Berry connection and quantum geometry in time-dependent systems with instantaneous quantum integrable field theory

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Letter

Berry connection and quantum geometry in time-dependent systems with instantaneous quantum integrable field theory

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Ministry of Science and Technology of the People's Republic of China

Berry connection

Consider a **time-dependent system** with instantaneous eigenstates $\{|\phi_n(t)\rangle\}$

Berry connection matrix entries

$$\gamma_{nm} = i\langle\phi_n(t)|\dot{\phi}_m(t)\rangle = i\left\langle\phi_n(t)\middle|\frac{d}{dt}\middle|\phi_m(t)\right\rangle$$

Berry phase $\beta = \oint_0^T \gamma_{nn}\, dt$

Quantum geometric potential $Q_{nj}(t) = \gamma_{nn}(t) - \gamma_{jj}(t) + \frac{d}{dt}\arg\gamma_{jn}(t), \forall\, n \neq j$

JW*, M. Zhao, J. Chen, and Y. Zhang, Phys. Rev. A 77, 062114 (2008)

Topological quantities can be further constructed…..

A theorem

Consider a **time-dependent system** whose **low-energy physics** can be organized by quantum integrable field theory (QIFT) instantaneously. At each moment we have **instantaneous** eigenfunction

$$|\phi_n(t)\rangle = U_a(t)|\phi_n(t=0)\rangle$$

$$U_a(t) = \sum_m |\phi_m(t)\rangle\langle\phi_m(0)|$$

Adiabatic Hamiltonian $\quad H_{adia} = iU_a^{-1}(t)\dot{U}_a(t)$

Yongde Zhang, *Advanced Quantum Mechanics* (Science Press, Beijing, China, 2010, in Chinese)

Focusing on the **low energy sector** if

$$H_{adia} = \sum_{i=1}^{N_{\mathcal{O}}} \int_0^L f_i(t)\mathcal{O}_i(x)dx$$

then in the thermodynamic limit $L \to \infty$ the corresponding Berry connection matrix entries

$$\gamma_{nm} = i\langle\phi_n(t)|\dot{\phi}_m(t)\rangle$$

can be analytically determined as excitations up to two particles of the initial QIFT.

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

A short background on QIFT

In QIFT,

The conserved charges $[P_s, H_{IFT}] = 0$

$$P_s|A_{a_1}(\theta_1)\dots A_{a_N}(\theta_N)\rangle_{(\text{in,out})} = \sum_{i=1}^{N} q_{s,a_i} e^{s\theta_i} |A_{a_1}(\theta_1)\dots A_{a_N}(\theta_N)\rangle_{(\text{in,out})}$$

$$E^2 = p^2 + m^2 \Rightarrow \begin{cases} E = m\cosh\theta \\ p = m\sinh\theta \end{cases}$$

e.g. $P_1 = \boldsymbol{p}^0 + \boldsymbol{p}^1 = E + p = me^{\theta} \to q_{1,a_i} = m_{a_i}$.

- Stable quasiparticle excitations

Normalization $\left\langle A_{a_i}(\theta_1) \middle| A_{a_j}(\theta_2) \right\rangle = 2\pi\delta_{a_i a_j}\delta(\theta_1 - \theta_2)$

- Systematic form factor bootstrap

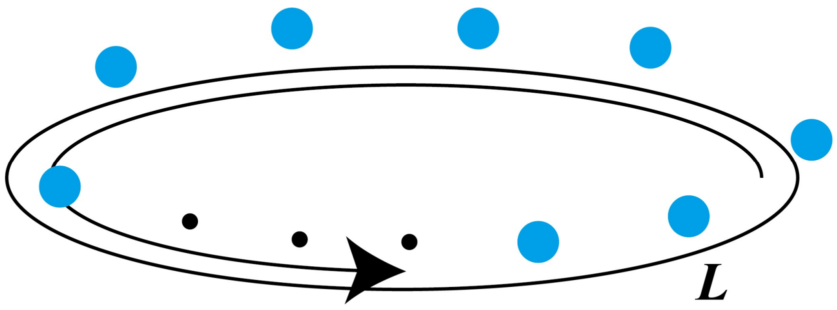
Dorey, P. Exact S-matrices. Lecture Notes in Physics, vol 498. Springer (1997).
V. Fateev, et.al, Int. J. Mod. Phys. A 05, 1025 (1990).
A. B. Zamolodchikov, Int. J. Mod. Phys. A 04, 4235 (1989).

$$\langle 0|O|\theta_1 \dots \theta_n\rangle = F_n^O(\theta_1, \dots, \theta_n)$$

Form factor elements → Berry connection matrix → Quantum geometry

Preparation before proof: QIFT at finite size

QIFT with finite system size L: General formalism



$$\langle 0|O|A_{a_1}(I_1)\dots A_{a_n}(I_n)\rangle_L = \frac{F^O_{a_1,\dots,a_n}(\theta_1,\dots,\theta_n)}{\sqrt{\rho(\theta_1,\dots,\theta_n)}}$$

$$\sum_{I_1,\dots,I_n} = \int \frac{d\theta_1\dots d\theta_n}{(2\pi)^n}\,\rho(\theta_1,\dots,\theta_n)$$

$$|A_{a_1}(I_1)\dots A_{a_n}(I_n)\rangle_L = \frac{1}{\sqrt{\rho(\theta_1,\dots,\theta_n)}}\,|A_{a_1}(\theta_1)\dots A_{a_n}(\theta_n)\rangle$$

Normalization $\;{}_L\langle A_{a_i}(I_1)|A_{a_j}(I_2)\rangle_L = \delta_{a_ia_j}\delta_{I_1I_2}$

The Bethe-Yang quantization condition,

$$e^{iP_jL}\prod_{j\neq k}S_{jk}(\theta_j-\theta_k)=1$$

$$Q_j(\theta_1,\dots,\theta_n) = m_j\sinh\theta_j L - i\sum_{j\neq k}\log S_{jk}(\theta_j-\theta_k) = 2\pi I_j$$

$$\rho(\theta_1,\dots,\theta_n) = \mathrm{Det}\left[\frac{\partial Q_k(\theta_1,\dots,\theta_n)}{\partial\theta_j}\right]$$

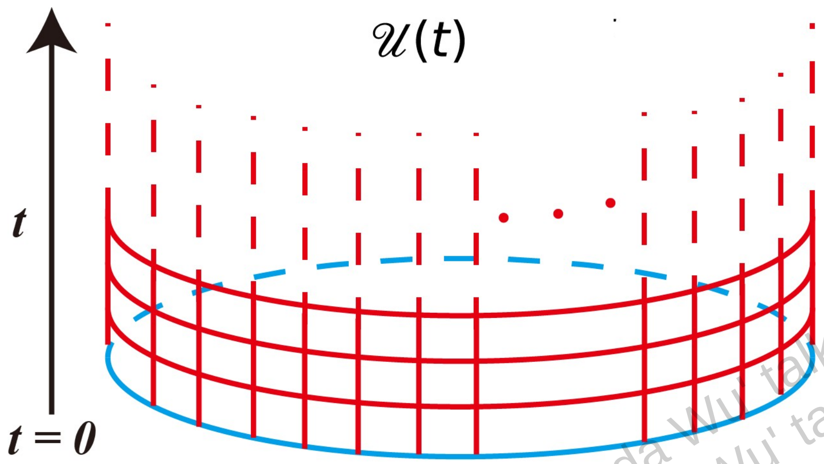
NOTE: the measure transformation for one particle

$$\rho\Delta\theta = \Delta Q = 2\pi\Delta I$$

B. Pozsgay and G. Takacs, Nucl. Phys. B 788, 167(2008).
B. Pozsgay and G. Takacs, Nucl. Phys. B 788, 209(2008).
K. Hódsági, et.al, JHEP (2019).
X. Wang, M. Oshikawa, M. Kormos*, and **JW***, Phys. Rev. B 110, 195101 (2024).

Proof

Quenching from finite size QIFT



The initial state $|\psi_n\rangle_{\text{ini },L} = \left|A_{a_1}(I_1) \ldots A_{a_N}(I_N)\right\rangle_L$

$|\phi_n(t)\rangle_L = U_a(t)|\phi_n\rangle_{\text{ini },L},$

$$H_{adia} = iU_a^{-1}\dot{U}_a, \quad H_{adia} = \sum_i^{N_O} \int_0^L dx\, f_i(t) O_i(x)$$

The Berry connection matrix element:

$$\gamma_{nm} = i\langle\phi_n(t)|\partial_t|\phi_m(t)\rangle$$

$$= \left\langle\phi_n(0)\middle|iU_a^{-1}\dot{U}_a\middle|\phi_m(0)\right\rangle = \langle\phi_n(t)|H_{adia}|\phi_m(t)\rangle$$

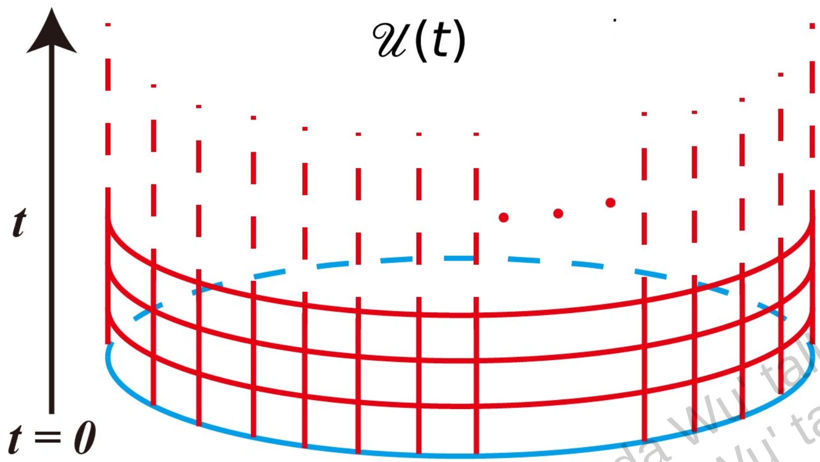
$$\gamma_{nm} = L\sum_i f_i(t)\ {}_L\langle\phi_n|O_i|\phi_m\rangle_L$$

$$= L\sum_i f_i(t)\ {}_L\langle A_1(I_1') \ldots A_N(I_N')|O_i|A_1(I_1) \ldots A_M(I_M)\rangle_L$$

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

Proof

Quenching from finite size QIFT



The initial state $|\psi_n\rangle_{\text{init},L} = \left|A_{a_1}(I_1)\dots A_{a_N}(I_N)\right\rangle_L$

$|\psi_n(t)\rangle_L = U_a(t)|\psi_n\rangle_{\text{init},L}$, $H_{adia} = \sum_i^{N_O}\int_0^L dx\, f_i(t) O_i(x)$

The Berry connection matrix element $\gamma_{nm} = \langle\psi_n(t)|H_{adia}|\psi_m(t)\rangle =$

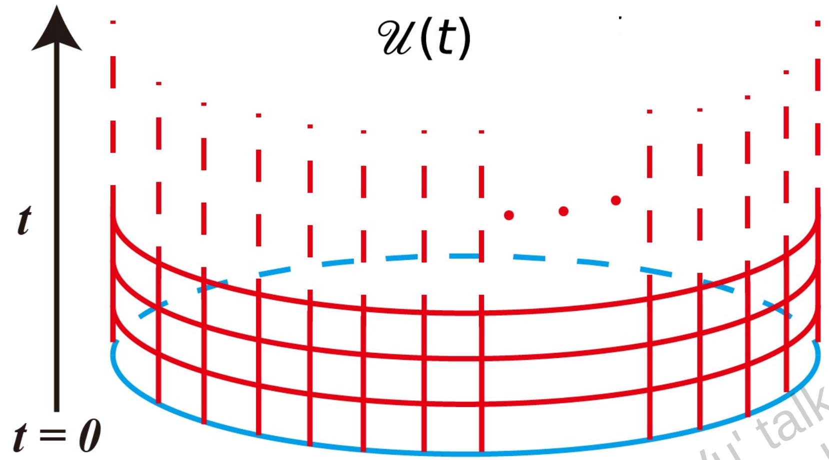
$$\begin{cases} n \neq m, \dfrac{\gamma_{nm}}{L} \approx \sum_i f_i(t) \dfrac{F_{NM}^{O_i}(\theta_1' + i\pi, \dots, \theta_N' + i\pi, \theta_1, \dots, \theta_M)}{\sqrt{\rho_N(\theta_1', \dots, \theta_N')}\sqrt{\rho_M(\theta_1, \dots, \theta_M)}} \\ n = m, \dfrac{\gamma_{nn}}{L} \approx \sum_i f_i(t) \left[{}_L\langle O_i\rangle_L + \sum_{A\subset\{A_1,\dots,A_N\}} \dfrac{\mathcal{F}^{O_i}(A)\rho(\{A_1,\dots,A_N\}\backslash \mathrm{A})_L}{\rho(\{A_1,\dots,A_N\})_L} \right] \end{cases}$$

$$\mathcal{F}^{O_i}(A) = F_{AA}^{O_i}(\{\theta_A + i\pi\}, \{\theta_A\})$$

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

Proof

Quenching from finite size QIFT



Theorem: *Consider time-dependent many-body quantum systems with emergent quantum integrable field theory instantaneously, if the effective adiabatic Hamiltonian follows* $H_{adia} = \sum_i^{N_O} \int_0^L dx\, f_i(t) O_i(x)$ *, then the corresponding BCM entries with total quasiparcticle number* Z *follow the asymptotic behavior* $L^{1-Z/2}$ *, corresponding to excitations up to two particles in the thermodynamic limit (*$L \to \infty$*).*

The initial state $|\phi_n\rangle_{\mathrm{init},L} = \left|A_{a_1}(I_1) \dots A_{a_N}(I_N)\right\rangle_L$

$$|\phi_n(t)\rangle_L = U(t)|\phi_n\rangle_{\mathrm{init},L},\ U(t){:}\ H_{adia} = \sum_i^{N_O} \int_0^L dx\, f_i(t) O_i(x)$$

The Berry connection matrix element $\gamma_{nm} = i\langle\psi_n(t)|\partial_t|\psi_m(t)\rangle =$

$$\begin{cases} n \neq m, \dfrac{\gamma_{nm}}{L} \approx \displaystyle\sum_i f_i(t) \dfrac{F_{NM}^{O_i}(\theta_1' + i\pi, \dots, \theta_N' + i\pi, \theta_1, \dots, \theta_M)}{\sqrt{\rho_N(\theta_1', \dots, \theta_N')}\sqrt{\rho_M(\theta_1, \dots, \theta_M)}} \\ n = m, \dfrac{\gamma_{nn}}{L} \approx \displaystyle\sum_i f_i(t) \left[{}_L\langle O_i \rangle_L + \sum_{A \subset \{A_1, \dots, A_N\}} \dfrac{\mathcal{F}^{O_i}(A)\rho(\{A_1, \dots, A_N\} \backslash A)_L}{\rho(\{A_1, \dots, A_N\})_L} \right] \end{cases}$$

$$\mathcal{F}^{O_i}(A) = F_{AA}^{O_i}(\{\theta_A + i\pi\}, \{\theta_A\})$$

Recalling $\rho(\theta_1, \dots, \theta_n) \sim L^n$, as $L \to \infty$,
only contributions up to ***two particle matrix elements*** will be left.

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Proof

QIFT with finite system size L: Expansion for diagonal matrix element

Crossing relation of form factor:

$$F^{O}_{mn}(\theta'_1,\dots,\theta'_m|\theta_1,\dots,\theta_n) = \langle A_{j_1}(\theta'_1)\dots A_{j_m}(\theta'_m)|O|A_{i_1}(\theta_1)\dots A_{i_n}(\theta_n)\rangle$$

$$= F^{O}_{m-1,n+1}(\theta'_1,\dots,\theta'_{m-1}|\theta'_m+i\pi,\theta_1,\dots,\theta_n) + \sum_{k=1}^{n} 2\pi\delta_{j_m i_k}\delta(\theta'_m-\theta_k)\prod_{l=1}^{k-1} S_{i_l i_k}(\theta_l-\theta_k)F^{O}_{m-1,n-1}(\theta'_1,\dots,\theta'_{m-1}|\theta_1,\dots,\theta_{k-1},\theta_{k+1},\dots,\theta_n)$$

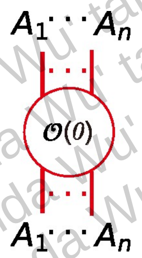
$$\mathcal{F}^{O}(A) = F^{O}_{AA}(\{\theta_A+i\pi\},\{\theta_A\})$$

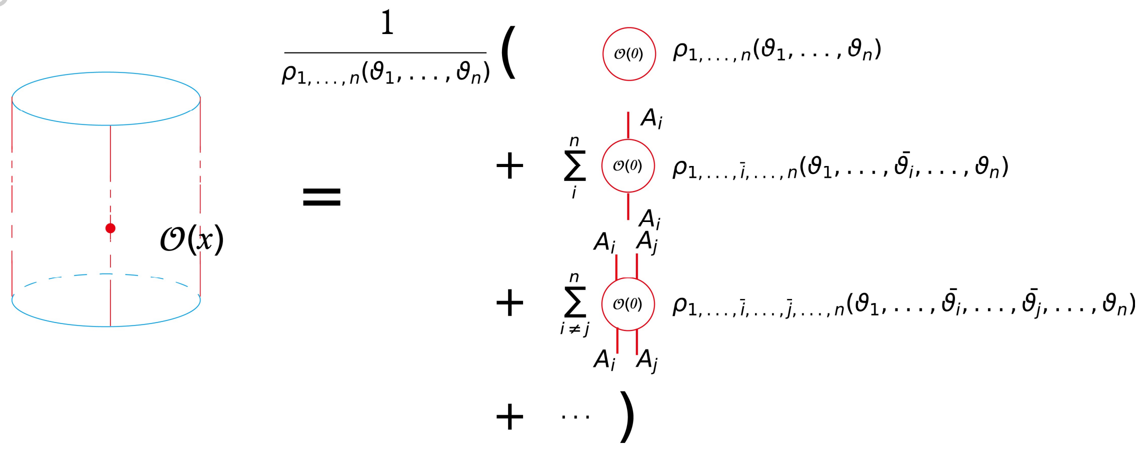
Finite size expansion:

$${}_L\langle I_1\dots I_n|O|I_1\dots I_n\rangle_L = {}_L\langle O\rangle_L + \sum_{A\subset\{A_1,\dots,A_n\}} \frac{\mathcal{F}^{O}(A)\rho(\{A_1,\dots,A_n\}\backslash A)_L}{\rho(\{A_1,\dots,A_n\})_L}$$

Diagonal matrix element: disconnected term

$$\langle A_1(\theta_1)\dots A_n(\theta_n)|O|A_1(\theta_1)\dots A_n(\theta_n)\rangle =$$





B. Pozsgay and G. Takacs, Nucl. Phys. B 788, 167(2008).
B. Pozsgay and G. Takacs, Nucl. Phys. B 788, 209(2008).
K. Hódsági, et.al, JHEP (2019).
X. Wang, M. Oshikawa, M. Kormos*, and **JW***, Phys. Rev. B 110, 195101 (2024).

Quantum geometric potential and effective energy gap

Quantum geometric potential (QGP) $Q_{nj}(t) = \gamma_{nn}(t) - \gamma_{jj}(t) + \frac{d}{dt}\arg\gamma_{jn}(t), \forall\, n \neq j$

- Local (in time) U(1)-invariant under transform $|\phi_n(t)\rangle \to e^{if_n(t)}|\phi_n(t)\rangle$
- $Q_{mn}(t)/2|\gamma_{mn}|$ characterizes geodesic curvature of Bloch sphere of 2-level system

$$H = A(t) + B(t)\vec{n}(t)\cdot\vec{\sigma}, \vec{n}(t) = [\sin\theta(t)\cos\varphi(t), \sin\theta(t)\sin\varphi(t), \cos\theta(t)]$$

$$\Rightarrow \begin{cases} |+,t\rangle = \cos\dfrac{\theta(t)}{2}|0\rangle + e^{i\varphi}\sin\dfrac{\theta(t)}{2}|1\rangle \\ |-,t\rangle = \sin\dfrac{\theta(t)}{2}|0\rangle - e^{i\varphi}\cos\dfrac{\theta(t)}{2}|1\rangle \end{cases}$$

$$\Rightarrow \frac{Q_{mn}(t)}{2|\gamma_{mn}(t)|} = \rho = \left(\vec{n}\times\frac{d\vec{n}}{ds}\right)\cdot\frac{d^2\vec{n}}{ds^2}, \text{ with } ds = |d\vec{n}|.$$

- Gives effective energy gap $\Delta_{jn}(t) = E_j(t) - E_n(t) + Q_{nj}(t)$

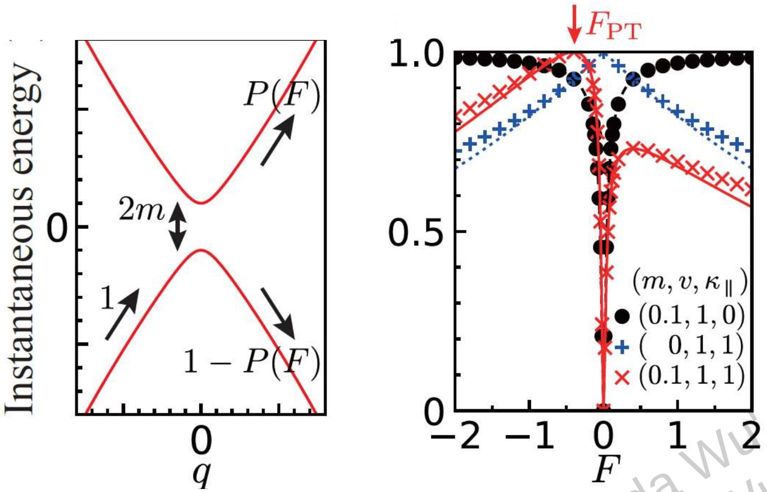
JW*, M. Zhao, J. Chen, and Y. Zhang, Phys. Rev. A 77, 062114 (2008)
C. Xu, **JW**, and C. Wu, Phys. Rev. A 97, 032124 (2018)

Quantum geometric potential and effective energy gap

Effective energy gap $\Delta_{jn}(t) = 0 \quad \rightarrow \quad$ Landau-Zener tunneling maximizes

$$H(F,t) = m\sigma^z + vq\sigma^x + \frac{1}{2}\kappa_{\parallel} v^2 q^2 \sigma^y$$

The tunneling probability ($q = -Ft$)



$$P(F) = \exp\left[-\frac{\pi\left(m + \frac{\kappa_{\parallel} vF}{4}\right)^2}{v|F|}\right] = \exp[-\frac{\pi}{4v|F|}\left(\Delta + \frac{FR_{+-}}{2}\right)^2]$$

Landau-Zener tunneling

$$\Delta_{eff} = E_+ - E_- + \frac{FR_{+-}}{2}, R_{+-} = \kappa_{\parallel} v$$

Calculate the QGP and effective energy gap

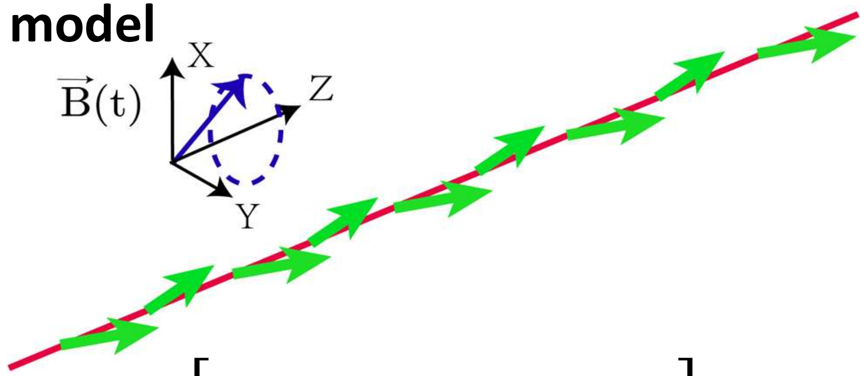
Perfect tunneling F_{PT} happens at $\Delta_{eff} = 0$

How about quantum many-body system?

S. Takayoshi, **JW**, T. Oka, SciPost Phys. 11, 075 (2021)

Quantum geometry in a time-dependent quantum Ising chain

The model



$$H(t) = -J\left[\sum_i \sigma_i^z \sigma_{i+1}^z + \vec{B}(t)\cdot\vec{\sigma}_i\right]$$

$$\vec{B} = (\cos\omega_0 t, -\sin\omega_0 t, h_z), \omega_0, h_z \to 0^+, J = 1$$

$|\psi(0)\rangle$ ground state of $H(0)$

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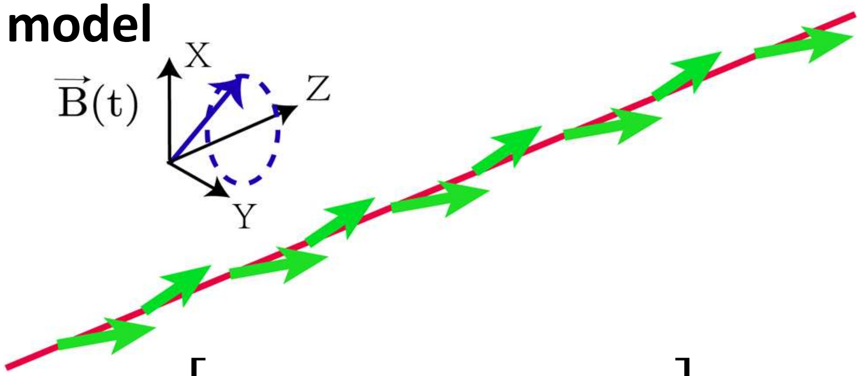
The model



$$H(t) = -J\left[\sum_i \sigma_i^z \sigma_{i+1}^z + \vec{B}(t)\cdot\vec{\sigma}_i\right]$$

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$|\psi(0)\rangle$ ground state of $H(0)$

$\downarrow$ scaling limit, $h \sim J^{\frac{15}{8}} h_z$

$$H_{E_8,t=0} = H_{c=1/2} - h\int \sigma(x)dx$$

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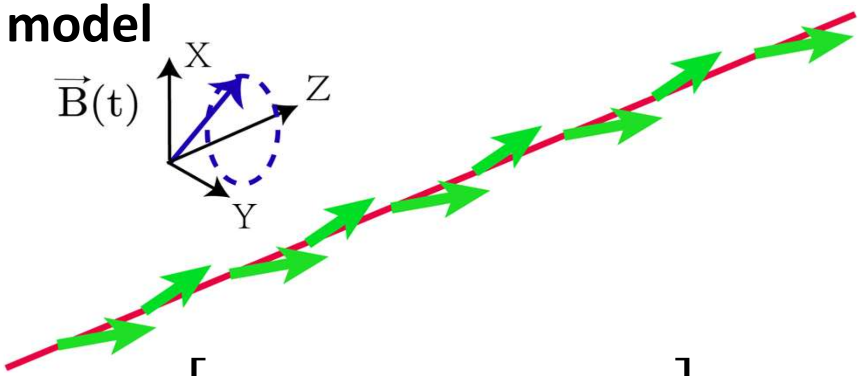
Quantum geometry in a time-dependent quantum Ising chain

The model



$$H(t) = -J\left[\sum_i \sigma_i^z \sigma_{i+1}^z + \vec{B}(t)\cdot\vec{\sigma}_i\right]$$

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$|\psi(0)\rangle$ ground state of $H(0)$

$\big\downarrow$ scaling limit, $h \sim J^{\frac{15}{8}} h_z$

$$H_{E_8,t=0} = H_{c=1/2} - h\int \sigma(x)dx$$

$\to$ an instantaneous quantum E_8 IFT

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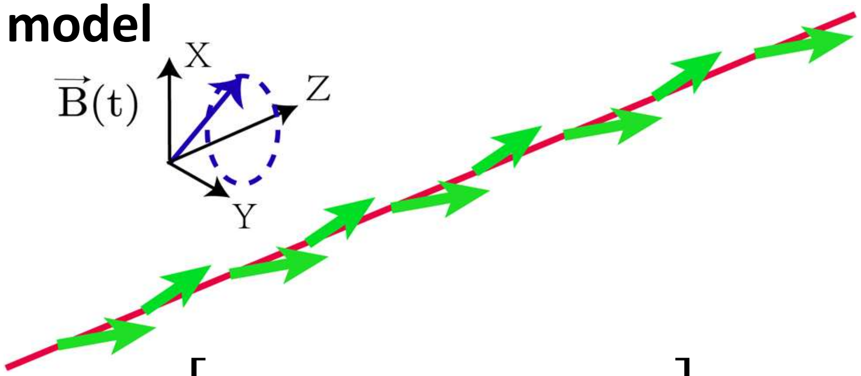
Quantum geometry in a time-dependent quantum Ising chain

The model



$$H(t) = -J\left[\sum_i \sigma_i^z\sigma_{i+1}^z + \vec{B}(t)\cdot\vec{\sigma}_i\right]$$

$\vec{B} = (\cos\omega_0 t, -\sin\omega_0 t, h_z), \omega_0, h_z \to 0^+, J = 1$

$|\psi(0)\rangle$ ground state of $H(0)$

scaling limit $h \sim J^{\frac{15}{8}} h_z$

$$H_{E_8,t=0} = H_{c=1/2} - h\int \sigma(x)dx$$

→ an instantaneous quantum E_8 IFT

Unitary transformation $U(t) = \exp\left\{-\frac{i}{2}\omega_0 t \sum_i \sigma_i^z\right\}$

Exact evolution $|\psi(t)\rangle = U^{-1}(t)\exp\{-iH_{\mathrm{eff}}t\}|\psi(0)\rangle$

$$H_{\mathrm{eff}} = -J\sum_i\left[\sigma_i^z\sigma_{i+1}^z + \sigma_i^x + \left(h_z - \frac{\omega_0}{2J}\right)\sigma_i^z\right]$$

$$H_{E_8,\mathrm{eff}} = H_{c=1/2} - \left(h - \frac{\omega}{2}\right)\int \sigma(x)dx$$

Instantaneous eigen functions

$$|\boldsymbol{\psi_n(t)}\rangle_{\boldsymbol{ins}} = \boldsymbol{e^{i\varphi_n(t)}U^{-1}(t)|\psi_n(0)\rangle}, E_n(t) = E_n(0)$$

U^{-1} is also an adiabatic transformation!

Reduced parameter $\kappa = h_z/(\omega_0/2)$.

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Quantum geometry in a time-dependent quantum Ising chain

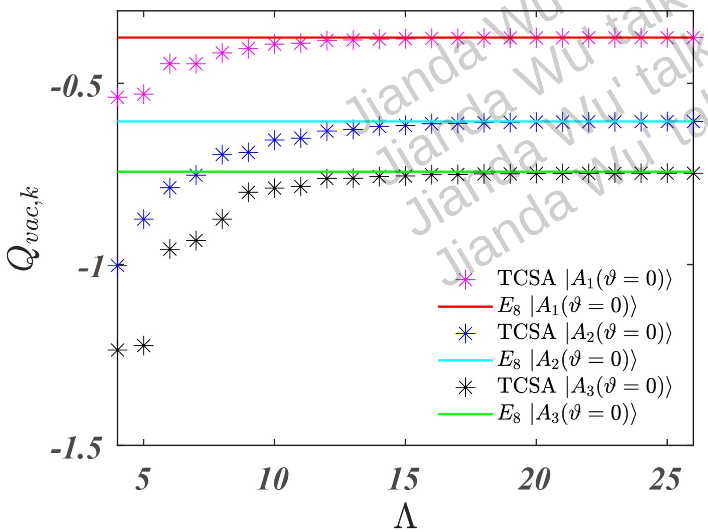
Adiabatic Hamiltonian $$H_{adia} = iU_a^{-1}(t)\dot{U}_a(t) = -\frac{\omega_0}{2}\sum_i \sigma_i^z = -\frac{\omega}{2}\int_0^{L\to\infty}\sigma(x)dx$$

Here, $\omega = J^{7/8}\omega_0$ and the continuum limit condition $2Ja = \hbar c = 1$

The QGP between $|\psi_n(t)\rangle_{ins}$ and $|\psi_m(t)\rangle_{ins}$ containing N and M E_8 particles, respectively

$$Q_{nm}(t) = \sum_{j=1}^{M}\frac{F_{jj}^{\sigma}(i\pi,0)}{m_j\cosh\theta_j} - \sum_{i=1}^{N}\frac{F_{ii}^{\sigma}(i\pi,0)}{m_i\cosh\theta_i} = -\frac{8}{15\kappa}\left(\sum_{j=1}^{M}\frac{m_j}{\cosh\theta_j} - \sum_{i=1}^{N}\frac{m_i}{\cosh\theta_i}\right)$$

e.g. the QGP between E_8 vacuum and single E_8 particle state: $Q_{vac,k} = \frac{F_{kk}^{\sigma}(i\pi,0)}{m_k}$,



Verified by truncated conformal space approach (TCSA)

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Quantum geometry in a time-dependent quantum Ising chain

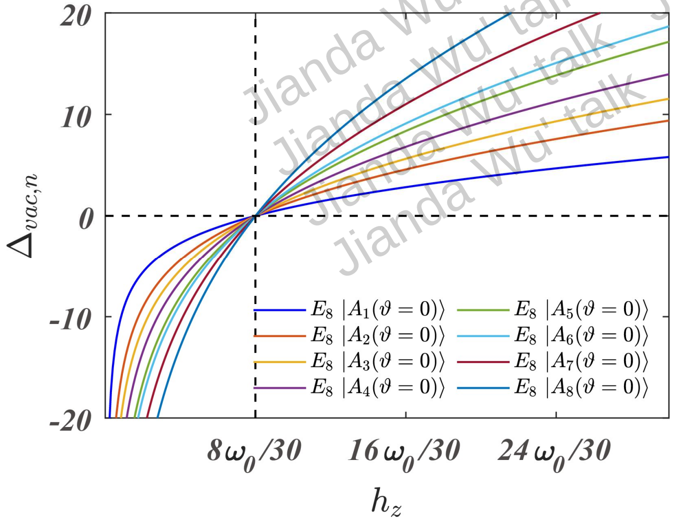
The effective energy gap (EEG) between $|\psi_n(t)\rangle_{ins}$ and $|\psi_m(t)\rangle_{ins}$ containing N and M E_8 particles

$$\Delta_{nm}(t) = \sum_{j=1}^{M} m_j \cosh\theta_j - \sum_{i=1}^{N} m_i \cosh\theta_i - \frac{8}{15\kappa}\left(\sum_{j=1}^{M} \frac{m_j}{\cosh\theta_j} - \sum_{i=1}^{N} \frac{m_i}{\cosh\theta_i}\right)$$

- Existence of zeros for EEG between E_8 vacuum and any number of E_8 particles

  upper bound $\kappa = \frac{8}{15}$

- Existence of zeros for EEG between E_8 vacuum and single E_8 particles



Independent of particle type!

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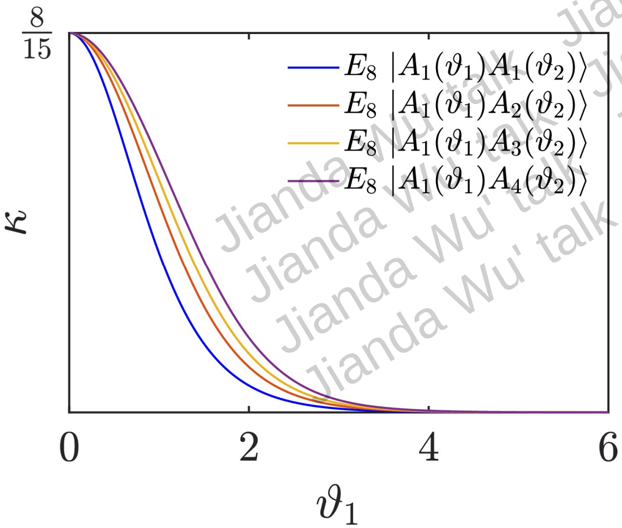
Quantum geometry in a time-dependent quantum Ising chain

The effective energy gap (EEG) between $|\psi_n(t)\rangle_{ins}$ and $|\psi_m(t)\rangle_{ins}$ containing N and M particles, respectively:

$$\Delta_{nm}(t) = \sum_{j=1}^{M} m_j \cosh\theta_j - \sum_{i=1}^{N} m_i \cosh\theta_i - \frac{8}{15\kappa}\left(\sum_{j=1}^{M} \frac{m_j}{\cosh\theta_j} - \sum_{i=1}^{N} \frac{m_i}{\cosh\theta_i}\right)$$

The zeros of EEG between E_8 vacuum and two-E_8-particle continuum: $\kappa = \frac{8}{15}$ serves as an upper bound



As the total energy increases, the zeros keep decreases.

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

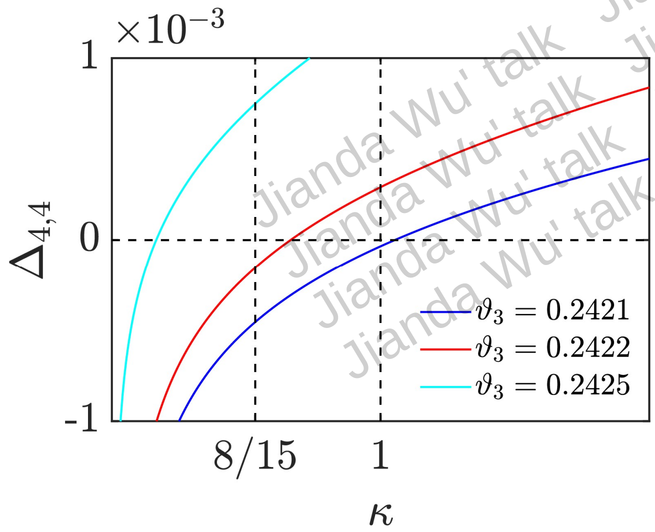
Quantum geometry in a time-dependent quantum Ising chain

The effective energy gap (EEG) between $|\psi_n(t)\rangle_{ins}$ and $|\psi_m(t)\rangle_{ins}$ containing N and M E_8 particles, respectively:

$$\Delta_{nm}(t) = \sum_{j=1}^{M} m_j \cosh\theta_j - \sum_{i=1}^{N} m_i \cosh\theta_i - \frac{8}{15\kappa}\left(\sum_{j=1}^{M} \frac{m_j}{\cosh\theta_j} - \sum_{i=1}^{N} \frac{m_i}{\cosh\theta_i}\right)$$

The zeros of EEG between any two multi-E_8-particle states: the upper bound can be larger

e.g.



The density of states for such multi-particle states with large upper bound is small.

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

Spectral entropy of Loschmidt echo

Loschmidt echo $L(t) = {}_{ins}\langle\psi_0(t)|\psi(t)\rangle = \langle\psi(0)|e^{-iH_{\mathrm{eff}}t}|\psi(0)\rangle$

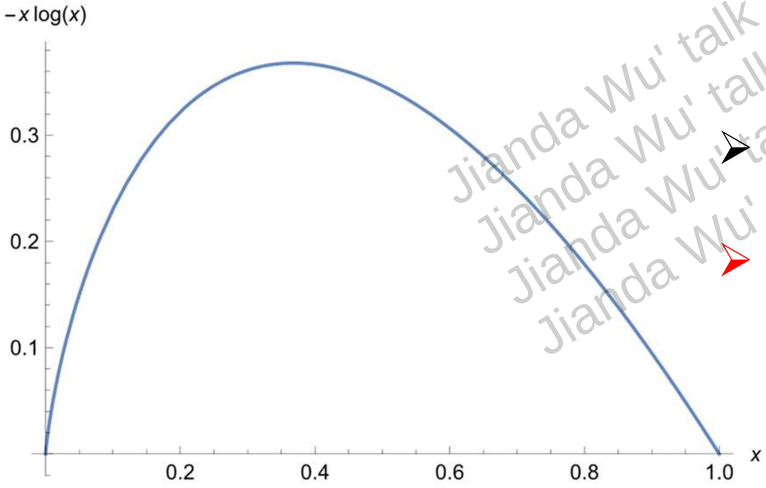
- Characterize strength of Landau-Zener tunneling
- May be detected through "image" cold-atom experiment

Loschmidt echo spectrum $L(\omega) = \int dt\, e^{i\omega t}L(t) = \sum_m |\langle\phi_m|\psi(0)\rangle|^2\delta(\omega - E_m)$

Eigen states and eigenenergies of H_{eff}

Introducing the Loschmidt echo spectral entropy

$$S_{LE} = -\sum_m p_m \ln p_m \ , p_m = |\langle\phi_m|\psi(0)\rangle|^2,\ \sum_m p_m = 1$$



- if $p_m \to 0$ or 1 then those states contribute negligibly to S_{LE}
- Increasing S_{LE} implies enhancement of MBLZT

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

S_{LE} in the time-dependent quantum Ising chain

Loschmidt echo spectral entropy: **The truncated lattice free fermion approach** (TLFFA)

$$H = -J\sum_i(\sigma_i^z\sigma_{i+1}^z + g\sigma_i^x + h_z\sigma_i^z) = H_{TFIC} - Jh_z\sum_i \sigma_i^z$$

Connect R and NS sectors

${}_R\langle P|\sigma^z|Q\rangle_{NS}$ is known

$$\boxed{H_{TFIC} = \sum_q \varepsilon_q(r_q^\dagger r_q - \frac{1}{2}), \qquad \varepsilon_q = 2J\sqrt{1+g^2-2g\cos q}}$$

$$\begin{pmatrix} H_{TFIC} & -Jh_z\sum_i \sigma_i^z \\ -Jh_z\sum_i \sigma_i^z & H_{TFIC} \end{pmatrix}$$

The Hilbert space: $\begin{cases} \text{Ramond Sector}: |P\rangle_R = r_{p_1}^\dagger r_{p_2}^\dagger \dots r_{p_n}^\dagger |0\rangle_R \\ \text{Neveu}-\text{Schwarz Sector}: |Q\rangle_{NS} = r_{q_1}^\dagger r_{q_2}^\dagger \dots r_{q_n}^\dagger |0\rangle_{NS} \end{cases}$

$$P:\left\{p = \frac{2\pi}{L}j\right\}, Q:\left\{q = \frac{2\pi}{L}\left(j-\frac{1}{2}\right)\right\}, j = -\frac{L}{2}+1, -\frac{L}{2}+2, \dots, \frac{L}{2}.$$

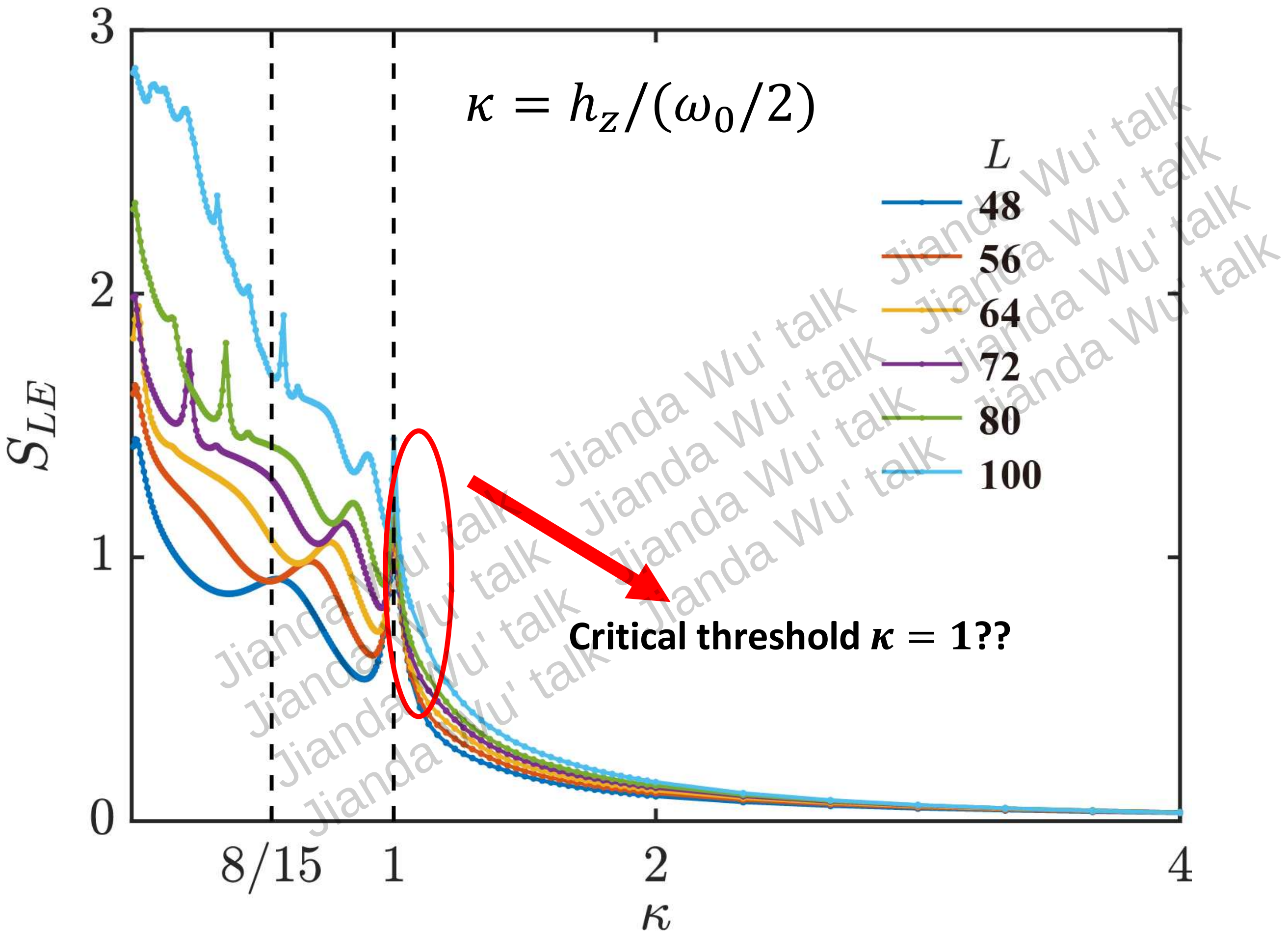
Truncation according to ***energy*** and ***# of states***

Xiaodong He, Xiao Wang, and **JW***, to appear very soon.

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

N Iorgov et al J. Stat. Mech. (2011) P02028.

S_{LE} in the time-dependent quantum Ising chain



Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

S_{LE} in the time-dependent quantum Ising chain

Many-body Landau-Zener tunneling (MBLZT)

Critical threshold $\boldsymbol{\kappa = 1}$

A physical understanding:

$$|\psi(t)\rangle = U^{-1}(t)e^{iH_{\mathrm{eff}}t}|\psi(0)\rangle$$

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

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$$|\psi(t)\rangle = U^{-1}(t)e^{i\underline{H_{\text{eff}}}t}|\psi(0)\rangle$$

$$H_{\text{eff}} = -\sum_i\left[\sigma_i^z\sigma_{i+1}^z + \sigma_i^x + \left(h_z - \frac{\omega_0}{2}\right)\sigma_i^z\right]$$

As $\kappa \leq 1, h_z \leq \omega_0/2$. The low energy excitations:

$\kappa = h_z/(\omega_0/2)$.

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

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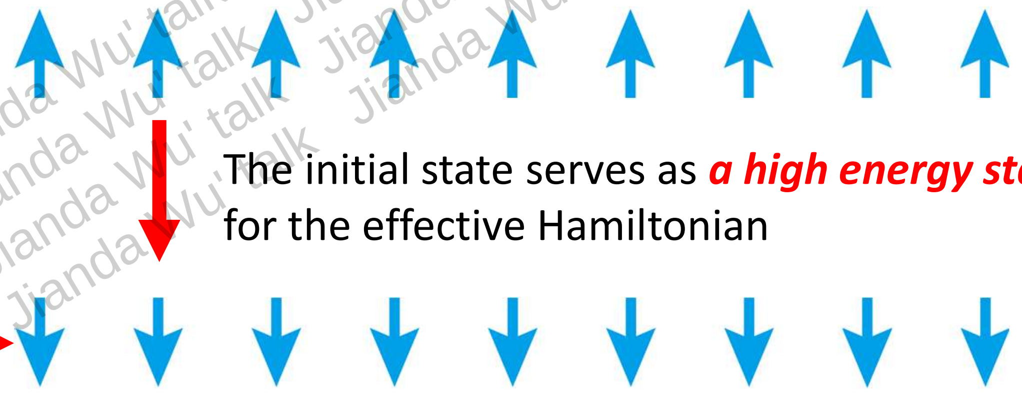


The initial state serves as ***a high energy state*** for the effective Hamiltonian

$$H_{\text{eff}} = -\sum_i \left[\sigma_i^z \sigma_{i+1}^z + \sigma_i^x + \left(h_z - \frac{\omega_0}{2}\right) \sigma_i^z\right]$$

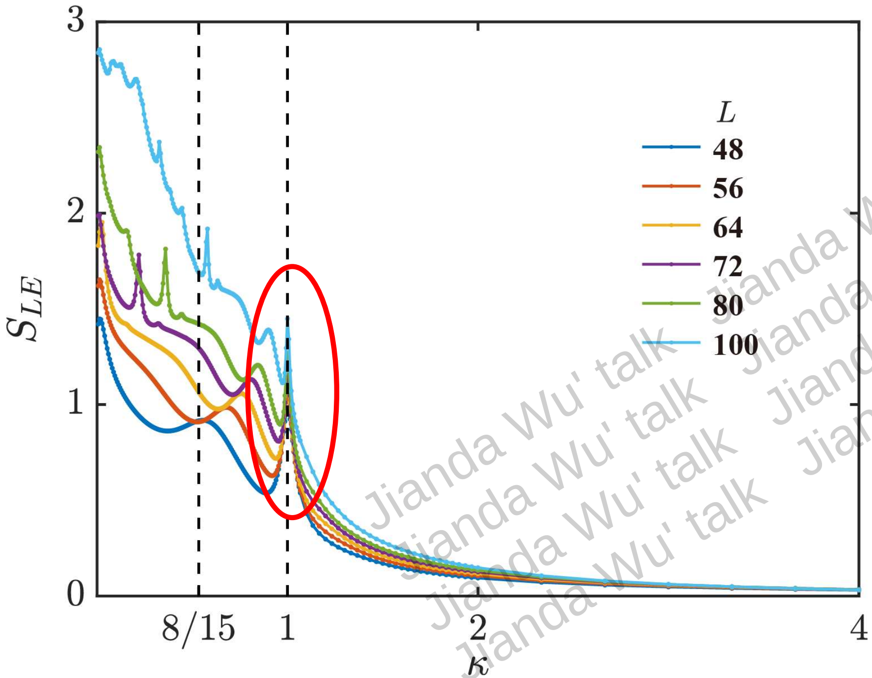
$\kappa \leq 1 \Rightarrow h_z \leq \omega_0/2$ The low energy excitations

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

S_{LE} in the time-dependent quantum Ising chain

Many-body Landau-Zener tunneling (MBLZT)

Scaling analysis near critical threshold $\kappa = 1$



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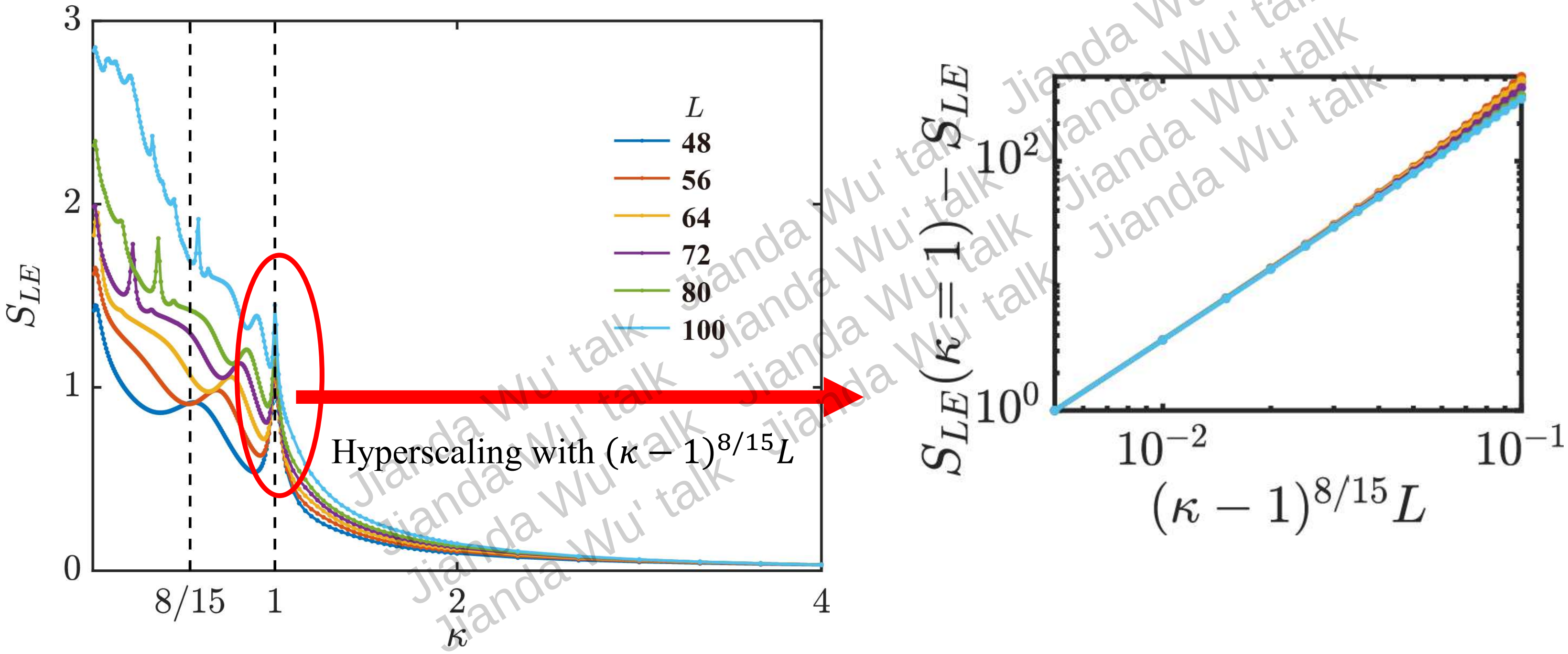

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

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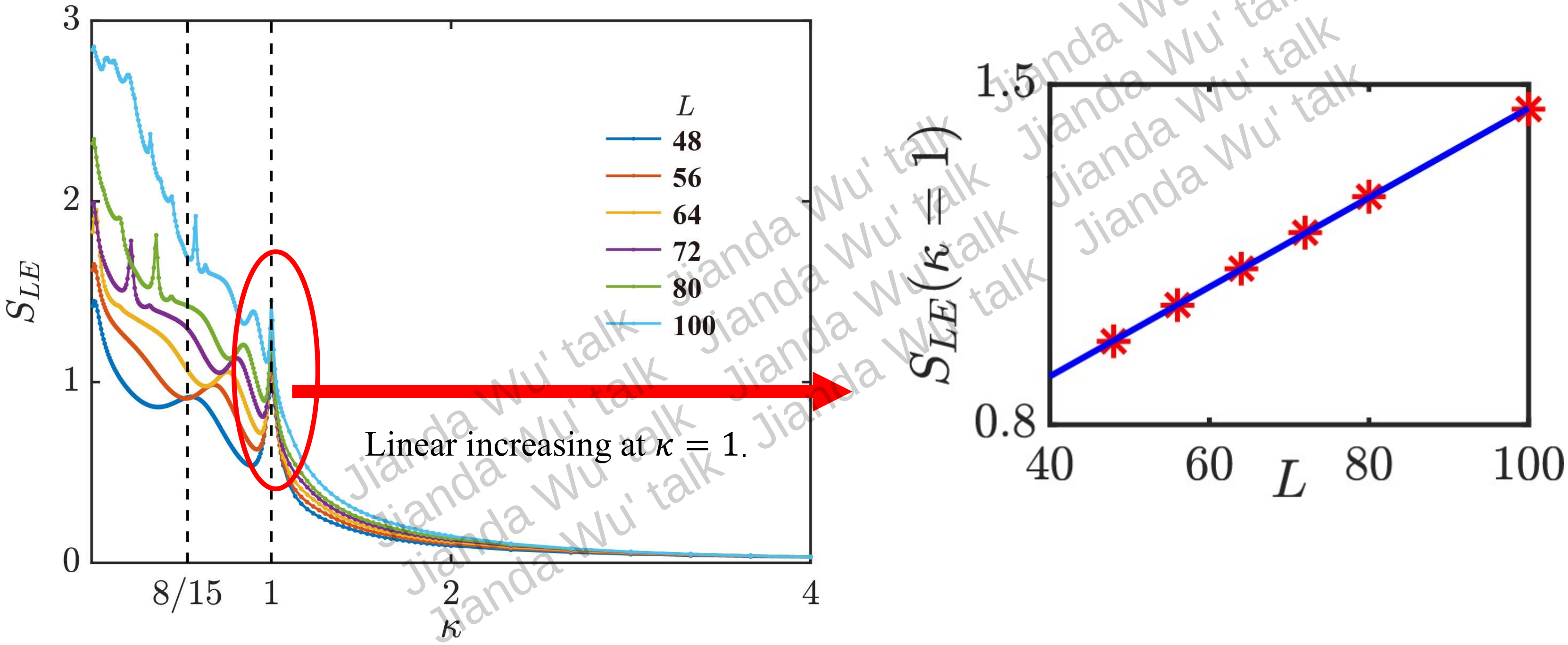


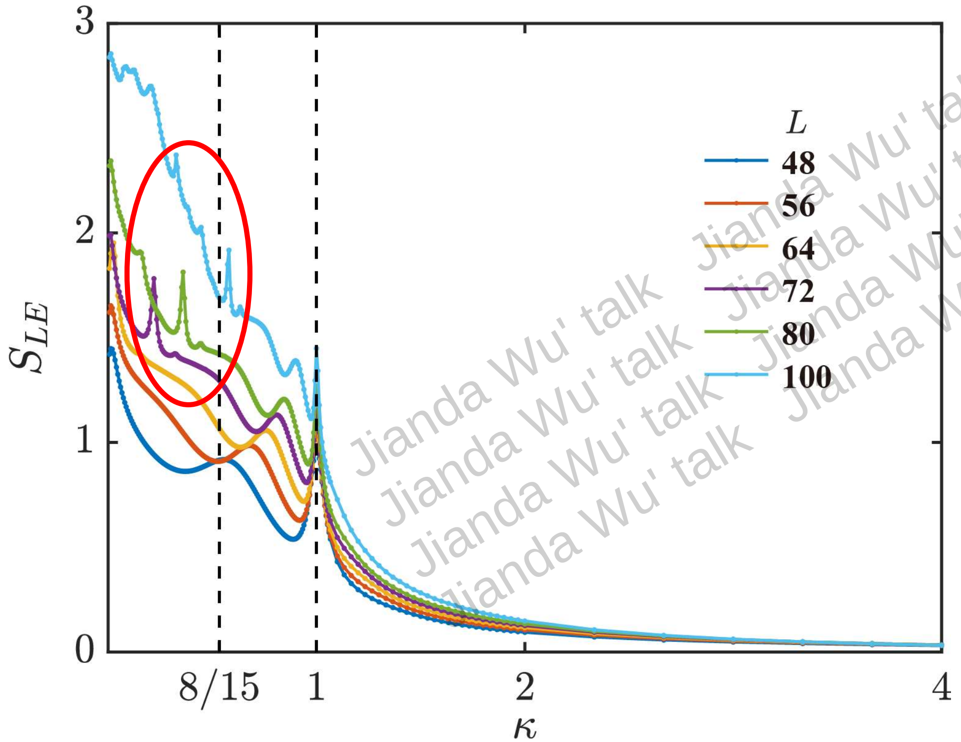
Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

S_{LE} in the time-dependent quantum Ising chain

Many-body Landau-Zener tunneling (MBLZT):

The second enhancement around $\kappa = \frac{8}{15}$ $\Leftarrow$ **Low energy sectors begin to involve**

the ***upper bound*** for EEG from ground state to excited states



Multi-new "divergent" behaviors?

Limited by the computational power

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

Summary

- The many-body Berry connection matrix can be determined for time-dependent systems whose low-energy physics can be organized by quantum integrable field theory instantaneously. And it can be explicitly determined up to at most two-particle process in the thermodynamical limit.

- The zeros of effective energy gap can serve as a good pointer for Landau-Zener tunneling.

- The Loschmidt echo spectral entropy is proposed to serve as a powerful tool to illustrate the many-body Landau Zener tunneling, and reveals critical behavior that follows hyperscaling in the time-dependent quantum Ising chain we study.

- The methodology of the spectral entropy analysis can also apply to other non-equilibrium systems.

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

Continuously seeking PhD students and Postdocs!

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Poster: Mesons in a quantum Ising ladder

N. Xi#, X. Wang#, Yunjing Gao, Y. Jiang, Rong Yu*, **JW***, accepted by Phys. Rev. B (Letter) (2025, in production)

Yunjing Gao, Y. Rong*, **JW*,** Sci. Chin. Phys. Mech. Astron. 69, 217401 (2026)

Yunjing Gao, Y. Jiang*, **JW***, JHEP 07, 072 (2025)

Yunjing Gao, X. Wang, N. Xi, Y. Jiang, R. Yu*, **JW*** Phys. Rev. B 111, L201117 (2025)

Yunjing Gao#, J. Yang#, H. Lin, R. Yu, **JW***, Phys. Rev. B 111, L241105 (2025)

Yunjing Gao, **JW*** Chin. Phys. Lett. 42, 047501 (2025)

Yunjing Gao, J. Yang, Z. Zhu, Y. Mizuno, **JW***, arXiv: 2205.08259.

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Poster: Perfect many-body tunneling and discrete time crystal in a periodic-driven Heisenberg spin chain

Xiaotong Chen and **JW***, arXiv: 2507.15565, submitted to Phys. Rev. Lett. (under Review)

HE, Xiaodong
何晓东

Poster: Renormalization of truncated lattice Hamiltonian spectrum

Xiaodong He, Xiao Wang, and **JW***, to appear very soon.

Xiao Wang, Xiaodong He, and **JW***, Phys. Rev. B (Letter) 112, L201102 (2025)

Tsung-Dao Lee Institute

Thank You!

合肥国家实验室
Hefei National Lab