The Last Polynomials

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Founded in 1997

Modeled after Princeton IAS

Professor Yang has been the honorary director since the beginning, and he considered this as a new chapter of his life after his eighties.

§4 翟荟，請找人帮我 test 下面猜想（6个），多谢。 Y5

振宁 2017年9月11日

Prob 1.

Consider the polynomial equation:

$$P_8(z) = (1+z^8) + 8a^7(z^7+z) + 28a^{12}(z^6+z^2) + 56a^{15}(z^5+z^3) + 70a^{16}z^4 = 0. \qquad (1)$$

(A) Take $a=1.1$, show that all 8 roots are real & <0.

(B) Take $a=2$, " the same

(C) Take $a=2.1$ " " "

Prob 2 Consider

$$P_7(z) = (z^7+1) + 7a^6(z^6+z) + 21a^{10}(z^5+z^2) + 35a^{12}(z^3+z^4)$$

z^3

2A)
2B) } Same as 1A, 1B, 1C
2C)

Zhai Hui, please find someone to help me test the following six conjectures. Thanks.

Chen Ning

2017.9.11

Decoding the mystery

[34] 程芸，请找人帮我test 下面猜想，多谢。
6个
振宁 2017年9月11日

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$$P_n = \sum_m \binom{n}{m} a^{m(n-m)} z^m$$

This is the partition function of the Ising model with all-to-all interactions

$$H = J\sum_{ij} \sigma_i\sigma_j + h\sum_i \sigma_i$$

with $a = e^{4\beta J}$

Decoding the mystery

$$P_n = \sum_m \binom{n}{m} a^{m(n-m)} z^m \qquad a = e^{4\beta J}$$

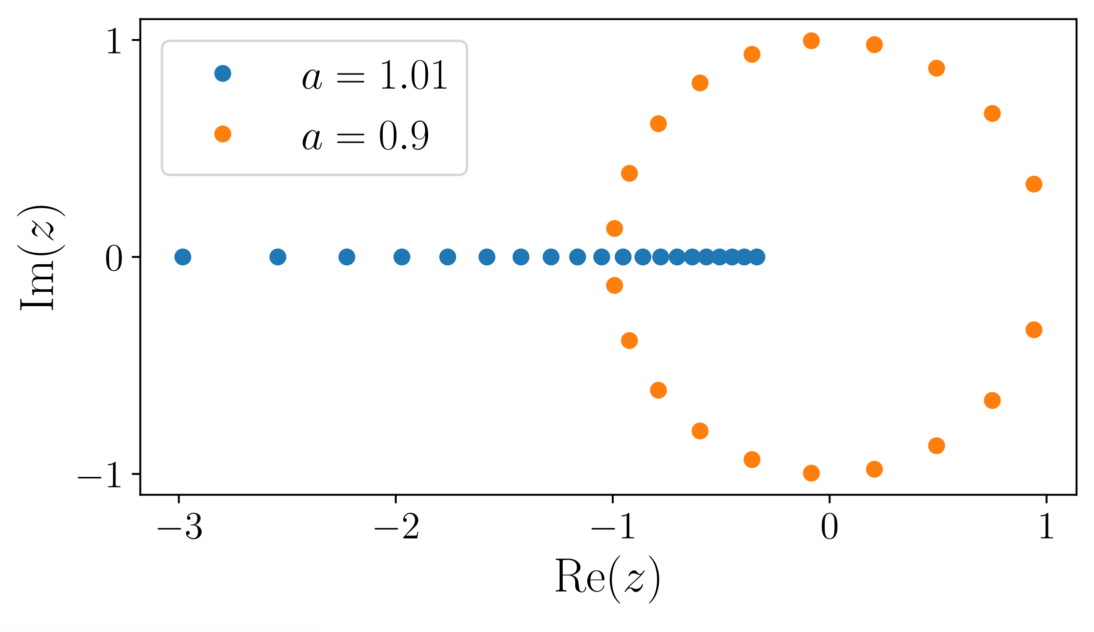
This is the partition function of the Ising model with all-to-all interactions

$$H = J \sum_{ij} \sigma_i \sigma_j + h \sum_i \sigma_i$$

It can be proved

1) all zeros are real and negative if $a = e^{4\beta J} > 1$ **for** $J > 0$

2) all zeros lie in the unit circle if $a = e^{4\beta J} < 1$ **for** $J < 0$ **Lee-Yang Zeros**



A Pure Scientist

His research is always driven by curiosity about the hidden beauty of nature.

The structure of everything

Often has a hidden beauty in it

If you can sense vaguely some of its beauty

Do not let it go

PHYSICAL REVIEW VOLUME 96, NUMBER 1 OCTOBER 1, 1954

Conservation of Isotopic Spin and Isotopic Gauge Invariance*

C. N. YANG † AND R. L. MILLS
Brookhaven National Laboratory, Upton, New York

(Received June 28, 1954)

It is pointed out that the usual principle of invariance under isotopic spin rotation is not consistant with the concept of localized fields. The possibility is explored of having invariance under local isotopic spin rotations. This leads to formulating a principle of isotopic gauge invariance and the existence of a **b** field which has the same relation to the isotopic spin that the electromagnetic field has to the electric charge. The **b** field satisfies nonlinear differential equations. The quanta of the **b** field are particles with spin unity, isotopic spin unity, and electric charge $\pm e$ or zero.

A Scientist with Unique Style and Taste

His research on style of Dirac and Heisenberg

Professor Yang is, after Einstein and Dirac, the preeminent stylist of the 20th-century physics

—- Dyson

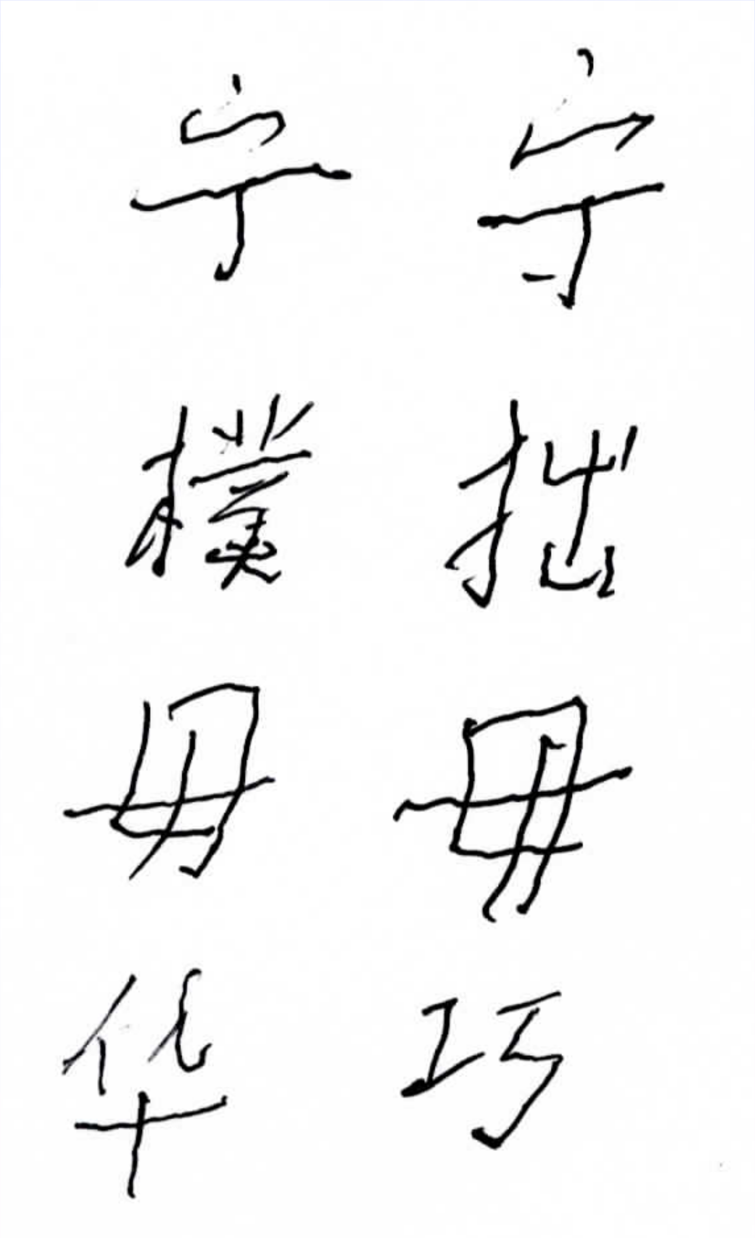
Your book follows your approach to physics

A Forward-Looking Scientist

Each generation of physicists defines their own problems

Entering a growing field

A Down-to-Earth Scientist



Never take so-called "self-evident" laws for granted

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

SOME EXACT RESULTS FOR THE MANY-BODY PROBLEM IN ONE DIMENSION WITH REPULSIVE DELTA-FUNCTION INTERACTION*

C. N. Yang
Institute for Theoretical Physics, State University of New York, Stony Brook, New York
(Received 2 November 1967)

The repulsive δ interaction problem in one dimension for N particles is reduced, through the use of Bethe's hypothesis, to an eigenvalue problem of matrices of the same sizes as the irreducible representations R of the permutation group S_N. For some R's this eigenvalue problem itself is solved by a second use of Bethe's hypothesis, in a generalized form. In particular, the ground-state problem of spin-$\frac{1}{2}$ fermions is reduced to a generalized Fredholm equation.

(1) Consider the one-dimensional N-body problem

$$H=-\sum_{1}^{N}\partial^2/\partial x_i^{\,2}+2c\sum_{i<j}\delta(x_i-x_j),\quad c>0, \tag{1}$$

with no limitation on the symmetry of the wave function ψ. For a given irreducible representation R_ψ of the permutation group S_N of the N coordinates x_i, we want to determine the wave function ψ. Assume Bethe's hypothesis[1] to be valid: Let $p_1,\cdots,p_N$ = a set of unequal numbers. For $0<x_{Q1}<x_{Q2}<\cdots<x_{QN}<L$,

$$\psi=\sum_P[Q,P]\exp i[p_{P1}x_{Q1}+\cdots+p_{PN}x_{QN}], \tag{2}$$

where $P=[P1,P2,\cdots,PN]$ and $Q=[Q1,Q2,\cdots,QN]$ are two permutations of the integers $1,2,\cdots,N$. $[Q,P]$ can be arranged as a $N!\times N!$ matrix. Denote the columns of this matrix by ξ_P. To satisfy the continuity of ψ and the proper discontinuity of its derivative as required with the aid of the following identities:

$$Y_{ij}^{ab}Y_{ji}^{ab}=1, \tag{7}$$

and

$$Y_{jk}^{ab}Y_{ik}^{bc}Y_{ij}^{ab}=Y_{ij}^{bc}Y_{ik}^{ab}Y_{jk}^{bc}, \tag{8}$$

which are easily verified. Thus given a set of unequal p's, and $\xi_0=\xi_P$ for P = identity, all ξ_P's are determined.

(2) The imposition of the periodic boundary conditions leads to equations which, upon expressing ξ_P in terms of ξ_0, become

$$\lambda_j\xi_0=X_{(j+1)j}\times X_{(j+2)j}\cdots X_{Nj}X_{1j}X_{2j}\cdots X_{(j-1)j}\xi_0, \quad j=1,\cdots,N, \tag{9}$$

where

Thank you very much !