fractional difference logistic map (FDLM). They are well investigated. The finite time evolution of the FLM and FDLM is characterized by cascade of bifurcations type trajectories (CBTT) and strong dependance on the initial conditions and the number of iterations. The map's asymptotic behavior is described by the conditions of the asymptotic stability, the rate of convergence to the asymptotically periodic points, asymptotic bifurcation points, and the transition to chaos. The maps were used to show numerically that the fractional Feigenbaum constant δ exists and is equal to its regular value. Fractional generalizations may also be used to naturally introduce the 2D and 3D logistic maps. Applications of the FLM and the FDLM include cryptography, distribution of ageing, population biology, etc. One of the most important opened problems is a theoretical analysis of the asymptotic universality in fractional dynamics which could be based on the derived equations defining the asymptotically periodic and bifurcation points.

sciforum-165258: On the dynamics of a nutrient-plankton-fish ecosystem with Caputo fractional operator

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In this study, we propose a nutrient-plankton-fish interaction model in a marine environment incorporating the Caputo fractional operator to investigate the role of various parameters, such as the mortality rate of zooplankton, the maximal ingestion rate of fish, and the nutrient consumption rate by phytoplankton, on stabilizing the system. To investigate the qualitative properties of the model, the fundamental properties of its solutions and the stability of its equilibria are examined. Further, to investigate the effect of the order of the fractional derivative on system stability, we have arbitrarily chosen four values: 0.91, 0.94, 0.97, and 1. To verify the theoretical results on the system's stability, we present numerical examples using MATLAB. The findings identify specific biological rates that contribute to system instability, providing critical information for control. Reducing zooplankton mortality, potentially through reduced pesticide use, may unintentionally disrupt population dynamics, leading to erratic fluctuations. Also, the negative relationship between fish ingestion rate and stability serves as a cautionary tale for fisheries managers. Policies that actively promote high ingestion rates risk causing the entire fish population to collapse into chaotic patterns. Moreover, the fractional-order derivative approach enhances realism by accounting for memory effects, offering deeper insights into stability and resilience compared to classical integer-order models.

sciforum-163108: Operator-based Comparative Study of Anomalous Thermal Diffusion

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For decades anomalous diffusion has attracted growing attention as a fundamental transport mechanism in complex and heterogeneous systems. In media such as porous materials, polymers and biological tissues transport often deviates from classical diffusion, so conventional diffusion models fail to describe these phenomena. These deviations are typically caused by memory effects, nonlocal interactions, structural heterogeneity and strong correlations between constituents. Fractional diffusion equations have emerged as powerful tools to overcome these limitations, extending classical approaches by incorporating memory kernels and nonlocality. In this study, we investigate and compare solutions of fractional diffusion equations formulated using Caputo, Caputo-Fabrizio and Atangana-Baleanu operators. Analytical solutions are derived and systematically analyzed to investigate the specific impact of different fractional operators on diffusion profiles, transport dynamics and memory-dependent behavior, highlighting how the choice of operator influences these characteristics. Our comparative analysis reveals notable differences in transport patterns between operators, offering further insights into the interplay of memory effects, structural heterogeneity and boundary-induced effects. This study provides a comprehensive framework for modeling subdiffusive transport and demonstrates the effectiveness of different fractional operators in capturing anomalous diffusion phenomena in complex media. The findings have potential applications in photothermal characterization and imaging of various functional materials and biological tissues.

sciforum-160026: Scatter plots on the dynamical system of coupled nonlinear Klein-Gordon equations

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The Klein-Gordon equation with cubic nonlinearity is studied in theoretical and numerical manners. The master equation is known to hold a spontaneous symmetry breaking. Based on the high-precision numerical method [1] consisting of the Fourier spectral method for space and the implicit Runge-Kutta method for time, the detail structure of dynamical systems around Lyapunov-stabile stationary states are studied by Ref. [2]. Here it is notable that the appearance of breather waves is suggested to be associated with the stability of Lyapunov-stability of stationary solutions [3]; more precisely, the breather solution appears in the critical situation when the constant stationary state starts to lose its stability. In this paper, based also on the high-precision numerical method [1], finite-dimensional scatter plots are introduced to infinite-dimensional dynamical systems of coupled nonlinear Klein-Gordon equations. The proposed plot is a kind of finite dimensional representation of the originally infinite-dimensional dynamical systems [4]. Scatter plots show a wide variety of geometric shapes, which are sometimes holds the fractal structure. In particular, the difference between the global existence and finite-time blow up of solutions are illustrated in a geometric way.

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sciforum-162160: Structural Optimization of Integer-Order Approximations for Fractional Differential Equations

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This study presents a computational framework designed to analyze the intrinsic structure of fractional differential equations (FDEs) by mapping them onto optimal ordinary differential equation (ODE) architectures. While FDEs are instrumental in modeling non-local dynamics and memory effects, analyzing their fundamental structural properties—distinct from obtaining numerical solutions—remains a complex challenge. The proposed methodology transforms FDEs involving iterated Caputo derivatives into a higher-order representation comprised of two distinct elements: a parameterizable integer-order polynomial component and a residual fractional power series.

Crucially, this approach posits that the integer-order component is not a static derivation but a flexible, parameterizable architecture. By employing global optimization techniques, specifically Particle Swarm Optimization (PSO), the framework searches for a polynomial structure that minimizes a dual objective: the deviation from a high-fidelity reference solution and the magnitude of the truncated residual series. This process effectively identifies the most parsimonious ODE that captures the dominant dynamics of the fractional system.

Numerical experiments conducted on both a linear FDE and a nonlinear fractional Riccati equation demonstrate the framework's efficacy. The results reveal that the optimal integer-order representation often requires a higher polynomial degree than the forcing function of the original FDE to compensate for the transformation of the fractional operator. For instance, a quadratic ODE architecture was identified as the optimal representation for a linear FDE. This work provides a novel tool for model reduction and structural analysis, offering deeper insights into the interplay between local integer-order dynamics and non-local fractional memory effects.

Session 5. Fractional Calculus in Machine Learning: Applications and Challenges

sciforum-167307: Beyond Integer-Order Learning: The Role of Fractional Calculus in Next-Generation Machine Learning

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Introduction: Data-driven modelling in the fields of science, engineering, and health has been revolutionized by the quick development of machine learning (ML). However, the majority of current machine learning frameworks rely on traditional integer-order calculus, which frequently falls short of capturing memory effects, long-term relationships, and inherited characteristics present in many real-world systems.

Method: The goal of this study is to develop and analyze fractional-order machine learning architectures, including fractional dynamic systems for time-series prediction, fractional neural networks, and fractional gradient-based optimization methods. Applications in biomedical signal processing, epidemiological modelling, and socio-behavioral systems, where memory effects and delayed reactions are critical, will receive special attention. The project aims to improve model accuracy, stability, and interpretability in complicated situations by integrating Caputo and Riemann-Liouville fractional derivatives into learning dynamics.

Results: Despite its potential, several obstacles hinder the application of fractional calculus in machine learning, including the lack of standardized training frameworks, increased computational costs, numerical instability, and parameter identifiability issues. In order to facilitate practical implementation, this study will methodically investigate these difficulties and suggest effective numerical methods, regularization techniques, and scalable algorithms. A unified theoretical framework for fractional-order learning systems, validated application case studies, and open-source computational tools to facilitate additional study in this developing subject are among the anticipated results.

Conclusion: This project intends to contribute to the next generation of intelligent systems that can more accurately model memory-dependent phenomena by bridging cutting-edge mathematical theory with contemporary artificial intelligence techniques. This will advance machine learning theory and practice in complex, real-world settings.

sciforum-133928: Discrete fractional order plant absorption carbon dioxide: Analysis by artificial intelligence

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A nonlinear mathematical model is proposed to examine the role of plants with varying abilities to absorb atmospheric carbon dioxide (CO_2) and its broader ecological impacts. Different plant species exhibit distinct CO_2 absorption capacities, which can significantly influence the overall reduction of atmospheric CO_2 levels. The study aims to analyze how disparities in plant absorption capacities affect atmospheric CO_2 concentrations, with a particular focus on the influence of plant growth and harvesting rates. The model is formulated using a discrete difference operator, facilitating a numerical exploration of the system's dynamics. A hybrid computational framework is employed namely, the Discrete Numerical Iterative Method integrated with the Levenberg-Marquardt neural network algorithm (DNIM-LM). Artificial intelligence techniques are used to assess model performance, including training progress, error distribution, regression accuracy, and overall fitness. The dataset is partitioned into 70% for training, 15% for validation, and 15% for testing. Results reveal that plant species with higher CO_2 absorption capacities lead to more rapid decreases in atmospheric CO_2 as their growth rates increase. Conversely, higher harvesting rate coefficients are associated with increased atmospheric CO_2 concentrations. The study concludes that differences in plant absorption abilities significantly shape the dynamic behavior of atmospheric CO_2 reduction. These findings underscore the critical role of plant growth and harvesting practices in regulating CO_2 levels, offering valuable insights for ecosystem management and carbon sequestration strategies.

sciforum-145071: Fractal and Machine Learning-Based Analysis of Shoreline Change in Storm-Affected Coastal Environments

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Understanding and forecasting coastline retreat under storm forcing is critical for coastal resilience planning. This study presents a multi-site, data-driven framework integrating fractal shoreline metrics, ERA5-derived storm parameters, and machine learning models to predict monthly shoreline retreat across three morphologically distinct Turkish coasts (Istanbul Karaburun, Karasu, and Sinop Boyabat). Satellite imagery (Sentinel-2, Landsat-8) was processed to extract historical shorelines, followed by computation of box-counting (D_box) and boundary-method (D_bm) fractal dimensions. Storm indicators—including storm index, high-wind hours, significant wave height exceedance durations, and wave energy metrics—were derived from ERA5 reanalysis. To address data sparsity, a block-bootstrap data augmentation strategy generated 300 synthetic years, yielding a final dataset of 10,815 monthly observations.

Five machine learning models were trained and evaluated: Random Forest, GRU, MLP, Ridge, and SVR. Results demonstrate that the Random Forest model achieved the highest performance (R^2 = 0.9866, MAE = 0.0113 m/month), followed by the GRU model (R^2 = 0.9552). Feature importance analysis revealed that shoreline-specific characteristics, fractal metrics, and storm intensity indicators are the dominant predictors. Cross-site generalization tests show that a single unified model can effectively predict retreat patterns across morphologically diverse coastlines.

The findings highlight: (1) the strong predictive capacity of storm indices and fractal shoreline complexity; (2) the suitability of tree-based and recurrent neural models for coastal change prediction; and (3) the viability of bootstrap-based synthetic augmentation for long-term coastal datasets. This framework provides a scalable and transferable methodology for early-warning systems and climate-adaptation strategies. Future work will explore scenario-based forecasting under extreme storm thresholds and SHAP-based interpretability to enhance model transparency.

sciforum-163288: Fractional Diffusion for Dinosaur Image Generation

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Generative modeling of dinosaur imagery is challenging due to large inter-species morphological variation, complex textures (scales, skin folds, armour, horns), and limited curated datasets. This paper investigates diffusion-based image generation for dinosaur images using a Denoising Diffusion Probabilistic Models (DDPM) and introduces fractional-order image analysis as a principled tool for evaluating the quality of generated images. Experiments are conducted on a 15-species dinosaur image dataset derived from the Jurassic Park franchise, consisting of two thousand images spanning diverse body structures, postures, and visual styles.

A DDPM model is trained to generate dinosaur images from Gaussian noise using a U-Net–based denoiser. To assess generation quality beyond conventional pixel-wise or perceptual metrics, we employ fractional calculus–based texture and structure measures. Fractional-order gradients, fractional Laplacian energy, and fractional spectral statistics are computed on both real and generated images across a range of fractional orders. These metrics enable continuous interpolation between fine-scale texture sensitivity and coarse morphological structure, providing a richer characterization of generative fidelity than integer-order operators.

Quantitative analysis reveals that fractional-order metrics are highly sensitive to differences in textural realism, edge continuity, and long-range spatial correlations between real and synthetic images. An optimal fractional order is observed at which the statistical distributions of generated images most closely match those of real dinosaur images, particularly for species with distinctive skin patterns and skeletal outlines. Visual inspection further confirms that fractional-order analysis highlights improvements in fine-scale detail preservation as diffusion training progresses.

The results demonstrate that fractional calculus provides an effective and interpretable framework for evaluating diffusion-generated images, especially for complex, texture-rich objects such as dinosaurs. This work highlights the synergy between diffusion models and fractional-order image analysis, offering new directions for generative modelling assessment in data-limited scientific and educational image processing domains.

sciforum-163284: Fractional-Order Face Analytics for Age and Gender

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Face analysis systems commonly rely on integer-order image operators that emphasize either local texture or global structure, limiting their ability to represent the continuous, multi-scale nature of facial appearance. This paper presents a fractional-order (FO) image processing and statistical learning framework for face analysis and evaluates its effectiveness on two large-scale tasks: age estimation and gender classification. Experiments are conducted on the UTKFace dataset and a second large gender-classification benchmark, demonstrating that fractional calculus provides a principled and flexible representation for modeling gradual facial variations across scales.

The proposed framework integrates three complementary FO components. First, variable-order fractional gradients are computed on perceptually meaningful colour channels, where the fractional order adapts spatially to local structural complexity, enabling a smooth transition between fine texture sensitivity and coarse shape information. Second, fractional spectral representations, including fractional Fourier and multi-resolution wavelet statistics, are employed to capture long-range spatial correlations and anisotropic facial patterns that are poorly represented by conventional integer-order operators. Third, fractional diffusion–based embeddings are constructed on feature similarity graphs to support exploratory data analysis, preserving non-Gaussian and anomalous diffusion characteristics in the resulting low-dimensional manifolds. The extracted FO statistics are combined with standard machine-learning regression and classification models for age prediction and gender recognition.

Experimental results on UTKFace show that fractional-order features improve age estimation accuracy across the full lifespan (0–116 years), with particularly notable error reductions in early childhood and older age groups where texture–shape transitions are subtle. For gender classification, the proposed representation demonstrates increased robustness to illumination, pose, and image quality variations compared with integer-order baselines. Exploratory analyses further reveal smooth manifolds aligned with age progression while avoiding over-segmentation caused by dataset artifacts. Although race annotations are available in UTKFace, they are not used for prediction.

sciforum-160244: Fractional-Order Learning Dynamics on SPD Manifolds

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In this study, we investigate the integration of fractional calculus and Riemannian geometry within the framework of machine learning. We propose a *fractional-order learning model* that operates on the Symmetric Positive Definite (SPD) manifold, a curved geometric space naturally suited for representing covariance-based data. Each data instance is modeled as an SPD matrix, ensuring that the learning dynamics respect the manifold's intrinsic geometry. The UCI Human Activity Recognition (HAR) dataset was employed to demonstrate the method, with multichannel sensor recordings transformed into SPD covariance representations. The learning algorithm extends classical gradient descent through a Caputo-type fractional derivative, introducing a controllable *memory effect* that allows updates to depend on both current and historical gradients. Experiments performed for fractional orders ν=1.0,1.2,1.5 show that increasing the fractional order produces smoother convergence trajectories, mitigates oscillations, and improves stability on the SPD manifold. This fractional–geometric coupling yields a regularized, memory-aware learning paradigm that enhances generalization capability, particularly for small or noisy datasets. The results suggest that fractional dynamics can act as an implicit temporal regularizer for Riemannian optimization, offering a principled approach to stabilize manifold-based learning algorithms. Overall, the study introduces a coherent framework bridging fractional calculus, information geometry, and modern machine learning, and also highlights the potential of fractional-order geometric learning as a unifying paradigm that bridges mathematical theory, computational modeling, and real-world sensor data analysis.

sciforum-164890: Fractional-order memristive and nonmeristive Hopfield neural networks: from model to circuit implementation.

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The circuit realization of fractional neural models is a well-known and essential approach in neuromorphic computing, advanced encryption methods, intelligent robotics, self-tuning control, massively parallel computation, and so forth. Indeed, a crucial step in moving forward with such applications in real-world scenarios is the electronic implementation of neural networks. However, fractional-order Hopfield neural networks (FO-HNN) with and without memristors present two main issues for circuit implementation. The first is that FO-HNNs rely on hyperbolic tangent activation functions, which lead to bulky and cumbersome hardware implementations. As the second issue, FO-HNNs require, on the one hand, frequency-approximation methods to emulate a fractional capacitor in analog circuit designs. But those methods suffer from equilibrium points and memory inconsistencies. On the other hand, FPGA and ARM hardware require appropriate numerical methods to address the inherent memory requirements of fractional derivatives. Both issues limit the applications of FO-HNNs. Therefore, in this talk, we present two novel approaches for achieving feasible, robust, and efficient implementation of FO-HNNs. To address the first problem, a novel approach that employs piecewise linear activation functions, rather than complex hyperbolic functions, is discussed. To address the second drawback, a new decomposition method is applied to obtain a semi-analytical solution for FO-HNNs. Similar to the Adomian approach, the Caputo-based fractional-order differential equation describing the underlying system is decomposed into n-pwl subsystems without explicit nonlinearities. As a result, we obtain an algorithm with low computational complexity, optimized simulation time, and no massive amounts of generated data in contrast to standard numerical approaches like Adams-Bashforth-Moulton and short-memory Grunwald-Letnikov. Based on the results and comparisons presented, both the piecewise-linear activation function and decomposition are excellent choices for the experimental realization of memristive and nonmeristive FO-HNNs, offering low hardware complexity, reduced form factor, and the fewest electronic components.

sciforum-163285: Fractional-Order Vision for Dinosaur Image Classification

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Image-based dinosaur classification presents unique challenges due to limited data, large intra-class shape variability, and strong pose and illustration-style differences. Conventional integer-order image operators often fail to capture both fine-scale texture and global morphological structure simultaneously, particularly in small-sample data science challenges. This work explores fractional-order (FO) image processing and pattern recognition as an alternative representation framework for dinosaur image analysis.

We propose a fractional-order computer vision pipeline combining three complementary components. First, fractional-order gradient and edge operators are applied to dinosaur images to capture multi-scale contour and texture information associated with skeletal structure, armour, horns, wings, and body outlines. Second, fractional spectral descriptors, derived from fractional Fourier and multi-resolution wavelet statistics, model long-range spatial correlations and anisotropic shape patterns. Third, fractional-order pattern recognition tools, including fractional diffusion embeddings and fractional-order feature statistics, are used for exploratory data analysis, visualization, and classification. The approach is evaluated on two datasets: (i) a five-class dinosaur dataset with limited samples per class, and (ii) a 40-class carnivorous–herbivorous dinosaur dataset with train/validation splits. Fractional features are coupled with classical machine-learning classifiers, avoiding data-hungry deep models.

Experimental analysis shows that fractional-order representations provide improved class separability as compared to integer-order baselines, particularly for visually similar dinosaur species. Fractional diffusion embeddings reveal smooth morphological manifolds that align with anatomical traits such as body mass, posture, and feeding type. In the carnivorous–herbivorous dinosaur classification task, FO features demonstrate robust discrimination despite large inter-species variation and small sample sizes.

This study demonstrates that fractional-order image processing and pattern recognition offer an effective, interpretable, and data-efficient framework for dinosaur image analysis. By introducing a continuous order parameter, fractional methods naturally capture multi-scale shape and texture variations, making them well suited for paleontological and educational vision tasks with limited image data.

sciforum-167512: Physics-Informed Neural Networks and Boundary Integral Methods on Fractal Domains

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Physics-Informed Neural Networks (PINNs) provide a mesh-free, optimization-based framework for the numerical solution of partial differential equations by enforcing the governing equations and boundary conditions through a unified loss functional. In parallel, classical numerical methods such as the Galerkin Boundary Element Method (BEM) offer a well-established discretization strategy based on boundary integral formulations and boundary-only representations.

In this work, we present a systematic comparison between PINNs and a Galerkin BEM for the numerical solution of the Laplace equation posed on Koch prefractal domains. These geometries form a sequence of polygonal approximations converging to a fractal limit, with increasing geometric complexity and decreasing boundary regularity as the prefractal level increases. Both approaches are implemented independently and tested on identical geometries and discretizations in order to ensure a fair and consistent comparison.

The numerical experiments investigate the influence of boundary refinement and prefractal level on the performance of each method. The comparison focuses on solution accuracy, computational cost, sensitivity to the prescribed boundary data, and robustness with respect to increasing geometric irregularity.

The goal of this study is to provide a balanced assessment of the capabilities and limitations of PINNs and classical boundary integral techniques when applied to non-smooth and highly structured computational domains, and to highlight their respective advantages in the context of fractal-like geometries.

sciforum-167567: Practical PINN Implementation for a Fractional-Order Damped Oscillator with CppAD-Computed Gradients

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This work presents a practical C++ implementation of a physics-informed neural network (PINN) for a fractional-order damped oscillator. A fully connected network outputs displacement and velocity, so the governing dynamics are enforced through a compact state-space residual involving first and second time derivatives. Integer-order derivatives are obtained via automatic differentiation, which removes finite-difference noise and preserves smooth, consistent gradients during training. The history-dependent fractional damping term is incorporated using the classical L1 discretization on a uniform time grid, which makes each residual evaluation depend on the entire predicted solution history and naturally captures memory effects. The training objective combines the squared residual norms at collocation points with a strongly weighted initial-condition penalty to control drift and stabilize early iterations. Gradients of the complete objective with respect to all network parameters are computed using reverse-mode automatic differentiation in CppAD by constructing a scalar loss function of a flat parameter vector, enabling efficient gradient-based optimization. Parameters are updated with the Adam algorithm using bias correction and double-precision moment accumulation for numerical robustness. The implementation includes deterministic parameter packing, explicit size checks, and lightweight diagnostics of boundary values during training, improving reproducibility and debuggability. Overall, the code provides an end-to-end baseline for PINN-based simulation of fractional-order oscillatory systems and can be readily extended to include external forcing, alternative loss weight schedules, and parameter identification from measurement data.

sciforum-163020: Recent Developments in Fractional Artificial Neural Network Approximation

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Fractional calculus (FC), with its roots dating back to 1965 and pioneered by Leibniz and L'Hospital, is increasingly attracting attention in the applications of machine learning, a rising star in artificial intelligence (AI). Machine learning (ML), particularly in artificial neural network (ANN)applications, captures an internal memory dynamic by utilizing patterns in datasets. Fractional differential calculus, fractional derivatives, and integrals are used in models developed to internalize this dynamic and make learning algorithms more effective. According to studies in the literature, the Caputo definition of the fractional derivative is used in artificial neural networks called Fractional-order Artificial Neural Networks (FAANs). Here, we observe that activation functions based on fractional derivatives stand out. These activation functions are more flexible than their classical counterparts due to the need to carry adjustable parameters, and they increase the learning capacity and accuracy of the network. Based on this, this study will introduce the activation functions found in the literature at the intersection of FC and ANN. This study shall discuss the convergence results in Banach spaces by considering the binary and multiple components of the relevant activation functions actively used in ANN operators. New activation functions obtained from the combination of these very general activation functions with more specific ones will be examined. Convergence analysis will be discussed using Banach space-valued Caputo fractional derivative and Caputo fractional Taylor formulas. Finally, a mathematical analysis of these activation functions—recently added to the literature—will be performed. The work closes with a nuanced examination of existing challenges and identifies several fruitful directions for future exploration in uniting fractional calculus with neural network methodologies.

sciforum-159867: Reconstruction of Fractional-Order Ellipsoidal Nanofluid Turbulent Flow Fields and Particle Orientation Evolution Based on PINNs

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To investigate the orientation of ellipsoidal nanoparticles in turbulence, a Physics-Informed Neural Network (PINN) framework for fractional order problems was constructed, in which the fluid was described by a fractional order fluid constitutive model in a periodically sinking channel. By introducing fractional derivative into the constitutive relation, the model combines the nonlocal effect and memory effect of complex turbulence, making the effective rheology more accurate than the classical integer order closure. In addition, the second-order moment tensor equation derived from Fokker-Planck equation is used to characterize the rotational diffusion behavior of ellipsoidal nanoparticles, and the variation of nanoparticle concentration in the channel is considered. The results show that the PINN based on fractional constitutive model can predict the flow field. Based on the reconstructed flow field, the data free neural network is used to predict the orientation and concentration distribution of nanoparticles in turbulence. The parameter analysis shows that with the increase of the rotating Péclet number, the shear effect controls the rotating diffusion more strongly, resulting in the ellipsoidal nanoparticles arranged faster and more clearly along the mainstream direction. Alignment near the wall precedes alignment at the center of the channel, usually accompanied by alignment overshoot. A smaller lateral Péclet number contributes to the formation of a steep directional gradient, which is coupled with wall migration, resulting in higher alignment near the wall and greater dispersion at the center. In general, this study emphasizes the effectiveness of combining the fractional fluid constitutive model with PINN to solve the transport and orientation of multi-scale nanoparticles in turbulence.

sciforum-160877: Solving Fractional Squeeze-Film Flow in the Knee Joint with Physics-Informed Neural Networks

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The knee joint is a complex biotribological system. Its healthy relies on the interaction between articular cartilage and synovial fluid. Specifically, cartilage acts as a poroelastic structure, while synovial fluid exhibits complex non-Newtonian behavior. To elucidate the lubrication mechanisms, it is essential to investigate the fluid-structure coupling between them. This study presents a computational framework using Physics-Informed Neural Networks (PINNs) to solve the fractional squeeze-film flow problem in a knee joint model. The model characterizes the synovial fluid as a fractional Maxwell fluid, capturing its viscoelastic memory effects, while the cartilage as a porous elastic layer and the femoral component as a rigid sphere. By directly embedding the governing equations into the loss function of the neural network, this complex flow behavior can be effectively solved. The results indicate that the permeability of the cartilage layer is a critical parameter regulating joint lubrication performance. High permeability accelerates the exudation of synovial fluid from the loaded region, leading to a rapid decline in film thickness and a significant reduction in fluid load-supporting capacity. Meanwhile, an increase in the fractional order and a decrease in the relaxation time also weaken the stability and load-bearing performance of the fluid film, further accelerating lubrication failure. Collectively, these mechanisms exacerbate cartilage wear and elevate the risk of osteoarthritis. This study provides novel insights into the lubrication failure of diseased cartilage. It also highlights the potential of PINNs for tackling complex biomechanical problems.

sciforum-167562: Understanding the Black Box of Fractional Machine Learning Models

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Fractional Calculus (FC) has gained attention in machine learning (ML) field due to its ability to model long-term memory (LTM), complex system behavior, and non-local dynamics. Fractional machine learning models (FMLM) demonstrate increased convergence properties in diverse applications (e.g., financial forecasting, EEG, ECG, climate and environment, and robotic control theory) by combining classical learning frameworks with memory-aware dynamics, which improves realism, robustness, and interpretability, particularly in time-dependent settings. Thus, these advantages introduce significant challenges in terms of model explainability and interpretability. The main goal of this paper is to analyze the explainability associated with FMLM. Unlike classical models based on integer-order derivatives, Fractional Machine Learning Models (FMLMs) show non-local behavior, and their predictions are influenced by the complete historical input data or the model's optimization process. As a result, these characteristics offer important challenges for Explainable Artificial Intelligence (XAI) approaches that are primarily designed for local and memoryless models. Fractional neurons contribute to modeling expressiveness but also amplify the black box characteristics of the model by introducing further interpretability issues, which can be reduced by XAI that explains input contributions and highlights the role of fractional parameters. Moreover, the limitations of existing XAI techniques for FMLMs are investigated, and significant issues related to parameter interpretability, decision traceability, and model transparency are identified. It also proposes possible research directions toward the development of explainability frameworks based on fractional learning models. By integrating fractional-order calculus in AI/ML applications, the fractional neuron retains memory effects, while XAI makes it easier to understand the impact of historical data and fractional parameters on neuronal decisions.

sciforum-163297: Fractional-Order Differential Privacy

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DP-SGD is a standard tool for training machine learning models with differential privacy, but the Gaussian noise required to protect individual records can destabilize optimization and significantly reduce accuracy, especially under tight privacy budgets and small sampling rates. We propose **Fractional-DP-SGD**, a simple plug-in modification that preserves the core DP-SGD privacy mechanism—per-example gradient clipping and Gaussian noise injection—while changing only the update dynamics. Instead of updating parameters directly with the current noisy gradient, Fractional-DP-SGD applies a **finite-window, normalized fractional-memory filter** to the sequence of previously released DP gradients. This introduces a **power-law temporal aggregation** that behaves like a long-memory smoother, shaping the injected noise over time and potentially reducing the effective variance of parameter updates while keeping privacy accounting unchanged. We provide a formal privacy guarantee using the post-processing property of differential privacy together with standard Rényi DP accounting under subsampling, and we analyze the additional computational and memory costs introduced by the finite window. Finally, we outline an extensive experimental protocol comparing Fractional-DP-SGD against DP-SGD and widely used DP optimizers such as DP-Adam, DP-Adagrad, and DP-FTRL across multiple datasets and models. Our evaluation includes systematic ablations over fractional order, window length, kernel family, clipping norm, and noise multiplier to isolate when and why fractional memory improves stability and utility at matched privacy.

Session 6. Fractal Geometry: Mathematical Foundations and Real-World Applications

sciforum-163105: Self-similar currents and their properties based on the general theory of fractal elements

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Fractal growth processes frequently emerge in systems characterized by anomalous transport, memory effects, and long-range correlations, features that exponential relaxations fail to capture. This motivates a central question: does fractal pattern formation give rise to power-law (fractional) kinetics, and if so, how is the fractional order linked to geometric properties such as fractal dimension? The authors explore this question through various experiments and proposed theory of fractal elements.

The study focuses on Lichtenberg figures generated by high-voltage discharges on wood surfaces, a striking example of fractal pattern formation in a heterogeneous dielectric medium. When a wooden substrate, with anisotropic conductivity and moisture variability is subjected to electric fields above its breakdown threshold, the discharge propagates through branching streamers and carbonization fronts. These fronts display long-tailed waiting-time distributions, and memory-dependent evolution.

Experimental evidence reveals that the discharge does not advance with constant velocity or classical exponential behaviour. This raises a key question: Does the electrical current measured during breakdown also exhibit fractional behaviour?

To investigate this, the authors use the recent theoretical framework of Nigmatullin and Chen (2023), which provides a rigorous method for analyzing complex, self-similar signals. Their “general theory of fractal elements” shows that an experimental waveform can be decomposed into elementary self-similar modes, each associated with a power-law scaling exponent and amplitude. This decomposition reveals that the current signal is a structured combination of fractal components encoding the underlying fractional dynamics.

Actually, the essence of this theory is decomposition of a complex self-similar signal on a combination of fractal modes governed by a set of power-law exponents that can be confirmed on many self-similar processes (in time and space). The relationship between fractal dimension and power-law exponents governed by fractal dynamics is not found due to the influence of the medium structure, where the discharge current is propagated.

sciforum-162557: A Hybrid Fuzzy Logic System Leveraging Higuchi Fractal Dimension for Transparent and Predictive Control of Adaptive TMS Protocols

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The outcomes of Transcranial Magnetic Stimulation (TMS) depend critically on the momentary intrinsic cortical state, resulting in high variability that compromises clinical efficacy. To enable reliable, personalised TMS, closed-loop systems are essential for predicting and targeting favourable brain states in real-time.

This study proposes a novel TMS-EEG approach based on a **Hybrid Fuzzy Logic System (FLS)** to predict single-trial brain responsiveness accurately. Inputs extracted from the pre-stimulus TEP included **Power Spectral Densities** across canonical bands (delta to gamma) and the non-linear measure **Higuchi Fractal Dimension (HFD)**, which reflects network complexity. The post-stimulus response was quantified using the **Area Under the Curve (AUC)**, with trials labelled as 'low' or 'high' responders.

The FLS is uniquely suited to model the non-linear relationships of biological data, providing **transparency and interpretability** absent in 'black box' models. It utilises a **hybrid rule-based inference system** integrating **Expert-defined rules** (neurophysiology-based) and **Data-driven rules** from a Random Forest classifier to generate understandable, linguistic mappings.

Across 1560 trials, the model achieved classification **accuracy of 73%** and a Cohen's Kappa score of **0.46**. Rule inspection confirmed that brain states characterised by **reduced HFD** and **reduced beta and gamma power** were associated with 'high' responsiveness. This research validates the FLS as a transparent, high-performance computational engine, representing a substantial step toward practical **adaptive TMS protocols** guided by real-time brain-state prediction to maximise therapeutic efficacy.

sciforum-157367: Advanced foundational tensors and fractional calculus and applications

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Building on the recently published framework of advanced tensor theories—where tensors are rigorously defined via standard and fractional derivatives—this work extends the formalism to foundational mathematics, fractional calculus, and fractional geometry. We introduce these *advanced tensors* as generalized objects that unify differential structures across integer and non-integer orders, enabling seamless interpolation between classical and fractional regimes.

Central to the presentation is the construction of *transformation laws* for mathematical objects under advanced tensor actions, revealing that possibly novel algebraic and geometric invariants preserved across fractional dimensions. These transformations are shown to induce natural fractional differential geometries on manifolds, yielding *fractional manifold–metric pairs* with curvature expressions involving Caputo–Fabrizio or Riemann–Liouville operators.

Real-world applications are explored in depth: modeling anomalous diffusion in heterogeneous media, designing fractional physical and engineering systems with memory-dependent and environmental-dependent dynamics, and developing scale-invariant image processing algorithms via fractional tensor convolutions. We further demonstrate how advanced tensors facilitate multi-scale physics simulations—bridging quantum microstates to macroscopic continuum behavior—through dimensionally hybrid tensor fields.

By establishing rigorous links between abstract fractional structures and actionable mathematical, physics, and engineering paradigms, this study positions advanced foundational tensors as a unifying language for next-generation mathematical modeling in sophisticated and complex systems, such as comology, gravity, and high energy physics.

sciforum-143665: Applications of Fractal Geometry for Antenna Design in Wireless Communications

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This paper presents a comprehensive review of the integration of fractal geometry in antenna engineering. It surveys key concepts, representative geometries, and major application domains of fractal antennas, arrays, and metasurfaces, highlighting their role in enabling multiband operation, miniaturization, enhanced directivity, and design flexibility in modern electromagnetic systems.

The integration of fractal geometry with electromagnetic theory has revolutionized antenna design, offering innovative solutions for modern telecommunications. Fractal antennas leverage unique geometric properties such as self-similarity and space-filling to achieve multifrequency operation, miniaturization, high directivity antennas and arrays, and meta surfaces. Practical applications include multi-frequency antennas. For example, geometries such as the Sierpinski monopole enable operation across multiple frequency bands, thereby reducing the need for separate antennas in applications including cellular communications, localization, and short-range wireless links.

In addition to supporting multiple resonances, certain fractal configurations—particularly space-filling curves like Hilbert and Koch—allow for significant miniaturization by lowering resonant frequencies without compromising performance. This capability is especially valuable in space-constrained scenarios such as automotive systems, portable devices, and embedded IoT platforms.

Beyond multiband operation and size reduction, fractal antennas can also support localized electromagnetic modes, often referred to as fractons and fractinos, which enhance directivity. By concentrating radiation in specific directions, these modes improve link reliability and signal strength in long-distance applications, including satellite communications.

Fractal concepts have also been successfully applied to metasurfaces, whose self-similar features confer distinct advantages over conventional designs. These include enabling multiband or wideband operation, achieving further size reduction, and providing a high degree of design flexibility for integration into complex system architectures.

As a result of these combined benefits—multiband capability, compactness, directivity control, and design versatility—fractal antenna engineering has been adopted in industries such as mobile communications and automotive, where high performance in compact and adaptable form factors is essential.

sciforum-163194: Brain dynamics fractality as a biomarker for drug-responsive epilepsy

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One-third of people with epilepsy (PwE) live with drug-resistant epilepsy (DRE), a condition in which seizures persist despite adequate medication. Epilepsy is now understood as a network disorder that disrupts large-scale neuronal coordination and alterations in the *scale-free* properties and *fractal* organization of brain dynamics may emerge as signatures of pathological brain functions. This study investigates whether the temporal sequence of EEG microstates (the brief, quasi-stable topographies that reflect ongoing functional configurations) shows altered fractal properties in DRE compared with drug-responsive epilepsy (nDRE), and whether these properties change after therapeutic neuromodulation. We tested the following hypotheses: 1) Fractal features of brain dynamics are different between people with DRE and people with drug-responsive epilepsy (nDRE), thus qualifying as a potential diagnostic biomarker; 2) Fractal features of brain dynamics can be modulated in people with DRE, thus qualifying as a potential response biomarker.

Resting-state EEG (eyes closed) was examined in 60 DRE and 60 nDRE patients, and in a subgroup of 10 DRE patients recorded before and after Vagus Nerve Stimulation (VNS). The study was approved by the Ethics Committee ‘Lazio Area 2’ (number 99.24CET2 CBM, April 11, 2024). Microstate sequences were extracted, and their temporal structure was characterized using scale-free metrics, including the Higuchi Fractal Dimension and Hurst exponent, which quantify short-time complexity properties and long-range dependencies. While conventional microstate metrics and transition patterns did not differentiate DRE from nDRE, fractal analyses revealed significantly higher scale-free disruption in DRE, indicating less self-similar and more erratic state trajectories. Notably, similar fractal shifts were observed after VNS and accompanied clinical improvement, suggesting that fractal markers are sensitive to therapeutic modulation. These findings indicate that the scale-free structure of brain-state sequences may capture fundamental alterations in cortical dynamics associated with DRE. Identifying fractal biomarkers for DRE could support earlier diagnosis, reduce disease burden, and advance precision approaches tailored to individual network dysfunction.

sciforum-158752: Construction of fractal stiffness and mass matrices for two-dimensional self-similar frames

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In this work we constructed the dynamics equilibrium equation for fractal manifolds based on the concept of fractal calculus introduced recently by Balankin with their fractal continuum derivatives, as an extension of traditional differential calculus concepts to spaces and functions characterized by fractal geometry.

The generalized form of the equation of motion for two-dimensional self-similar frames subjected to forced vibration incorporates both the stiffness and mass matrices, which account for the hierarchical structure and scaling properties of the frame's geometry; allowing the accurate modeling of dynamic responses by considering the influence of fractal continuity and irregular distribution of mass and rigidity, reflecting the distinctive physical behavior of such frames under external influence.

The fundamental interrelation between Balankin fractal derivatives and ordinary derivatives establishes a connection that enables the transformation of vector differential operators defined in the fractal domain R_x^3 into their counterparts within the fractal continuum R_ξ^3. This relationship behave as a mathematical bridge, for the mapping from complex fractal structures (characterized by non-integer dimensions and self-similariy) into a generalized fractal continuum framework.

The alpha and beta parameters of order $\alpha, \beta \in (0,1]$ are added to the new fractal matrices with this transformation, preserving essential fractal properties (as geometry and topology) and presenting new analytical tools and models for physical phenomena ocurring within fractal continua. The influence of these parameters on the vibrational characteristic of the frames is analyzed graphically. Some mechanical implications are also discussed.

sciforum-163344: Fractal and Fractional Approaches to the Morphological Analysis of Helianthus annuus

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Helianthus annuus exhibits complex morphological structures, including leaves, stems, and flower heads, which often display self-similarity and multiscale organization. Understanding these patterns is essential for insights into plant development, growth dynamics, and ecological adaptation. Fractal geometry and fractional calculus provide robust frameworks for analyzing such structural complexity and modeling the spatial and temporal dynamics of plant morphology. This work explores the application of fractal measures and fractional-order modeling techniques to Helianthus annuus morphology, emphasizing their ability to capture multiscale patterns, symmetry, and memory effects in growth processes. Different approaches reported in the literature, including fractal dimension estimation and fractional modeling, are conceptually compared in terms of biological interpretability, analytical robustness, and potential applications in plant growth studies. Special attention is given to how non-integer order operators can represent adaptive structural development, variability in leaf and flower arrangement, and complex spatial correlations. Furthermore, the discussion includes potential implications for agricultural optimization and ecological research. The synthesis identifies key methodological trends, advantages, and current limitations of fractal and fractional approaches in plant morphogenesis research. By integrating theoretical frameworks with observed sunflower structures, this work provides a coherent perspective on plant morphology and supports the broader use of advanced mathematical tools in quantitative botany.

sciforum-167207: Fractal Characterization of Multi-Scale Pore Structures in Marine-Continental Transitional Shale of the Upper Permian Longtan Formation, South Yellow Sea Basin

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The heterogeneity of pore systems across micro- to macro-scales critically controls gas storage and transport in shale, yet remains challenging to quantify accurately, especially in complex marine-continental transitional facies. Here, we introduce a novel pore-volume-weighted comprehensive fractal dimension (D_t) model that integrates high-pressure mercury intrusion, low-temperature nitrogen adsorption, and carbon dioxide adsorption data. This approach overcomes the limitations of traditional single-fractal models by enabling a unified quantification of pore complexity from micro- to macro-pores. Applied to the Upper Permian Longtan Formation shale of the South Yellow Sea Basin, our model indicates that mesopores dominate the pore network. Samples with high total organic carbon (TOC) content exhibit distinctive dual-segment fractal features in nitrogen adsorption data, revealing the controlling role of organic matter on pore complexity. Furthermore, the D_t shows strong positive correlations with TOC, quartz content, and the Brunauer–Emmett–Teller specific surface area, suggesting that organic richness and brittle minerals synergistically govern pore-system complexity. This study demonstrates that D_t model provides a robust mathematical framework for the full-scale pore characterization. Our findings offer a fractal-based theoretical foundation for predicting reservoir quality in transitional shale systems. The understanding of the influence of geological conditions on pore systems has been deepened by applying fractal theory.

sciforum-162564: Fractal Dimension Analysis: Unlocking Ageing-Related Changes in Brain Criticality

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Fractal Dimension (FD) is a powerful computational neuroscience method that captures local oscillations and network topology through univariate measures. A central concept in neuroscience is **criticality**, which describes the brain's optimal operating state at the edge of chaos, balancing stability and flexibility. Quantifying this delicate balance is a frontier in neuroscience.

The **Hurst Exponent (HE)** is one of the most popular FD measures and a key way to quantify the balance of criticality. The HE measures scale-free, long-term memory in temporal dynamics and has been confirmed as an essential performance indicator of brain dynamics.

In this research, we investigated the effect of healthy ageing on known resting-state functional networks (RSNs) using the HE. Using Group Independent Component Analysis (GICA) on fMRI data from young adults (YA) and older adults (OA), we characterised baseline fractality in the YA group. We found that increasing HE values differentiate between subcortical, primary, and cognitive networks. This suggests that greater temporal complexity may reflect increased integrative processing in the brain.

Comparing the groups revealed **widespread significant differences** between young and old adults, with a general loss of scale-free long-term persistence (a decline in HE) across networks. This change may be specific to a loss of integration capability in higher cognitive functions.

Specifically, the **visuospatial** and **dorsal default mode (dDMN) networks** were the most affected by ageing. Machine learning classifiers highlighted them as the best predictors of ageing based on the HE. Interestingly, these high-order networks exhibited a signal complexity level in OAs that resembled that of subcortical structures. We also observed increased segregation (functional decoupling) within the dDMN, as the ventral part (vDMN) showed no significant difference in HE between YA and OA.

The HE can detect early deviations from optimal function and may mark the transition from healthy ageing to pathology.

sciforum-168055: Fractal Mathematics of Global Deprivation: New Evidence for Scale-Invariant Complexity

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This study presents a novel application of fractal geometry and complex systems theory to a mufti-layered socioeconomic dataset: NASA's GRDI v1, a global high-resolution (≈1 km) index of multidimensional deprivation. We present evidence that the spatial architecture of multidimensional deprivation is self-similar across scales, exhibiting a robust fractal geometry. Analyzing data from the Global Relative Deprivation Index (GRDI v1), we compute the box-counting dimension across six continental regimes. The analysis reveals a mean fractal dimension of $D = 1.60 \pm 0.40$, demonstrating significant regional heterogeneity (ANOVA $F = 122.27$, $p = 7.28 \times 10^{-10}$), from densely clustered deprivation in South Asia ($D = 1.904$) to sparse, fragmented patterns in the South America ($D = 0.781$). This power-law scaling is exceptionally strong (mean $R^2 = 0.997$) and is validated within 5% tolerance by correlation dimension analysis in 90% of regions.

The observed scaling is not an artifact of methodology. Fractal dimensions are stable across deprivation thresholds (coefficient of variation = 3.40%) and are statistically preferred over log-normal or stretched-exponential alternatives (likelihood ratio test, p 0.001). Controlling for spatial autocorrelation, we identify a significant association between fractal dimension and governance indicators (Mantel test $r = 0.73$, $p = 0.003$).

Theoretically, these findings characterize deprivation as a spatially embedded complex system with a non-trivial topological dimension (1 D 2). This indicates a sparse, hierarchical network structure that is nested across scales, implying that local interventions may propagate through self-similar pathways.

Practically, D serves as a scale-invariant metric for comparative analysis. It delineates distinct policy arenas: regions with $D > 1.7$ necessitate multi-scale strategies addressing hierarchical clustering, while regions with D 1.2 may be more responsive to targeted, nodal interventions. We further demonstrate that higher D correlates with slower recovery from economic shocks ($r = -0.61$, $p = 0.02$), suggesting that fractal complexity quantifies systemic fragility.

We argue that fractal mathematics provides an essential analytical tool for dismantling persistent deprivation structures, offering the first rigorous proof that poverty functions as a complex adaptive system with scaleinvariant properties.

sciforum-166767: Fractal-Inspired Multiscale Surface Topography Characterization via ISO 25178 Sdr

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Surface topography characterization increasingly relies on areal parameters (ISO 25178) to relate manufacturing processes to functional performance (e.g., wettability, adhesion, tribology). Yet, most roughness parameters remain scale-dependent, making multiscale frameworks essential for describing complex, fractal-like surfaces. In this work, we introduce a standard-friendly multiscale approach based on the interfacial area ratio Sdr (ISO 25178-2) iterated across scales using low-pass Gaussian filtering (ISO 16610), yielding an area–scale curve that quantifies surface complexity through relative area development. This protocol provides a practical alternative to the well-established Richardson Patchwork (triangular tiling) area-scale method used in standards, while relying exclusively on widely implemented standardized operators (Gaussian filter + ISO parameter), enabling straightforward deployment in conventional surface metrology software.

We validate the approach on grit-blasted TA6V surfaces produced under controlled variations of blasting pressure and media, showing that the multiscale Sdr procedure reproduces Patchwork trends over the investigated scale ranges and remains discriminant with respect to process conditions.

To objectively identify the most informative observation scale, we further propose an uncertainty-based scale selection strategy using bootstrap regression, highlighting a relevant cut-off length around 120 μm for capturing the pressure–topography relationship across media.

Overall, this work bridges fractal-inspired area-scale analysis with ISO-compliant metrology practice, providing a robust and accessible route to multiscale characterization and to process–topography interaction studies.

sciforum-157436: Fractional cosmology: Background dynamics and first order perturbations

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Various approches for constructing fractional cosmological models with additional dynamical degrees of freedom are presented. Focusing on one of these approaches, the corresponding fractional equations of motion are explicitly derived. In this fractional framework, the standard continuity equation of the corresponding relativistic cosmological model is no longer preserved, as energy is continuously exchanged between the fractional sector and the additional dynamical components. To address this issue, effective energy density and pressure are consistently defined in such a way that an effective continuity equation is satisfied. This procedure plays a crucial role not only in obtaining an appropriate dynamical system for the background evolution, but also in the systematic derivation of the perturbation equations.

The differences, advantages, and limitations of our fractional framework are examined in direct comparison with the corresponding standard cosmological models, both at the level of the field equations and their exact solutions. This comparative analysis is carried out not only at the background level, but also within first-order cosmological perturbation theory. In particular, the fractional generalization of the Mukhanov–Sasaki equation is derived, providing a consistent description of scalar perturbations in the presence of fractional effects and memory contributions. The resulting framework offers a coherent extension of relativistic cosmology in which fractional dynamics naturally influence both the background evolution and the perturbative sector.

sciforum-165844: Intelligent Fractal Video Compression and Super-Resolution Zooming

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Fractal Image Compression (FIC) provides a distinctive advantage of resolution independence, enabling deep zooming and super-resolution without the pixelation artifacts commonly observed in traditional Discrete Cosine Transform (DCT)–based methods. Despite these benefits, the high computational cost of exhaustive domain block searching has limited the practical adoption of FIC for real-time video applications. In this paper, we propose a novel high-performance architecture for fractal video compression. The proposed approach incorporates a variance-based intelligent search heuristic to substantially reduce the domain search space, along with a massively parallel GPU kernel for efficient affine block matching. According to experimental findings, the suggested approach maintains a high structural similarity index measure (SSIM) and supports resolution-independent zooming capabilities while achieving notable performance improvements over CPU-based implementations.

Despite its theoretical benefits, the computational difficulty of FIC has severely limited its practical usage, especially for video compression. To find the optimal affine match for each range block, the encoding method necessitates a thorough search across a vast pool of domain blocks. The computational cost of this brute-force matching is on the order of $O(N_r\, x\, N_d)$, where N_R and N_D represent the number of range and domain blocks, respectively. On conventional CPU-based systems, this complexity makes real-time encoding unfeasible. To overcome this fundamental bottleneck, this paper proposes a high-performance fractal video compression framework that leverages both algorithmic optimization and hardware acceleration.

By combining intelligent search heuristics with GPU parallelism, the proposed approach substantially accelerates fractal encoding while maintaining high reconstruction quality and preserving the intrinsic resolution-independent benefits of fractal compression.

The study demonstrated successful implementation and testing of a CUDA-Accelerated Intelligent Fractal Video Compressor which resolves the traditional performance limitations of Fractal Image Compression (FIC). Our key achievement came from combining an intelligent variance-based heuristic with a parallel GPU kernel that we optimized highly.

sciforum-143031: Investigating Scale-Dependent Clustering and Flow Behaviour in Multifractal Permeability Fields

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Lacunarity analysis has proven effective in distinguishing between multifractal grayscale patterns that share the same correlation dimension but exhibit differing degrees of scale-dependent clustering. In our earlier studies, we demonstrated that such scale-dependent clustering significantly influences fluid flow behavior in fractal-fracture networks, even when the fractal dimensions remain constant. Building on this foundation, the present research explores whether lacunarity, a metric quantifying heterogeneity and spatial clustering can serve as a predictor of fluid flow behavior in multifractal permeability fields. To investigate this, we consider a set of four multifractal patterns (three deterministic and one random), each characterized by identical correlation dimensions and defined by the parameters: b = 7, p = 40/49, and three iterations. These patterns are subjected to lacunarity analysis using the gliding-box algorithm to quantify their scale-dependent clustering, where lacunarity (L) is related to the correlation dimension, D_2, by the equation: $d\log(L)/d\log(r) = D_2 - 2$. We empirically tested this equation using two-dimensional multifractal grayscale patterns with known correlation dimensions. Subsequently, flow simulations are performed on these permeability fields using Trace3D, a streamline-based simulator that generates streamlines in 3D space and then solves the 1D equations analytically or numerically along the streamlines. In these simulations, the occupied regions of the multifractal fields are assumed to be highly porous and permeable, enabling all fluid flow to occur exclusively through these zones. The simulation results reveal a strong positive correlation of 95% between lacunarity and fluid recovery, indicating that increased clustering (as measured by lacunarity) enhances flow efficiency in such heterogeneous systems. These findings suggest that lacunarity is a valuable parameter for predicting fluid flow behavior in multifractal permeability fields and can potentially generate an informed descisions on reservoir characterization and field development modeling strategies.

sciforum-163109: Meditation-Related Increases in EEG Fractal Dimension Reveal Enhanced Brain Complexity

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Meditation has been proposed to alter large-scale brain dynamics, but its impact on neural complexity remains subtly yet incompletely understood. In this study, we investigated how three contemplative traditions, Himalaya Yoga Tradition, Vipassana, and Isha Shoonya Yoga, modulate electroencephalography (EEG) complexity, quantified via Higuchi's Fractal Dimension (HFD). Based on the Entropic Brain Hypothesis, we hypothesized that regular meditative practice would be associated with increased EEG complexity relative to a non-meditative condition. We used a publicly available EEG dataset. In the original study, written informed consent was obtained, and the protocol was approved by the local MRI (India) ethics committee and the UC San Diego IRB (#090731). EEG data were recorded from 48 participants (16 per meditation group) and a non-meditating control group during two 20-minute blocks: meditation (including an initial breath-focused phase) and instructed mind-wandering. After standard preprocessing (filtering, artifact rejection, ICA) and segmentation into short overlapping epochs, HFD was computed for each electrode. The parameter Kmax was optimized by inspecting the HFD–Kmax curve across subjects, yielding a stable plateau around Kmax = 10, which was adopted for all subsequent analyses. Cluster-based paired permutation tests were used to compare breath focus, full meditation, and mind-wandering within each group. Across meditation traditions, we observed a robust increase in HFD during meditation relative to instructed mind-wandering, with the strongest effects during the breath-focus phase, indicating higher brain signal complexity in meditative states. These findings support the Entropic Brain Hypothesis by suggesting that meditation shifts neural activity toward regimes of enhanced complexity, and highlight EEG fractal metrics as sensitive markers of meditation-induced changes in brain dynamics.

sciforum-145843: Research on the Structure and Modulus Compatibility of Biomimetic Porous Bone Scaffold Based on Fractal Theory

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Introduction: The repair of large segmental bone defects has always been a major challenge in clinical practice, and "stress shielding" caused by the mismatch in elastic modulus between the bone implant and the native bone is one of the key factors leading to implant failure.

Methods: Inspired by the branching structure of tree branches and trabecular bones, this study constructs a multi-level fractal scaffold geometric model with controllable fractal dimension, porosity, and pore size distribution. The influence of key structural parameters such as fractal iteration times, pore size, and trabecular diameter on the overall elastic modulus, compressive strength, and yield behavior of the scaffold was systematically studied using a combination of finite element calculations and mechanical experiments. Furthermore, scanning electron microscopy (SEM) was used to characterize the microstructure of the scaffolds fabricated by 3D printing.

Results: The porosity distribution of the 2-iteration and 3-iteration models is consistent with that of natural bone, featuring a radial gradient distribution with larger internal porosity and smaller external porosity. The elastic modulus of the 3-iteration model is 21% lower than that of the 2-iteration model, while its compressive strength is 17.3% higher than that of the 2-iteration model. The pore size of the fabricated scaffolds is consistent with that of the designed model, with a pore size error within 10%, indicating that the multi-level fractal scaffold designed in this study has good processability.

Conclusions: The 3-iteration fractal scaffold has higher strength and lower elastic modulus, which is similar to the mechanical properties of natural bone and can effectively alleviate stress shielding. The bionic scaffold based on fractal theory and the research on its structure-modulus matching proposed in this study can provide a new idea for alleviating the stress shielding of implants.

sciforum-160539: Temporal Variation in Fractal Dimension as a Precursory Signal for the 2015 Nepal Mw 7.8 Earthquake

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Earthquakes in the Himalayan region primarily result from the ongoing convergence of the Eurasian and Indian tectonic plates, with the Indian plate being underthrust beneath the Eurasian plate, leading to complex seismic activity. The Nepal region, located within a seismic gap, is especially significant due to its potential for large earthquakes. In this study, we investigate the temporal evolution of seismicity in the Himalayan region to identify precursor signals preceding the 2015 Mw 7.8 earthquake in Nepal Himalaya, with a focus on the fractal characteristics of seismic events. The magnitude of completeness (Mc) for the dataset was found to be 3.9 and above. The sliding window analysis using 100-event windows showed variation in fractal correlation dimension (Dc) values over time. During the first nine years, the trend increases; later, it follows a significant decreasing trend. Increasing and decreasing trends demarcate the two time periods during which the fractal behaviour of earthquake occurrences differs. We detected unusual patterns in the fractal correlation dimension (Dc) of seismic events, with values ranging from 0.94 to 1.66. A sinusoidal-like pattern emerged in the initial years of the dataset (within the first 9 years), followed by a notable change in the fractal dimension (after 9 years), just prior to the main earthquake. This indicates a shift towards stronger clustering of earthquakes as stress built up in the region. The patterns in the fractal dimension suggest significant changes in seismic behaviour before 11 years leading up to the 2015 earthquake, indicating stress accumulation and the likelihood of a large seismic event. Notably, the time period of the decreasing trend exhibits a comparatively higher crustal stress than the period of the increasing trend. This study provides insight into long-term stress accumulation patterns and their application in seismic hazard assessments.

sciforum-161941: Testing the time series for multifractality

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Multifractality has become a central concept for characterizing complexity across scientific domains, yet its reliable detection in empirical time series remains challenging due to the interplay of temporal correlations and heavy-tailed fluctuations. Recent analytical and numerical results demonstrate that genuine multifractality cannot arise in the absence of long-range temporal correlations, while the apparent multifractal spectra observed for shuffled or short uncorrelated data are finite-size artifacts or manifestations of bifractality in Lévy-stable regimes. Motivated by these findings, we revisit multifractal detrended fluctuation analysis (MFDFA) as the most stable and widely used tool for testing multiscaling properties in complex systems, from financial markets to natural language. We show, using controlled multiplicative cascades, how the distribution of fluctuations—modulated via q-Gaussian transformations—can broaden the singularity spectrum only when correlations are already present. This provides a principled way to separate true multifractal effects from those induced by fatter tails. We further demonstrate that empirical systems often exhibit multiscale organization consistent with correlation-driven multifractality, including financial markets where nonlinear dependencies and volatility clustering contribute to the observed spectra . Additional analogies arise in linguistic time series, where long-range dependencies and hierarchical structure produce fractal or multifractal patterns even in symbolic data. Building on these theoretical and empirical insights, we propose a robust protocol for testing multifractality that incorporates surrogate data, tail-stabilized reference distributions, and scale-range diagnostics. This procedure allows researchers to quantify how much of the spectrum width originates from temporal organization versus the heaviness of fluctuations. The talk concludes by emphasizing that properly disentangling these components is essential for interpreting multifractality as a genuine marker of complexity in natural and socio-economic systems.

sciforum-163054: The Morphological Mark of Inequality: Fractal Dimension Reveals Structural Spatial Segregation in Alagoinhas (BA)

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The understanding of urban morphology has been enhanced by mathematical and computational approaches capable of quantifying the spatial complexity of cities. This investigates the hypothesis that the fractal complexity of the urban fabric varies significantly as a function of socioeconomic stratification in Alagoinhas, Bahia, Brazil. To this end, the analysis of the Fractal Dimension (FD) was employed on residential samples from three income groups (Class A: R$ 2,000; Class B: R$ 1,000 - 1,500; Class C: R$ 600), aiming to fill a gap in the understanding of the articulation between urban structure and spatial justice. The Box-Counting methodology was applied to 30 georeferenced samples (ten per class), obtained from high-resolution images (Google Earth/CNES) and standardized in square buffers of 200 meters side. The log-log regression analysis for the FD estimation demonstrated high statistical robustness, with the R-squared consistently at 1.000 for all samples. The results revealed that all classes exhibit high density, with FD values close to 2.0. Class A (mean FD 1.992) showed the highest average spatial complexity and the lowest intrinsic variation, suggesting more consolidated and detailed occupation patterns. Class C (mean FD 1.990) demonstrated the highest morphological uniformity (standard deviation of 0.002), reflecting homogeneous and dense occupation patterns, typical of peripheral areas with lower infrastructure. Class B (mean FD 1.988) exhibited the greatest internal variation, characterizing itself as a morphological transition zone. The comparative analysis (regression and K-means) highlighted a complete absence of spatial correlation between Class A and Class C, confirming the hypothesis that socioeconomic segregation manifests in structurally distinct urban patterns. We conclude that the fractal dimension is an effective metric for quantifying inequality in the production of space, demonstrating how social stratification in Alagoinhas is mapped onto its urban morphology.

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