Hidden Symmetries of 4D Gauge Theories

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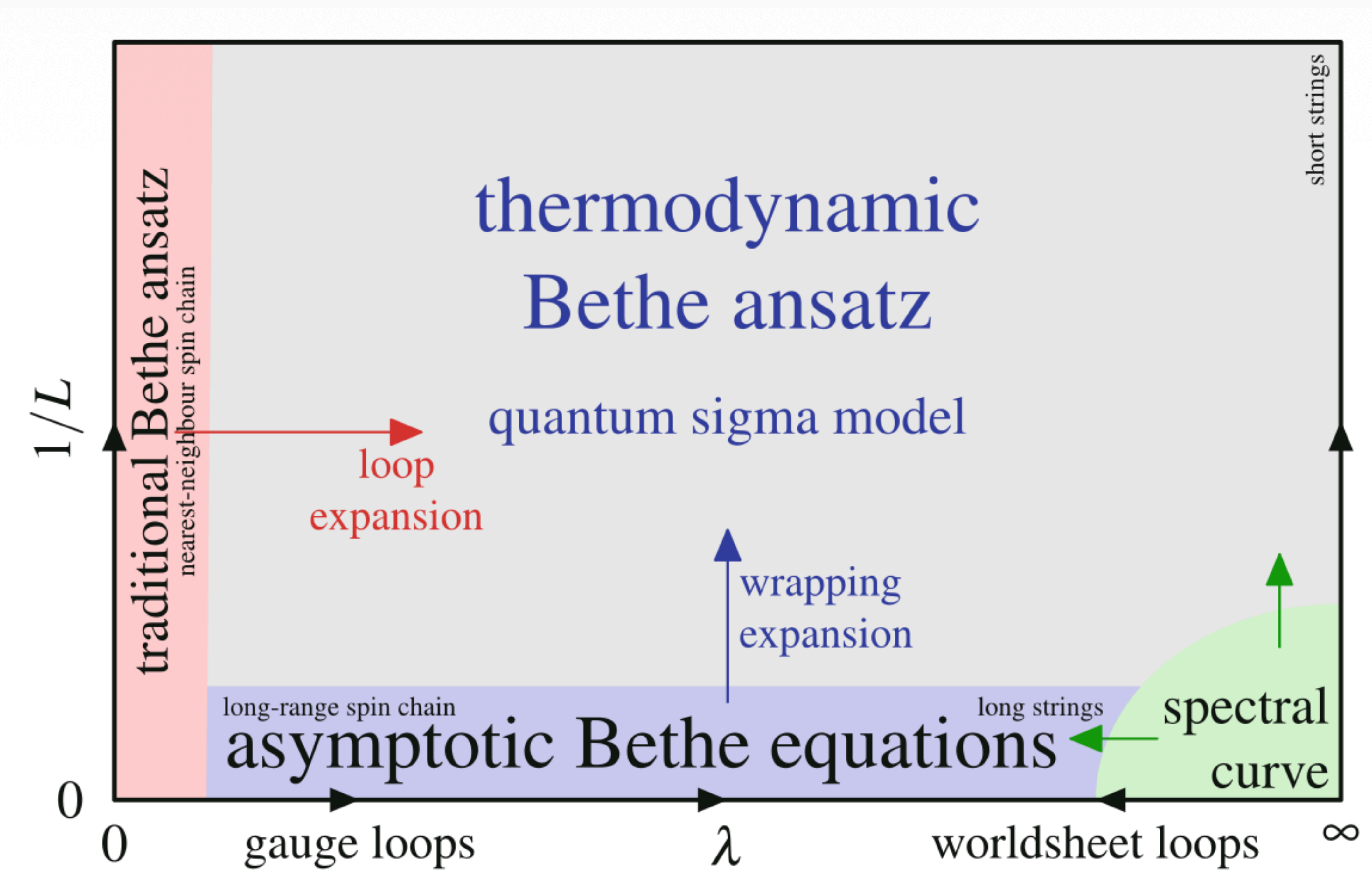
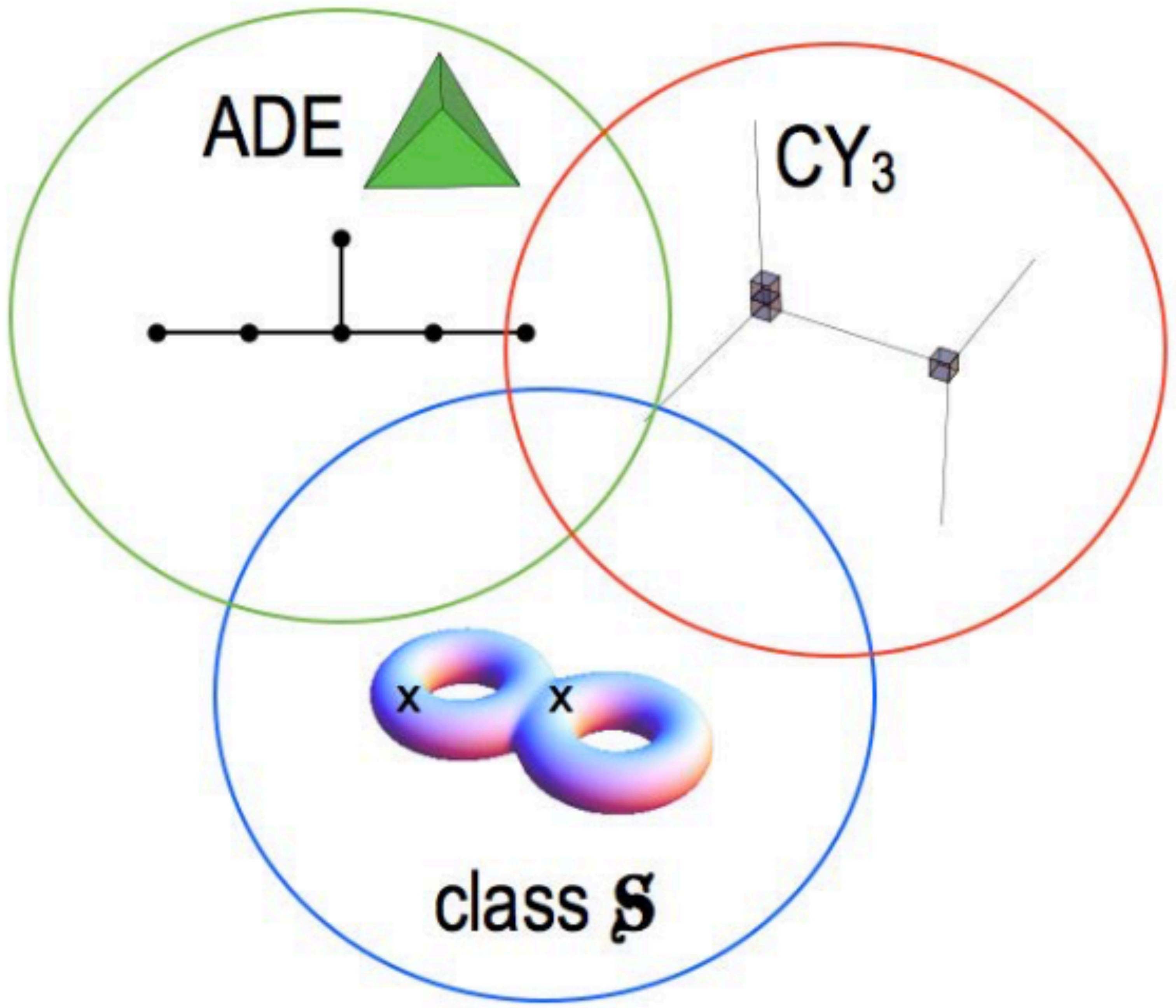
Based on work with Hanno Bertle, Elli Pomoni, Konstantinos Zoubos, 2411.11612
and work in progress with Kaizheng Xu, Zhengwen Liu

Interesting connections between
2d integrable systems and 4d supersymmetric gauge theories
have been discovered over the past few decades:



$\mathcal{N} = 4$ SYM
in the planar limit



$\mathcal{N} = 2$ gauge theories
on the Coulomb branch

We study a particular $\mathcal{N}=2$ gauge theories, namely the $\mathbb{Z}_2$-orbifold of $\mathcal{N}=4$ SYM and its marginal deformations, in the planar limit.

Mainly from the perspective of SYMMETRY,

which plays a fundamental role in modern physics:

- govern the formulation of physical theories
- characterize the phase structure
- determine the degeneracy structure of physical observables

Traditional mathematical framework for (continuous) symmetries:
(Lie) group theory.

For integrable systems, we also have Yangian, quantum groups, etc.

Recently, the notion of symmetry has been refined and generalized in many interesting ways:

Higher-form/ higher-group symmetry

Non-invertible symmetry

Subsystem symmetry

A key lesson: Relax certain group axioms while still capturing the essential notions of symmetry.

Our key discovery:

Lie algebra symmetry $\mathfrak{su}(4)_{\mathscr{R}}$ in $\mathcal{N} = 4$ SYM

$\xrightarrow{\text{orbifolding}} \mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R \oplus \mathfrak{u}(1)_r$

$\rightarrow$ "quantum Lie algebroid $\mathfrak{su}(4)_{\mathscr{R}}$"

Abandoned requirements:

- Any two elements can be composed
- The coproduct is trivial

$\mathcal{N} = 4$ SYM

Arguably the most symmetric gauge theory in 4d.

Field content in $\mathcal{N} = 1$ superspace: a vector multiplet V and three chiral multiplet $\overrightarrow{\Phi} = (X, Y, Z)$, all in the adjoint representation of the gauge group $G = \mathrm{SU}(N)$

$$\mathscr{L}_{\mathcal{N}=4} = \int d^4\theta \,\mathrm{tr}\left(\bar{\Phi}_i e^{gV} \Phi_i e^{-gV}\right) + \frac{1}{2}\left(\int d^2\theta \,\mathrm{tr} W^\alpha W_\alpha + \text{h.c.}\right) + \left(\int d^2\theta \,\frac{g}{3\,!}\epsilon_{ijk}\mathrm{tr}\left(\Phi^i\Phi^j\Phi^k\right) + \text{h.c.}\right)$$

Global symmetry: $\mathcal{N} = 4$ superconformal symmetry $\mathfrak{psu}(2,2\,|\,4) \supset \mathfrak{so}(4,2) \oplus \mathfrak{su}(4)_{\mathscr{R}}$

Under $\mathfrak{su}(4)_{\mathscr{R}}$, A_μ, ψ^a, $\phi^i \simeq \phi^{ab}$ transform in the irreps $\mathbf{1}$, $\mathbf{4}$, and $\mathbf{6} = \wedge^2\, \mathbf{4}$, respectively.

$$\phi^{ab} = -\,\phi^{ba} = \frac{1}{2}\epsilon^{abcd}\bar{\phi}_{cd} = \begin{pmatrix} 0 & Z & X & Y \\ -Z & 0 & \bar{Y} & -\bar{X} \\ -X & -\bar{Y} & 0 & \bar{Z} \\ -Y & \bar{X} & -\bar{Z} & 0 \end{pmatrix} \qquad R^a{}_b\phi^{cd} = \delta^c_b\phi^{ad} + \delta^d_b\phi^{ca} - \frac{1}{2}\delta^a_b\phi^{cd}$$

Gauge-invariant single-trace operators:

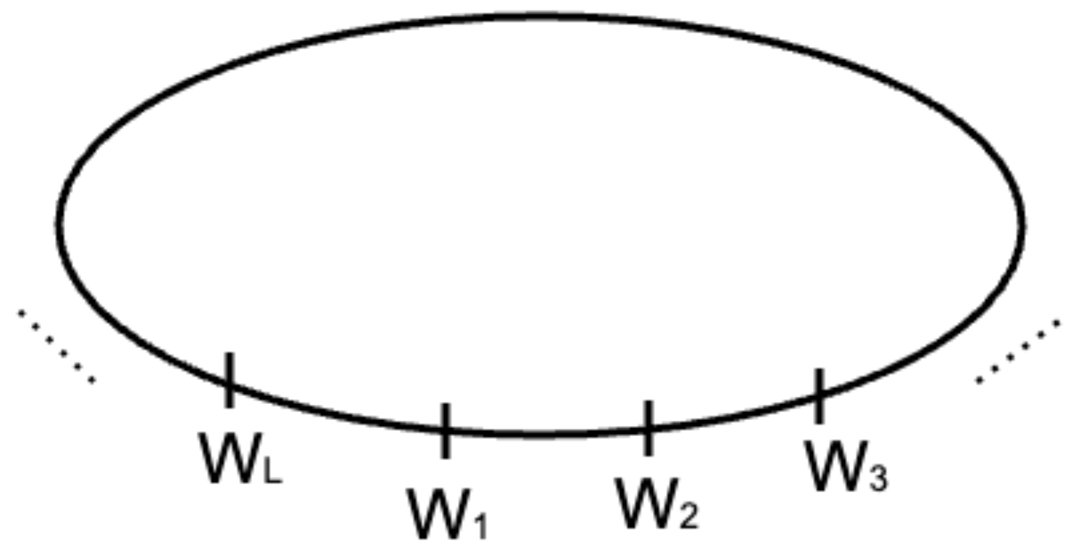
$$\mathscr{O}(x) = \mathrm{tr}\left(\mathscr{W}_1(x)\mathscr{W}_2(x)\cdots\mathscr{W}_L(x)\right), \quad \mathscr{W}_i \in \left\{D^kF^+, D^k\psi, D^k\phi, D^k\bar{\psi}, D^kF^-\right\}$$

$\Longleftrightarrow$ cyclic $\mathfrak{psu}(2,2|4)$ supersymmetric spin chain states

[Minahan, Zarembo, 2002]
[Beisert, Staudacher, 2003]

$$|\mathscr{W}_1, \mathscr{W}_2, \cdots, \mathscr{W}_L\rangle_{\text{cyclic}}$$

$$= |\mathscr{W}_2, \mathscr{W}_3, \cdots, \mathscr{W}_1\rangle_{\text{cyclic}} = \cdots$$



Interested in the correlation functions:

$$\left\langle \mathscr{O}_1\left(x_1\right)\cdots\mathscr{O}_n\left(x_n\right)\right\rangle$$

SO(6) sector

Focus on single-trace operators composed of scalar fields without derivatives,

$$\mathcal{O}[\chi] = \chi_{i_1 i_2 \cdots i_L} \mathrm{tr}\left(\phi^{i_1}\phi^{i_2}\cdots\phi^{i_L}\right) \Longleftrightarrow \chi_{i_1 i_2 \cdots i_L} | i_1, i_2, \cdots, i_L \rangle_{\text{cyclic}}$$

$\chi_{i_1 i_2 \cdots i_L}$: an SO(6) tensor with L indices

Correlation functions contain UV divergences and need to be regularized and renormalized.

Choose a basis $\{\mathcal{O}_A, A = 1,\cdots,6^L\}$ for bare operators.

The renormalized operators are their linear combinations

$$\mathcal{O}_{A,\text{ren}} = \mathbf{Z}_A{}^B \mathcal{O}_B$$

In the planar limit, multi-trace operators are suppressed, and $\{\mathcal{O}_A, A = 1,\cdots,6^L\}$ will not mix with other operators at the one-loop order.

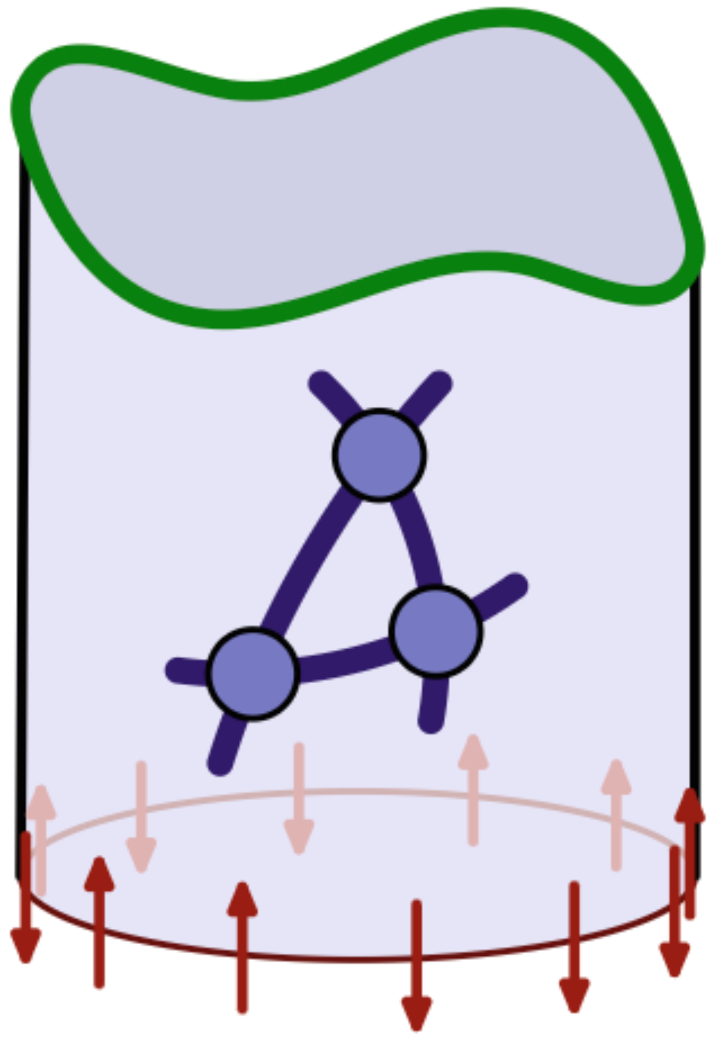
$$\Gamma = \frac{d\mathbf{Z}}{d\log\Lambda_{\text{uv}}}\mathbf{Z}^{-1} = \frac{g^2N}{16\pi^2}H + \mathcal{O}(g^4)$$

H: coincides with an integrable spin chain Hamiltonian solvable by Bethe ansatz.

[Minahan, Zarembo, 2002]

Diagonalizing the $6^L \times 6^L$ matrix Γ:

$$\Gamma\hat{\mathcal{O}}_A(0) = \gamma_A\hat{\mathcal{O}}_A(0)\,,\quad \gamma_A\text{: anomalous dimension}$$



$\Longrightarrow$ 2-point functions:

$$\langle\hat{\mathcal{O}}_A(x)\hat{\mathcal{O}}_B(0)\rangle = \frac{\delta_{AB}}{|x|^{2(L+\gamma_A)}}$$

4d $\mathcal{N}=2$ SCFTs via orbifolding

[Kachru, Silverstein, 1998] [Lawrence, Nekrasov, Vafa, 1998]

A family of $\mathcal{N}=2$ SCFTs can be constructed by orbifolding $\mathcal{N}=4$ SYM with respect to finite subgroups of $\mathrm{SU}(2)\subset\mathrm{SU}(4)_{\mathcal{R}}$.

Example: starting from $\mathcal{N}=4$ SYM with gauge group $\mathrm{SU}(2N)$, perform a simultaneous $\mathbb{Z}_2$ projection in the R-symmetry space and in the color space

$$\mathscr{P}:(V,X,Y,Z)\to(\tau^{-1}V\tau,-\tau^{-1}X\tau,-\tau^{-1}Y\tau,\tau^{-1}Z\tau),\quad \tau=\begin{pmatrix} I_{N\times N} & 0\\ 0 & -I_{N\times N}\end{pmatrix}$$

$\Longrightarrow$ The gauge group $\mathrm{SU}(2N)\longrightarrow\mathrm{SU}(N)_1\times\mathrm{SU}(N)_2$

$$A=\begin{pmatrix}A_1 & 0\\ 0 & A_2\end{pmatrix},\ Z=\begin{pmatrix}Z_1 & 0\\ 0 & Z_2\end{pmatrix},\ X=\begin{pmatrix}0 & X_{12}\\ X_{21} & 0\end{pmatrix},\ Y=\begin{pmatrix}0 & Y_{12}\\ Y_{21} & 0\end{pmatrix}$$

At the orbifold point of the conformal manifold, two gauge couplings $g_1 = g_2$.

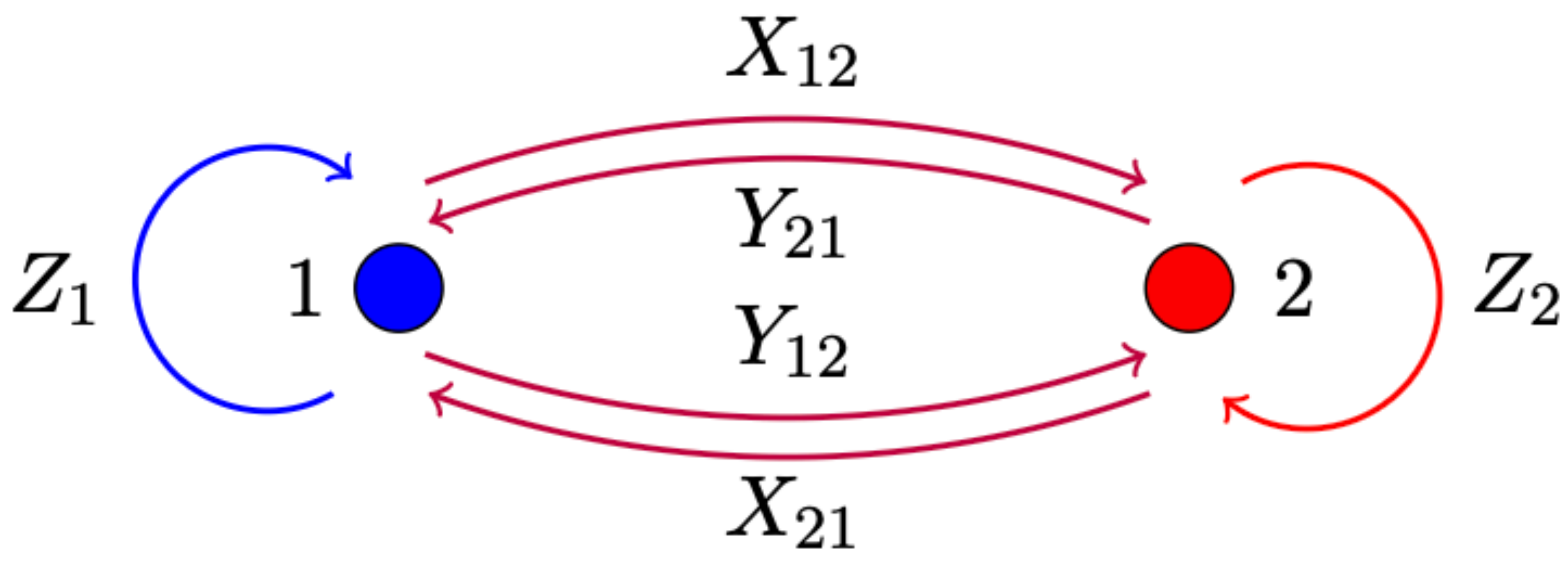
In general, two gauge couplings can take different values.

$$\mathscr{L}_{\text{orbifold}} = \frac{1}{2}\sum_{i=1}^{2}\left(\int d^2\theta \mathrm{tr}_i W_i^{\alpha} W_{i\alpha} + \text{h.c.}\right) + \sum_{i=1}^{2}\int d^4\theta \mathrm{tr}_i \left(\bar{Z}_i e^{g_i V_i} Z_i e^{-g_i V_i}\right)$$

$$+\int d^4\theta \mathrm{tr}_1 \left(\bar{X}_{12} e^{g_2 V_2} X_{21} e^{-g_1 V_1} + Y_{12} e^{-g_2 V_2} \bar{Y}_{21} e^{g_1 V_1}\right)$$

$$+\int d^4\theta \mathrm{tr}_2 \left(\bar{X}_{21} e^{g_1 V_1} X_{12} e^{-g_2 V_2} + Y_{21} e^{-g_1 V_1} \bar{Y}_{12} e^{g_2 V_2}\right)$$

$$+\left(\int d^2\theta \mathscr{W} + \text{h.c.}\right)$$



$$\mathscr{W} = g_1 \mathrm{tr}_1\Big((X_{12}Y_{21} - Y_{12}X_{21})Z_1\Big) + g_2 \mathrm{tr}_2\Big((X_{21}Y_{12} - Y_{21}X_{12})Z_2\Big)$$

Global symmetry:

- $\mathcal{N} = 2$ superconformal symmetry $\mathfrak{su}(2,2\,|\,2) \supset \mathfrak{so}(4,2) \oplus \mathfrak{su}(2)_R \oplus \mathfrak{u}(1)_r$
- Extra symmetry transforming between X and Y: $\mathfrak{su}(2)_L$
- $\mathbb{Z}_2$ automorphism of the quiver: switching all indices $1 \leftrightarrow 2$

The unbroken R-symmetry generators are

$$\mathfrak{su}(2)_R : \sigma_R^+ = R^1{}_2, \sigma_R^- = R^2{}_1, \sigma_R^3 = \frac{1}{2}\left(R^1{}_1 - R^2{}_2\right)$$

$$\mathfrak{su}(2)_L : \sigma_L^+ = R^3{}_4, \sigma_L^- = R^4{}_3, \sigma_L^3 = \frac{1}{2}\left(R^3{}_3 - R^4{}_4\right)$$

$$\mathfrak{u}(1)_r : \sigma_r = -R^1{}_1 - R^2{}_2 = R^3{}_3 + R^4{}_4$$

Why do we expect symmetries hidden in N=2 orbifold SCFTs?

Gauge-invariant single-trace operators in the orbifold theory:

$$\mathcal{O}_\omega(x) = \mathrm{tr}\left(\omega\left(\mathcal{P}\mathcal{W}_1(x)\right)\left(\mathcal{P}\mathcal{W}_2(x)\right)\cdots\left(\mathcal{P}\mathcal{W}_L(x)\right)\right),\ \omega \in \mathbb{Z}_2$$

$\omega = I_{2N}$: untwisted sector

$\omega = \tau$: twisted sector

Correlation functions in $\mathcal{N} = 4$ SYM and in the orbifold theory are different.

However, in the planar limit ($N \to \infty$, $g \to 0$, $g^2 N = \lambda$ fixed),

$$\left\langle \mathcal{O}_1\left(x_1\right)\cdots\mathcal{O}_n\left(x_n\right)\right\rangle^{\mathcal{N}=4} = \left\langle \mathcal{O}_{1,I}\left(x_1\right)\cdots\mathcal{O}_{n,I}\left(x_n\right)\right\rangle^{\text{orbifold}}_{g_1=g_2=f(g)}$$

This result was proved using the string theory perturbation technique [Bershadsky, Kakushadze, Vafa, 1998] and Feynman diagram analysis [Bershadsky, Johansen, 1998].

For example, in the SO(6) sector of $\mathcal{N} = 4$ SYM,

$$H = \sum_{\ell=1}^{L} \left(6\Pi^{\mathbf{1}}_{\ell,\ell+1} + 4\Pi^{\mathbf{15}}_{\ell,\ell+1} + 0\Pi^{\mathbf{20}'}_{\ell,\ell+1} \right)$$

Here $\Pi^{\mathbf{r}}_{\ell,\ell+1}$ is the projection operators: vector spaces at sites ℓ and $\ell + 1 \longrightarrow$ irrep $\mathbf{r}$

$$\mathbf{6} \times \mathbf{6} = \mathbf{1} + \mathbf{15} + \mathbf{20}'$$

This degeneracy structure is precisely predicted by $\mathfrak{su}(4)_{\mathscr{R}}$.

In the orbifold theory, the branching rules for $\mathfrak{su}(4)_{\mathscr{R}} \to \mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R \oplus \mathfrak{u}(1)_r$ are

$$\mathbf{1} \to (\mathbf{1},\mathbf{1})_0$$

$$\mathbf{15} \to (\mathbf{1},\mathbf{1})_0 + (\mathbf{3},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{2},\mathbf{2})_1 + (\mathbf{2},\mathbf{2})_{-1}$$

$$\mathbf{20}' \to (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{1})_2 + (\mathbf{1},\mathbf{1})_{-2} + (\mathbf{2},\mathbf{2})_1 + (\mathbf{2},\mathbf{2})_{-1} + (\mathbf{3},\mathbf{3})_0$$

A priori, there can be mixing among states in different $\mathfrak{su}(4)_{\mathscr{R}}$ irreps but have the same $\mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R \oplus \mathfrak{u}(1)_r$ quantum numbers.

Group theory prediction: H is a linear combination of $\mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R \oplus \mathfrak{u}(1)_r$ projectors.

A direct computation shows that H in the untwisted sector of the orbifold theory is

$$H = \sum_{\ell=1}^{L} \left(6\Pi^{\mathbf{1}}_{\ell,\ell+1} + 4\Pi^{\mathbf{15}}_{\ell,\ell+1} + 0\Pi^{\mathbf{20}'}_{\ell,\ell+1} \right)$$

which is the same as that of $\mathcal{N} = 4$ SYM.

⚠️ No splitting of $\mathfrak{su}(4)_{\mathscr{R}}$ representations into $\mathfrak{su}(2)_L \oplus \mathfrak{su}(2)_R \oplus \mathfrak{u}(1)_r$ irreps.

A big surprise from a group theoretic perspective of symmetry.

Our proposal: The naively broken $\mathfrak{su}(4)_{\mathscr{R}}$ generators, $R^{a=1,2}_{b=3,4}$ and $R^{a=3,4}_{b=1,2}$, are not lost.

🤔 How should we modify the definition of $\mathfrak{su}(4)_{\mathscr{R}}$ generators?

Step 1: Upgrade the generators

In $\mathcal{N} = 4$ SYM, all the fields are in the adjoint representation of the gauge group.

So, for example, X and Z can form an $\mathfrak{su}(2)_{XZ}$ doublet:

$$\sigma^{+}_{XZ} X = Z, \quad \sigma^{-}_{XZ} Z = X$$

However, after orbifolding, X_{12} and X_{21} are in bifundamental representations while Z_1 and Z_2 are in adjoint representations

$\Longrightarrow X_{12}, X_{21}, Z_1, Z_2$ do not form an $\mathfrak{su}(2)_{XZ}$ multiplets

To revive the broken generators: Lie group/algebra $\longrightarrow$ Lie groupoid/algebroid.

A brief introduction to groupoid

Roughly speaking, a groupoid can be thought of as a generalization of a group in which not all pairs of elements can be composed.

Mathematically, a groupoid $\mathcal{G}$ is a category where every morphism is an isomorphism. Explicitly, $\mathcal{G}$ consisting of a set $\mathcal{G}_0$ of objects, a set $\mathcal{G}_1$ of morphisms, and five structure maps $\mathcal{S}, \mathcal{T} : \mathcal{G}_1 \rightrightarrows \mathcal{G}_0$, $\circ : \mathcal{G}_1 \times \mathcal{G}_1 \rightharpoonup \mathcal{G}_1$, $\mathcal{I} : \mathcal{G}_0 \to \mathcal{G}_1$, ${}^{-1} : \mathcal{G}_1 \to \mathcal{G}_1$:

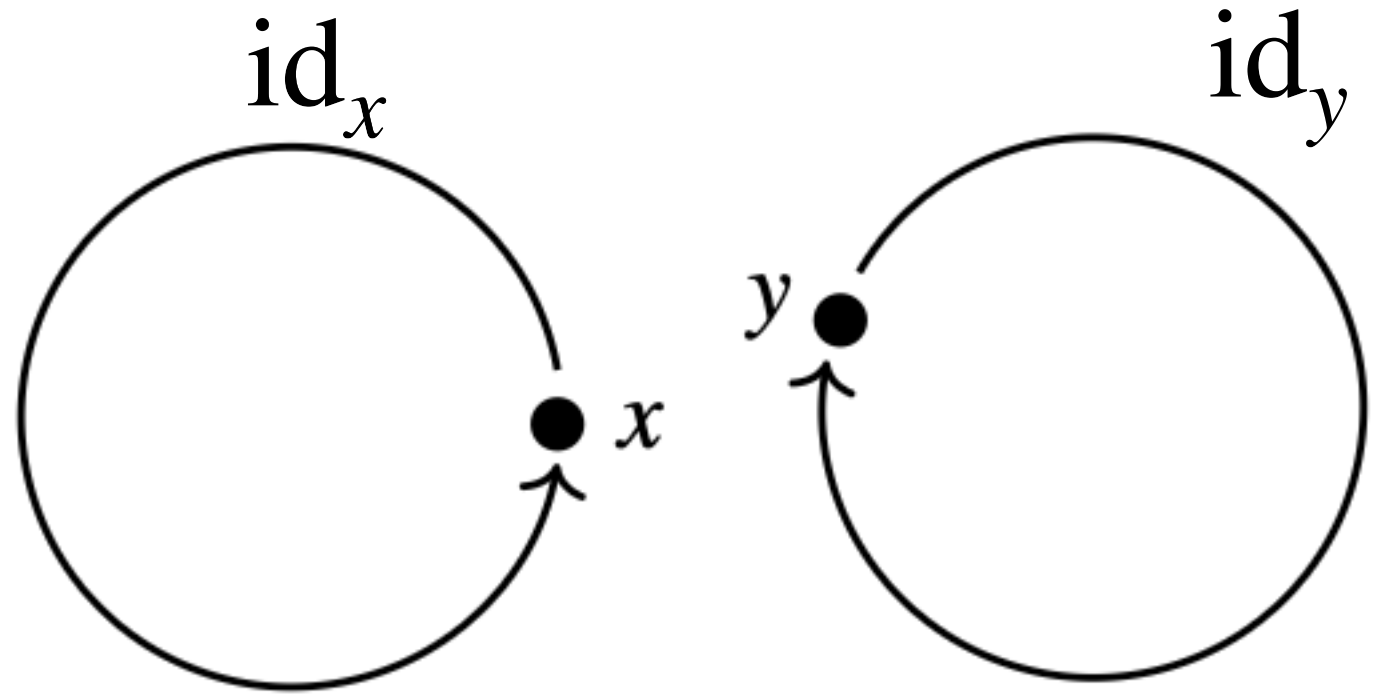
- For each morphism $g \in \mathcal{G}_1$, its source and target objects are respectively $\mathcal{S}(g)$ and $\mathcal{T}(g)$

$$\begin{tikzcd} \mathcal{S}(g) \arrow[r, "g"] & \mathcal{T}(g) \end{tikzcd}$$

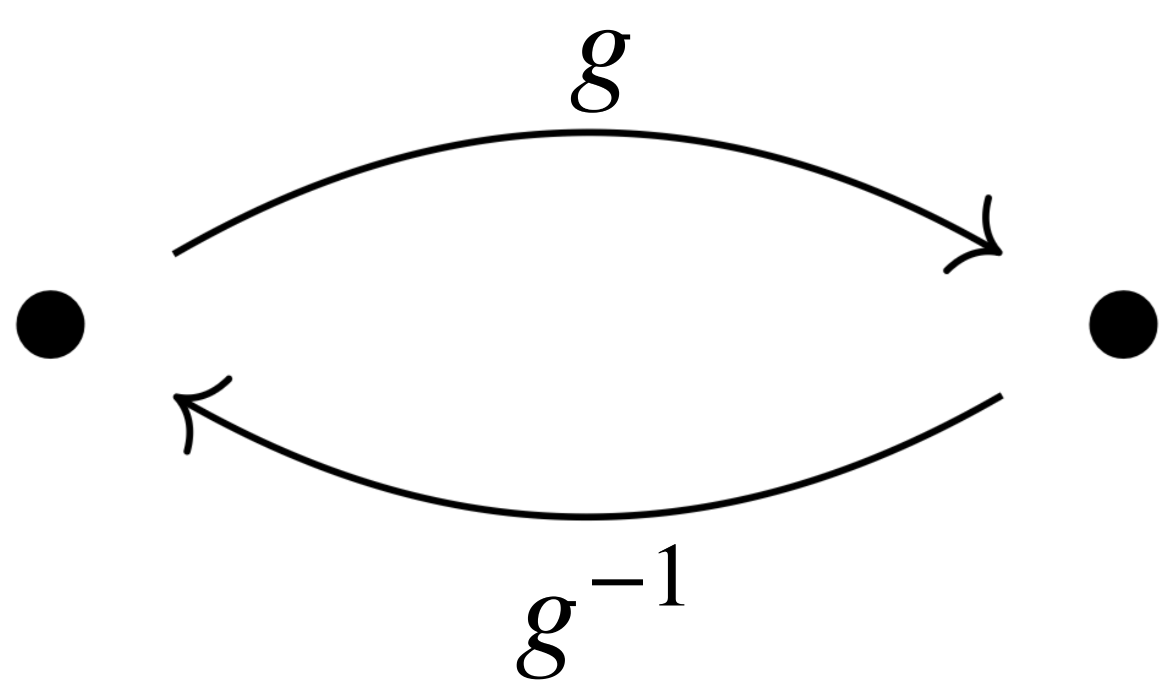
- A pair of morphisms $\left(g_2, g_1\right) \in \mathcal{G}_1 \times \mathcal{G}_1$ is composable when $\mathcal{T}\left(g_1\right) = \mathcal{S}\left(g_2\right)$, and the set of composable morphisms is denoted by $\mathcal{G}_2 \subseteq \mathcal{G}_1 \times \mathcal{G}_1$. The map $\circ$ is the composition, such that $\mathcal{S}\left(g_2 \circ g_1\right) = \mathcal{S}\left(g_1\right)$, $\mathcal{T}\left(g_2 \circ g_1\right) = \mathcal{T}\left(g_2\right)$. The composition is associative.

$$\begin{tikzcd} \bullet \arrow[r, "g_1"] \arrow[dr, "g_2 \circ g_1"'] & \bullet \arrow[d, "g_2"] \\ & \bullet \end{tikzcd}$$

- The unit map $\mathscr{I}$ sends every object $x \in \mathscr{G}_0$ to the identity morphism $\mathrm{id}_x \in \mathscr{G}_1$ at x, such that for every $g \in \mathscr{G}_1$, $\mathrm{id}_{\mathscr{T}(g)} \circ g = g \circ \mathrm{id}_{\mathscr{S}(g)} = g$.



- The inverse map ${}^{-1}$ sends every morphism $g \in \mathscr{G}_1$ to its inverse g^{-1}, such that $g^{-1} \circ g = \mathrm{id}_{\mathscr{S}(g)}$ and $g \circ g^{-1} = \mathrm{id}_{\mathscr{T}(g)}$.

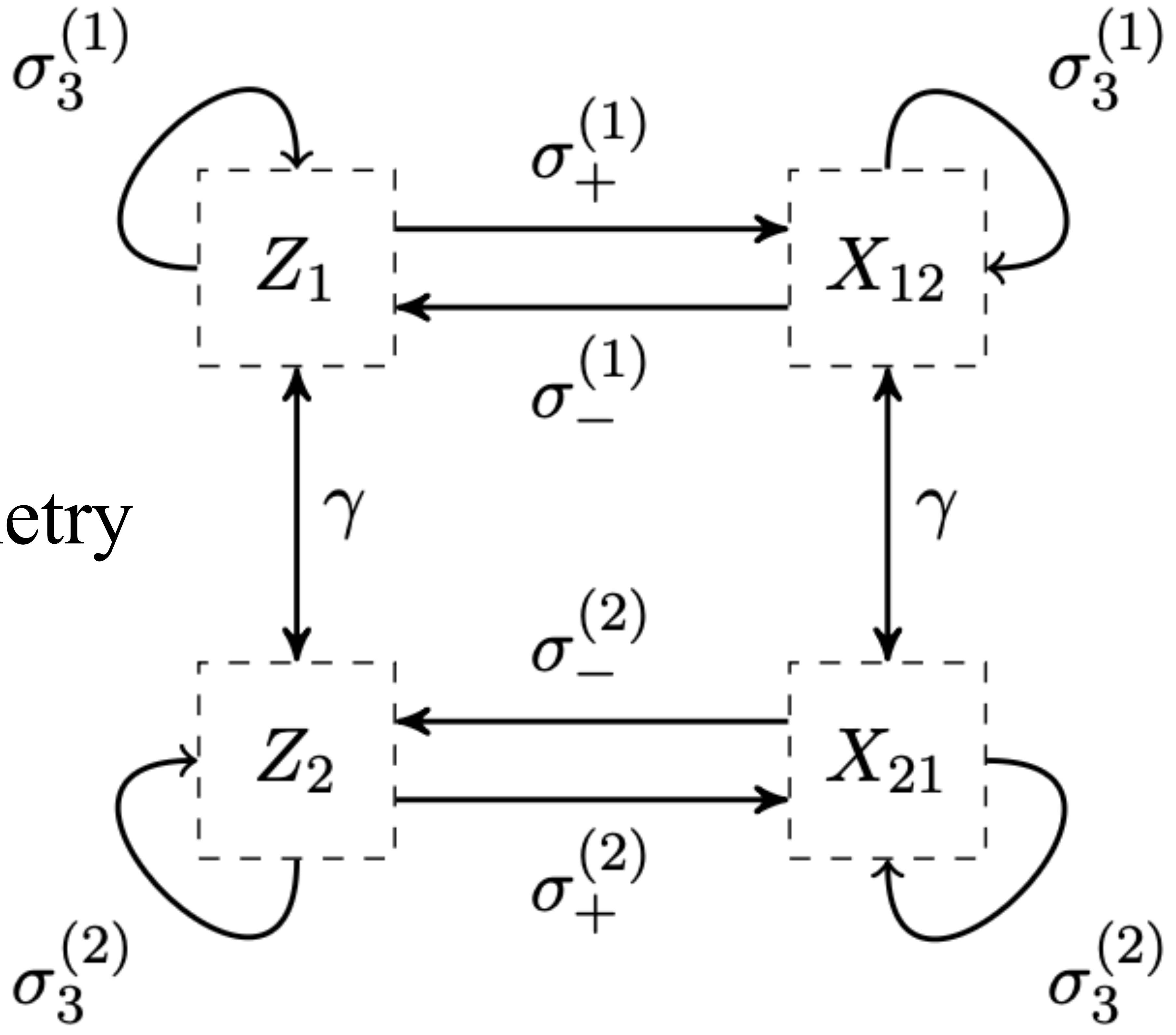


If $\mathscr{G}_0$ contains only a single object, then this definition reduces to that of a group.

In physics, the symmetry group is the set of all symmetry transformations which isomorphically relates one object to itself, endowed with the group operation of composition. A groupoid is a collection of symmetry transformations acting between possibly more than one object.

To enable a transformation between bifundamental fields and adjoint fields at the level of single-particle states, we upgrade all generators so that they are allowed to act not only on the internal space but also on the color space.

For example, the Lie algebra $\mathfrak{su}(2)_{XZ}$ is replaced by the following Lie algebroid, and the corresponding Lie groupoid is given by



$\mathscr{G}_0^{(L=1)} = \{Z_1, Z_2, X_{12}, X_{21}\}$

$\mathscr{G}_1^{(L=1)}$ is exponential maps of the generators $\{\sigma_+^{(1)}, \sigma_-^{(1)}, \sigma_3^{(1)}, \sigma_+^{(2)}, \sigma_-^{(2)}, \sigma_3^{(2)}\}$ and the discrete symmetry generator γ,

with natural structure maps.

Step 2: Modify the action on multi-particle states

When we extend the action of the generators from the space of single-particle states to the space of states of arbitrary length, we should consider the universal enveloping algebra of the Lie algebroid.

⚠️ A new problem arises because the color structure of the orbifold theory is constrained in the planar limit.

$$|XYZY\cdots\rangle^{\mathcal{N}=4} \nearrow |XYZY\cdots\rangle_1^{\text{orbifold}} = |X_{12}Y_{21}Z_1Y_{12}\cdots\rangle^{\text{orbifold}}$$

$$(\square_1,\bar{\square}_2)\otimes(\square_2,\bar{\square}_1)\otimes(\square_1,\bar{\square}_1)\otimes(\square_1,\bar{\square}_2)\otimes\cdots$$

$$|XYZY\cdots\rangle^{\mathcal{N}=4} \searrow |XYZY\cdots\rangle_2^{\text{orbifold}} = |X_{21}Y_{12}Z_2Y_{21}\cdots\rangle^{\text{orbifold}}$$

$$(\square_2,\bar{\square}_1)\otimes(\square_1,\bar{\square}_2)\otimes(\square_2,\bar{\square}_2)\otimes(\square_2,\bar{\square}_1)\otimes\cdots$$

A sequence of fields like $X_{12}Y_{21}Z_2Y_{12}\cdots$ is not allowed, since it is impossible to contract the color indices to obtain a single-trace operator, and is therefore excluded in the planar limit.

Path groupoid

To describe the space acted on by the generators, we introduce a quiver path groupoid:

$\mathcal{G}_0 = \{①, ②\}$: two color groups

$\mathcal{G}_1 = \{V_{11}, V_{12}, V_{21}, V_{22}\}$: four classes of allowed states of arbitrary length, where

$$V_{11} = \left\{ \{Z_1, \bar{Z}_1\}, \{X_{12}X_{21}, Z_1Z_1, \cdots\}, \{X_{12}X_{21}Z_1, Z_1Z_1Z_1, \cdots\}, \cdots \right\}$$

$$V_{12} = \left\{ \{X_{12}, \bar{X}_{12}, Y_{12}, \bar{Y}_{12}\}, \{X_{12}Z_2, Z_1X_{12}, \cdots\}, \{X_{12}X_{21}X_{12}, Z_1X_{12}Z_2, \cdots\}, \cdots \right\}$$

and their $\mathbb{Z}_2$ conjugates V_{22}, V_{21}.

Groupoid structure: a state in V_{ab} and another state in V_{cd} can be composed into a valid state iff $b = c$.

Introduce a nontrivial coproduct

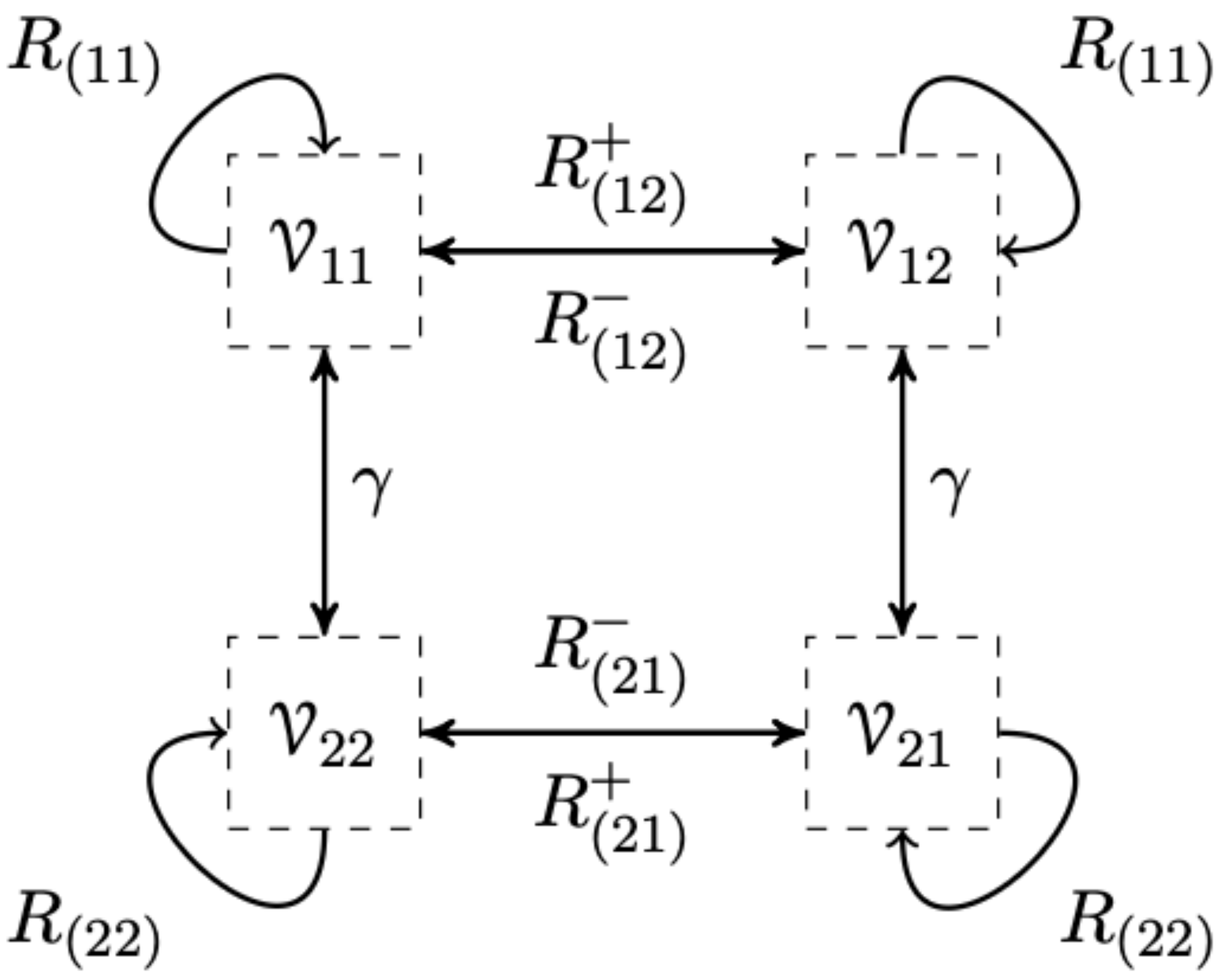
The naive Leibniz rule does not work, since a broken generator will transform a well-defined multi-particle state to an ill-defined state.

$$\sigma_+^{(1)} |\cdots X_{12} Y_{21} Z_1 Y_{12} \cdots\rangle = |\cdots X_{12} Y_{21} X_{12} Y_{12} \cdots\rangle + \cdots$$

There is a natural way to resolve the problem: shift all the color indices afterwards. Mathematically, this is realized by regarding the universal enveloping algebra as a Hopf algebra, and then introducing a nontrivial coproduct which defines the action on states of arbitrary length,

$$\Delta_o^{(L)}\left(R_b^a\right) = \sum_{\ell=1}^{L} \left(I \otimes \cdots \otimes I \otimes \overset{\ell}{R_b^a} \otimes \Omega_b^a \otimes \cdots \otimes \Omega_b^a \right), \quad \Omega_b^a = \begin{cases} I, & R_b^a \text{ is unbroken} \\ \gamma, & R_b^a \text{ is broken} \end{cases}$$

Putting everything together, we have defined a quantum deformation of the universal enveloping algebra of the Lie algebroid:



graded structure:

[(unbroken), (unbroken)] = (unbroken)

[(broken), (unbroken)] = (broken)

[(broken), (broken)] = (unbroken)

This structure generalizes straightforwardly to all generators of $\mathfrak{su}(4)_{\mathscr{R}}$.

🤔 The path groupoid combined with the R-symmetry Lie groupoid into a 2-category?

Step 3: Decomposition into open states

A single action of the broken generators acts naturally on open states instead of gauge-invariant closed states,

$$
\begin{tikzcd}
0 & \boxed{Z_1 Z_1} \arrow[l, "\Delta_\circ(\sigma_-^{(1)})"'] \arrow[r, shift left, "\Delta_\circ(\sigma_+^{(1)})"] & \boxed{X_{12} Z_2 + Z_1 X_{12}} \arrow[l, shift left, "\Delta_\circ(\sigma_-^{(1)})"] \arrow[r, shift left, "\Delta_\circ(\sigma_+^{(1)})"] & \boxed{X_{12} X_{21}} \arrow[l, shift left, "\Delta_\circ(\sigma_-^{(1)})"] \arrow[r, "\Delta_\circ(\sigma_+^{(1)})"] & 0
\end{tikzcd}
$$

$$
\begin{tikzcd}
0 & \boxed{X_{12} Z_2 - Z_1 X_{12}} \arrow[l, "\Delta_\circ(\sigma_-^{(1)})"'] \arrow[r, "\Delta_\circ(\sigma_+^{(1)})"] & 0
\end{tikzcd}
$$

To bypass this issue when dealing with closed states, we cut open the trace and average over all possible cutting points, in a cyclic manner,

$$
\mathrm{tr}_i\left(\varphi_1 \varphi_2 \dots \varphi_L\right) \to \frac{1}{L} \sum_{\tau \in \mathbb{Z}_L} \varphi_{\tau(1)} \varphi_{\tau(2)} \cdots \varphi_{\tau(L)}
$$

After opening up $\mathscr{L}_{\text{orbifold},g_1=g_2}$ and dropping the gauge fields (R-symmetry singlets), we obtain the following pieces:

$$|K_1\rangle_o = X_{12}\bar{X}_{21} + \bar{X}_{12}X_{21} + Y_{12}\bar{Y}_{21} + \bar{Y}_{12}Y_{21} + Z_1\bar{Z}_1 + \bar{Z}_1 Z_1 \qquad + \mathbb{Z}_2 \text{ conjugates}$$

$$|\mathscr{W}_1\rangle_o = \left(X_{12}Y_{21} - Y_{12}X_{21}\right) Z_1 + Z_1 \left(X_{12}Y_{21} - Y_{12}X_{21}\right) + \left(Y_{12}Z_2X_{21} - X_{12}Z_2Y_{21}\right)$$

An explicit calculation shows that

$$\Delta_o(R^a_b)\,|K_1\rangle_o = 0 \quad \forall R^a_b \in \mathfrak{su}(4)_{\mathscr{R}}$$

$$\Delta_o(R^a_b)\,|\mathscr{W}_1\rangle_o = 0 \quad \forall R^a_b \in \mathfrak{su}(3)_{XYZ}$$

This proves the invariance of the Lagrangian in the superspace formalism.

A straightforward calculation also confirms the invariants in components.

🏅 Recover the broken $\mathfrak{su}(4)_{\mathscr{R}}$ generators at the orbifold point.

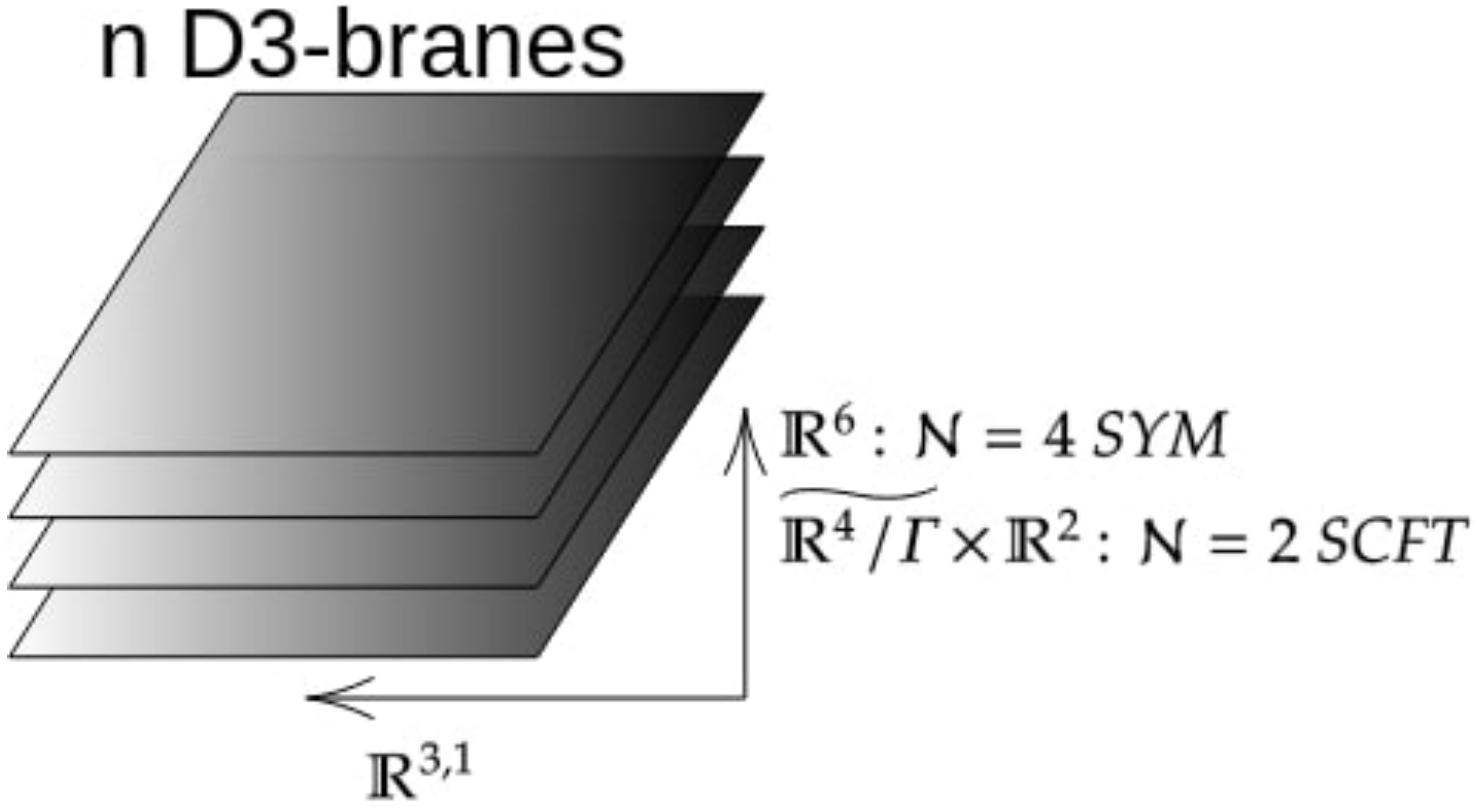
The procedure can be extended straightforwardly to the full $\mathfrak{psu}(2,2|4)$.

$\Longrightarrow$ Explains the planar equivalence of the correlation functions in the general sector of $\mathcal{N}=4$ SYM and its orbifolds.

⚠️ Neither $\mathcal{N}=2$ supersymmetry nor the subgroup $\mathbb{Z}_2 \subset \mathrm{SU}(2) \subset \mathrm{SU}(4)_{\mathcal{R}}$ is essential for our arguments

$\Longrightarrow$ Analogous definitions of the broken generators apply to more general $\mathcal{N}=1$ supersymmetric quiver gauge theories descended from $\mathcal{N}=4$ SYM via orbifolding/orientifolding.

String theory origin?



- Orbifolds are naturally related to groupoids [Moerdijk, Pronk, 1997]
- Background B-field $\Longrightarrow$ noncommutativity $\Longrightarrow$ quantum deformation [Seiberg, Witten, 1999]

Groupoid deformation of Yangian?

Planar integrability persists in the orbifold theory [Beisert, Roiban, 2005], e.g. the one-loop Bethe equations are modified from $\mathcal{N}=4$ SYM to its orbifolds

$$\left(\frac{u_{j,k}-\frac{i}{2}V_j}{u_{j,k}+\frac{i}{2}V_j}\right)^L \prod_{\substack{j'=1 \\ (j',k')\neq(j,k)}}^{J} \prod_{k'=1}^{K_{j'}} \frac{u_{j,k}-u_{j',k'}+\frac{i}{2}M_{j,j'}}{u_{j,k}-u_{j',k'}-\frac{i}{2}M_{j,j'}} = 1, \qquad \prod_{j=1}^{J}\prod_{k=1}^{K_j} \frac{u_{j,k}+\frac{i}{2}V_j}{u_{j,k}-\frac{i}{2}V_j} = 1.$$

$$\Downarrow$$

$$e^{2\pi i T s_j/S}\left(\frac{u_{j,k}-\frac{i}{2}V_j}{u_{j,k}+\frac{i}{2}V_j}\right)^L \prod_{\substack{j'=1 \\ (j',k')\neq(j,k)}}^{J} \prod_{k'=1}^{K_{j'}} \frac{u_{j,k}-u_{j',k'}+\frac{i}{2}M_{j,j'}}{u_{j,k}-u_{j',k'}-\frac{i}{2}M_{j,j'}} = 1.$$

$$e^{2\pi i T s_0/S}\prod_{j'=1}^{J}\prod_{k'=1}^{K_{j'}} \frac{u_{j',k'}+\frac{i}{2}V_{j'}}{u_{j',k'}-\frac{i}{2}V_{j'}} = 1$$

$$e^{-2\pi i L s_0/S}\prod_{j'=1}^{J} e^{-2\pi i K_{j'} s_{j'}/S} = 1.$$

reminiscent of fractional magnetic flux surrounded by the spin chain

🤔 Is there a groupoid deformation of Yangian $Y[\mathfrak{psu}(2,2|4)]$ for the orbifold theories?

⚡ Consider the variation of the equations of motion under Yangian generators:

$$\hat{J} \triangleright \check{\mathcal{O}} \left(= \frac{\delta S}{\delta \mathcal{O}} \right) \stackrel{?}{=} 0, \quad \mathcal{O} \in \{A_\mu, \psi, \bar{\psi}, \phi\}$$

Simplification: only a single level-one generator $\hat{P}$ (the dual conformal generator) along with the level-zero generators is sufficient to generate the whole Yangian algebra.

$$\begin{aligned}\Delta \widehat{\mathrm{P}}_{\dot{\alpha}\beta} = \widehat{\mathrm{P}}_{\dot{\alpha}\beta} \otimes 1 + 1 \otimes \widehat{\mathrm{P}}_{\dot{\alpha}\beta} \\ - \mathrm{L}^{\gamma}{}_{\beta} \wedge \mathrm{P}_{\dot{\alpha}\gamma} - \bar{\mathrm{L}}^{\dot{\gamma}}{}_{\dot{\alpha}} \wedge \mathrm{P}_{\dot{\gamma}\beta} - \mathrm{D} \wedge \mathrm{P}_{\dot{\alpha}\beta} - \tfrac{i}{2}\, \mathrm{Q}_{c\beta} \wedge \bar{\mathrm{Q}}_{\dot{\alpha}}{}^{c}\end{aligned}$$

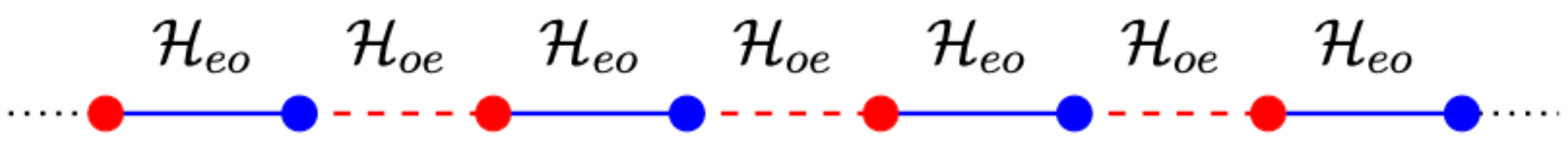
with the generators Q and $\bar{Q}$ deformed by the groupoid structure.

[Working in progress with Kaizheng Xu]

What if we marginally deform the couplings away from the orbifold point?

$$H_\kappa = \sum_{\ell=1}^{L} H_{\ell,\ell+1},\ \kappa = \frac{g_2}{g_1}$$

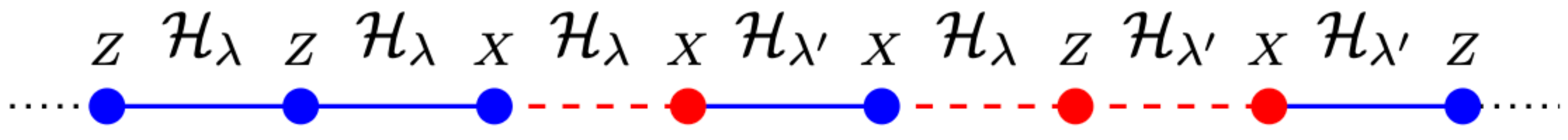
XY sector:
XXX spin chain with alternating couplings



$$\mathcal{H}_{eo} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa & -\kappa & 0 \\ 0 & -\kappa & \kappa & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{H}_{oe} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/\kappa & -1/\kappa & 0 \\ 0 & -1/\kappa & 1/\kappa & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2-magnon: [Bell, Loly, Southern, 1989]
[Medved, Southern, Lavis, 1991]
multi-magnon (not a Bethe-type approach):
[Southern, Lee, Lavis, 1994][Southern, Martínez Cuéllar, Lavis, 1998]

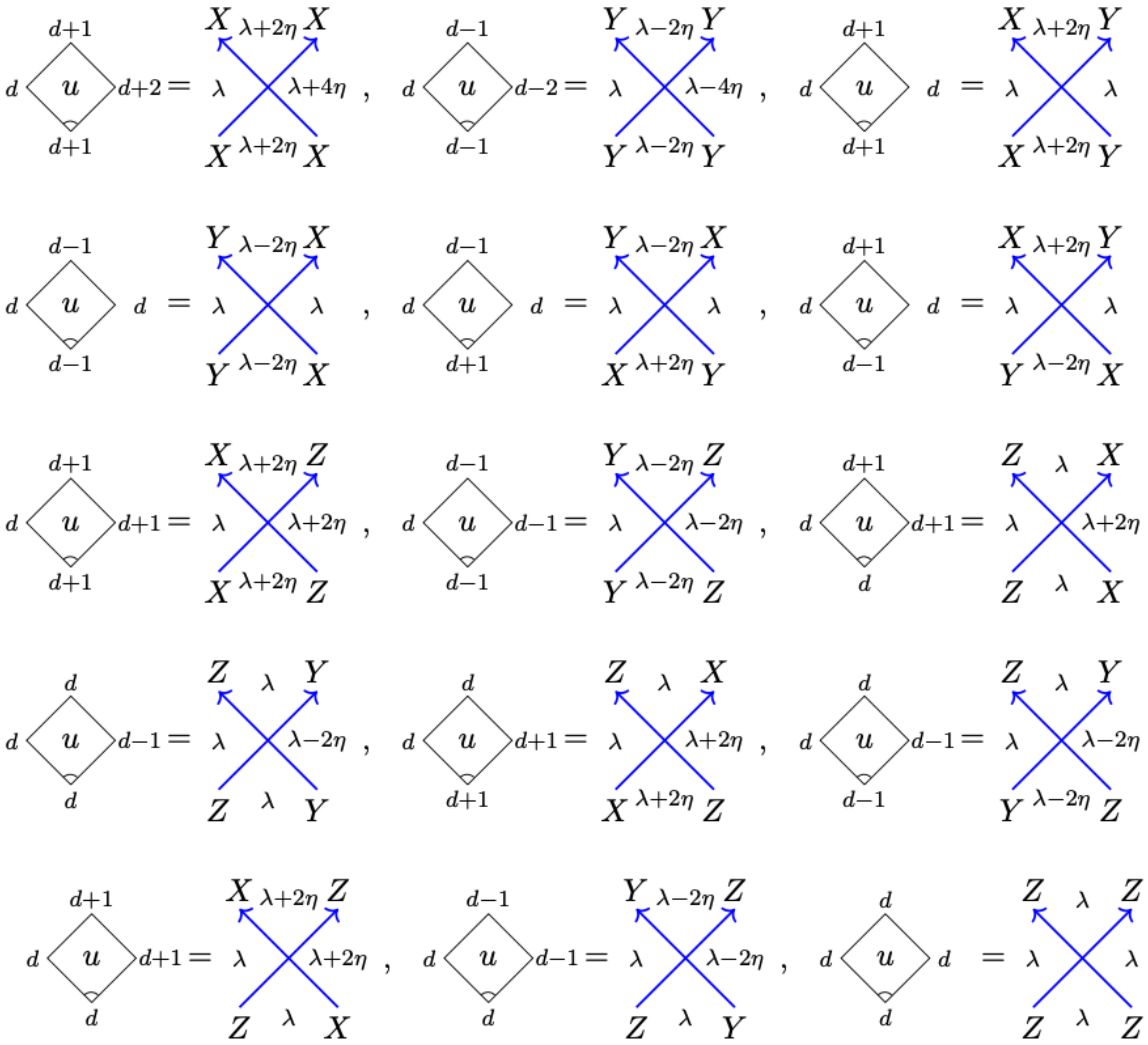
XZ sector:
dynamical dilute Temperley-Lieb model

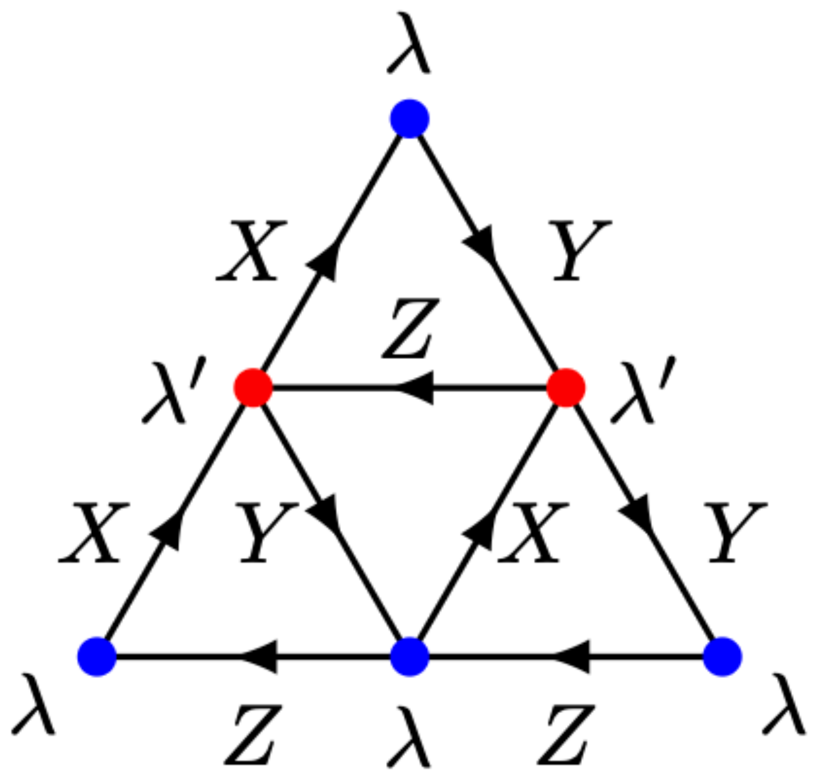


$$\mathcal{H}(\lambda) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \kappa & -1 & 0 \\ 0 & -1 & 1/\kappa & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathcal{H}(\lambda') = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1/\kappa & -1 & 0 \\ 0 & -1 & \kappa & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

1 to 4-magnon: [Pomoni, Rabe, Zoubos, 2021]
[Bozkurt, Nieto García, Pomoni, 2024]
[Bozkurt, Nieto García, Kong, Pomoni, 2025]

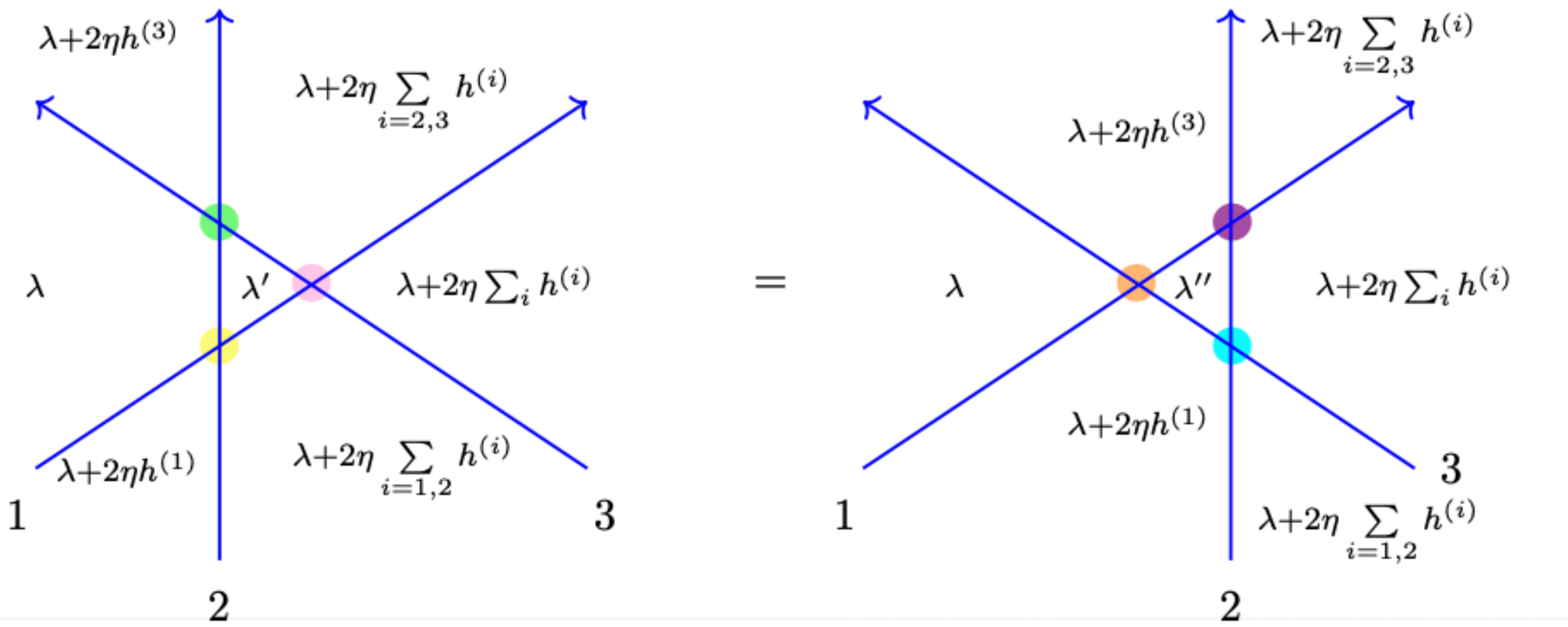
The holomorphic XYZ sector is captured by a dynamical 15-vertex model.





The adjacency diagram of the dilute RSOS model, which is also the dual graph to the brane-tiling diagram of the quiver theory.

- For a long time, it was believed that the model is not integrable for generic couplings as it does not satisfy the YBE. [Gadde, Pomoni, Rastelli, 2010]
- However, it obeys a dynamical YBE



$$R_{12}(u_1-u_2;\lambda+2\eta h^{(3)})R_{13}(u_1-u_3;\lambda)R_{23}(u_2-u_3;\lambda+2\eta h^{(1)})$$
$$=R_{23}(u_2-u_3;\lambda)R_{13}(u_1-u_3;\lambda+2\eta h^{(2)})R_{12}(u_1-u_2;\lambda)$$

The R-matrix of elliptic models, depending on the choice of basis, may obey a dynamical YBE, while that of rational/ trigonometric models always obey the usual YBE.

Expectation: the spectral problem of $\mathcal{N}=2$ SCFTs $\Longleftrightarrow$ dynamical elliptic models.

⚠️ An explicit elliptic parametrization is still missing!

⚠️ Solving the model exactly is still an open question.

For short spin chains in the SO(6) sector, we can brute-force diagonalize H.

😯 Can be expressed in terms of deformed $\mathfrak{su}(4)_{\mathscr{R}}$ projectors,

$$H^{(1)}_{\ell,\ell+1}(\kappa) = H^{(2)}_{\ell,\ell+1}(\kappa^{-1}) = 6\kappa^{-1}\tilde{\Pi}^{\mathbf{1}}_{\ell,\ell+1} + 2\left(\kappa+\kappa^{-1}\right)\tilde{\Pi}^{\mathbf{15}}_{\ell,\ell+1} + 0\tilde{\Pi}^{\mathbf{20}'}_{\ell,\ell+1}$$

The extra degeneracy indicates that certain deformation of $\mathfrak{su}(4)_{\mathscr{R}}$ exists even for $\kappa \neq 1$.

General strategy to deform $\Delta_o(R^a_b)$ to $\Delta_\kappa(R^a_b)$ via a Drinfeld twist:

$$\Delta_\kappa = \mathscr{F}\Delta_o\mathscr{F}^{-1}$$

where $|h,\mathfrak{R}\rangle = \mathscr{F}\,|h,\mathfrak{R}\rangle_o, \quad \lim_{\kappa\to 1}|h,\mathfrak{R}\rangle = |h,\mathfrak{R}\rangle_o$

🤔 We have not found a general formula for $\mathscr{F}^{(L)}_{\mathfrak{R}}$.

F- and D-terms:

$$F_{12}^{Y} = X_{12}Z_2 - \frac{1}{\kappa}Z_1X_{12}, \qquad \bar{F}_{12}^{\bar{Y}} = \bar{X}_{12}\bar{Z}_2 - \frac{1}{\kappa}\bar{Z}_1\bar{X}_{12},$$

$$F_{12}^{X} = Y_{12}Z_2 - \frac{1}{\kappa}Z_1Y_{12}, \qquad \bar{F}_{12}^{\bar{X}} = \bar{Y}_{12}\bar{Z}_2 - \frac{1}{\kappa}\bar{Z}_1\bar{Y}_{12},$$

$$F_1^{Z} = \frac{1}{\sqrt{\kappa}}(X_{12}Y_{21} - Y_{12}X_{21}), \qquad \bar{F}_1^{\bar{Z}} = \frac{1}{\sqrt{\kappa}}(\bar{X}_{12}\bar{Y}_{21} - \bar{Y}_{12}\bar{X}_{21}),$$

$$D_1 = \frac{1}{\sqrt{\kappa}}\left(\bar{X}_{12}X_{21} + \bar{Y}_{12}Y_{21} - X_{12}\bar{X}_{21} - Y_{12}\bar{Y}_{21} + \bar{Z}_1Z_1 - Z_1\bar{Z}_1\right)$$

$+\mathbb{Z}_2$ conjugates

Add terms from mixed sectors and organize using the unbroken $\mathfrak{su}(2)_R$:

$$\vec{F}_{12}^{Y} = \begin{pmatrix} F_{12}^{Y} \\ \Delta(\mathcal{R}^2{}_1)F_{12}^{Y} \end{pmatrix} = \begin{pmatrix} X_{12}Z_2 - \frac{1}{\kappa}Z_1X_{12} \\ \bar{Y}_{12}Z_2 - \frac{1}{\kappa}Z_1\bar{Y}_{12} \end{pmatrix}$$

and similar doublets for $F^X, F^{\bar{X}}, F^{\bar{Y}}$

$+\mathbb{Z}_2$ conjugates

$$G_1 \coloneqq \begin{pmatrix} G_1^{+} \\ G_1^{0} \\ G_1^{-} \end{pmatrix} = \frac{1}{\sqrt{\kappa}} \begin{pmatrix} X_{12}Y_{21} - Y_{12}X_{21} \\ \frac{1}{\sqrt{2}}\left(\bar{X}_{12}X_{21} + \bar{Y}_{12}Y_{21} - X_{12}\bar{X}_{21} - Y_{12}\bar{Y}_{21}\right) \\ \bar{X}_{12}\bar{Y}_{21} - \bar{Y}_{12}\bar{X}_{21} \end{pmatrix}$$

$$E_1 \coloneqq \frac{1}{\sqrt{2\kappa}}(Z_1\bar{Z}_1 - \bar{Z}_1Z_1)$$

XZ sector

Introduce a $\mathbb{Z}_2$-valued function: $s(\ell) = \begin{cases} 1 & \text{if the first gauge index on site } \ell \text{ is } 1 \\ \text{-}1 & \text{if the first gauge index on site } \ell \text{ is } 2 \end{cases}$

Perform Drinfeld twist with $\mathcal{F} = \kappa^{-\frac{s}{2}} \otimes \kappa^{-\frac{s}{2}}$

twist the states:

$$\mathcal{F} \triangleright (X_{12}Z_2 - Z_1X_{12}) = X_{12}Z_2 - \frac{1}{\kappa}Z_1X_{12}$$

$$\mathcal{F} \triangleright (X_{12}X_{21}) = X_{12}X_{21}$$

$$\mathcal{F} \triangleright (Z_1Z_1) = \kappa^{-1}\ Z_1Z_1$$

twist the broken generators: $\Delta_\kappa(\sigma^\pm) = \mathcal{F}\Delta_\circ(\sigma^\pm)\mathcal{F}^{-1} = \left(\kappa^{-\frac{s}{2}} \otimes \kappa^{-\frac{s}{2}}\right)(\mathbb{1} \otimes \sigma^\pm + \sigma^\pm \otimes \gamma)\left(\kappa^{\frac{s}{2}} \otimes \kappa^{\frac{s}{2}}\right)$

$$= \mathbb{1} \otimes \sigma^\pm + \sigma^\pm \otimes \kappa^{-\frac{s}{2}}\gamma\kappa^{\frac{s}{2}} = \mathbb{1} \otimes \sigma^\pm + \sigma^\pm \otimes \gamma\kappa^s$$

keep the unbroken generator: $\Delta_\kappa(\sigma^3) = \mathbb{1} \otimes \sigma^3 + \sigma^3 \otimes \mathbb{1}$

XY sector

Perform Drinfeld twist with

$$\mathcal{F}^{XY} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\left(\frac{1}{\sqrt{\kappa}}+1\right) & \frac{1}{2}\left(1-\frac{1}{\sqrt{\kappa}}\right) & 0 \\ 0 & \frac{1}{2}\left(1-\frac{1}{\sqrt{\kappa}}\right) & \frac{1}{2}\left(\frac{1}{\sqrt{\kappa}}+1\right) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ in the basis } \begin{pmatrix} X_{12}X_{21} \\ X_{12}Y_{21} \\ Y_{12}X_{21} \\ Y_{12}Y_{21} \end{pmatrix}$$

$$\mathcal{F}^{XY} \triangleright (X_{12}Y_{21} - Y_{12}X_{21}) = \frac{1}{\sqrt{\kappa}}(X_{12}Y_{21} - Y_{12}X_{21})$$

leaving $X_{12}Y_{21} + Y_{12}X_{21}$, $X_{12}X_{21}$ and $Y_{12}Y_{21}$ invariant.

The generators are unbroken.

D-term sector

Perform Drinfeld twist with

$$\mathcal{F}^{G_0} = \frac{1}{4}\begin{pmatrix} 3+\frac{1}{\sqrt{\kappa}} & 1-\frac{1}{\sqrt{\kappa}} & \frac{1}{\sqrt{\kappa}}-1 & 1-\frac{1}{\sqrt{\kappa}} \\ 1-\frac{1}{\sqrt{\kappa}} & 3+\frac{1}{\sqrt{\kappa}} & 1-\frac{1}{\sqrt{\kappa}} & \frac{1}{\sqrt{\kappa}}-1 \\ \frac{1}{\sqrt{\kappa}}-1 & 1-\frac{1}{\sqrt{\kappa}} & 3+\frac{1}{\sqrt{\kappa}} & 1-\frac{1}{\sqrt{\kappa}} \\ 1-\frac{1}{\sqrt{\kappa}} & \frac{1}{\sqrt{\kappa}}-1 & 1-\frac{1}{\sqrt{\kappa}} & 3+\frac{1}{\sqrt{\kappa}} \end{pmatrix} \quad \text{in the basis} \quad \begin{pmatrix} \bar{X}_{12}X_{21} \\ X_{12}\bar{X}_{21} \\ \bar{Y}_{12}Y_{21} \\ Y_{12}\bar{Y}_{21} \end{pmatrix}$$

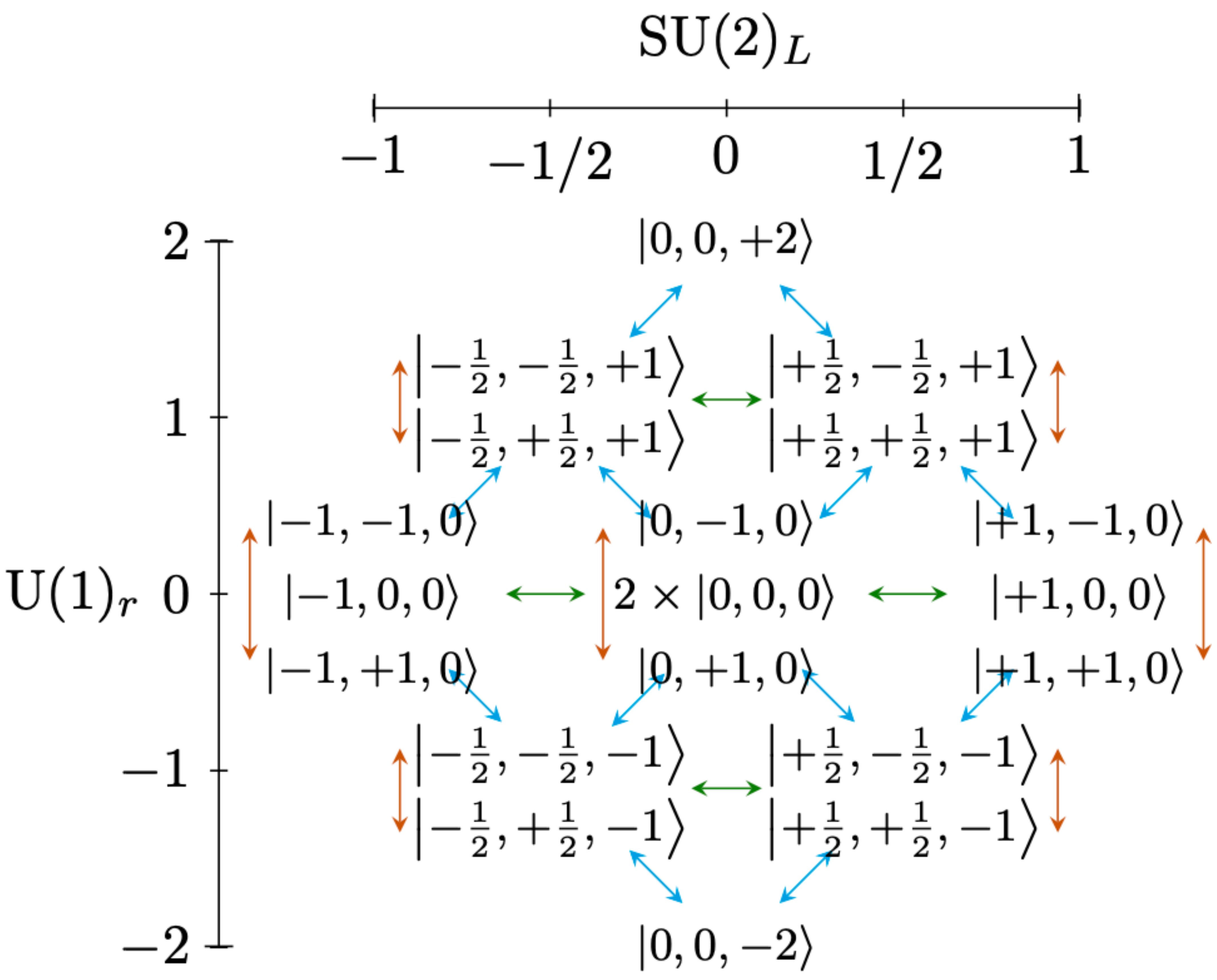
$$\mathcal{F}^{E} = \begin{pmatrix} \kappa^{-1} & 0 & 0 & 0 \\ 0 & \frac{1}{2}\left(\frac{1}{\sqrt{\kappa}}+1\right) & \frac{1}{2}\left(1-\frac{1}{\sqrt{\kappa}}\right) & 0 \\ 0 & \frac{1}{2}\left(1-\frac{1}{\sqrt{\kappa}}\right) & \frac{1}{2}\left(\frac{1}{\sqrt{\kappa}}+1\right) & 0 \\ 0 & 0 & 0 & \kappa^{-1} \end{pmatrix} \quad \text{in the basis} \quad \begin{pmatrix} Z_1Z_1 \\ Z_1\bar{Z}_1 \\ \bar{Z}_1Z_1 \\ \bar{Z}_1\bar{Z}_1 \end{pmatrix}$$

Correctly reproduce the quantum plane relations and leave $X_{12}\bar{X}_{21} + \bar{X}_{12}X_{21} + Y_{12}\bar{Y}_{21} + \bar{Y}_{12}Y_{21}\,,\;\; Z_1\bar{Z}_1 + \bar{Z}_1Z_1$ invariant.

Complete $20'$ multiplet in the 2-site $SO(6)$ sector

		primary of
$(\mathbf{1},\mathbf{1})_2$	$\lvert 0,0,+2\rangle$	$\mathcal{E}_{2(0,0)}$
$(\mathbf{1},\mathbf{1})_{-2}$	$\lvert 0,0,-2\rangle$	$\bar{\mathcal{E}}_{-2(0,0)}$
$(\mathbf{1},\mathbf{1})_0$	$\lvert 0,0,0\rangle$	$\hat{\mathcal{C}}_{0(0,0)}$
$(\mathbf{2},\mathbf{2})_1$	$\lvert \pm\frac{1}{2},\pm\frac{1}{2},+1\rangle$	$\mathcal{D}^{(\pm\frac{1}{2})}_{\frac{1}{2}(0,0)}$
$(\mathbf{2},\mathbf{2})_{-1}$	$\lvert \pm\frac{1}{2},\pm\frac{1}{2},-1\rangle$	$\bar{\mathcal{D}}^{(\pm\frac{1}{2})}_{-\frac{1}{2}(0,0)}$
$(\mathbf{3},\mathbf{3})_0$	$\lvert \pm 1,\pm 1,0\rangle, \lvert 0,0,0\rangle$	$\mathcal{B}_1$

$$
\begin{aligned}
\lvert(\mathbf{1},\mathbf{1})_2\rangle &= \bar{Z}_i\bar{Z}_i\\
\lvert(\mathbf{1},\mathbf{1})_{-2}\rangle &= Z_iZ_i\\
\lvert(\mathbf{1},\mathbf{1})_0\rangle &= X_i\bar{X}_{i+1}+\bar{X}_iX_{i-1}+Y_i\bar{Y}_{i-1}+\bar{Y}_iY_{i+1}-2\left(Z_i\bar{Z}_i+\bar{Z}_iZ_i\right)\\
\lvert(\mathbf{2},\mathbf{2})_1\rangle &= X_i\bar{Z}_{i+1}+\kappa^{(-1)^{i+1}}\bar{Z}_iX_i\\
\lvert(\mathbf{2},\mathbf{2})_{-1}\rangle &= X_iZ_{i+1}+\kappa^{(-1)^{i+1}}Z_iX_i\\
\lvert(\mathbf{3},\mathbf{3})_0\rangle &= X_iX_{i+1}.
\end{aligned}
$$



Complete 15 multiplet in the 2-site SO(6) sector

		primary of
$(\mathbf{1},\mathbf{1})_0$	$\lvert 0,0,0\rangle$	$\mathfrak{C}_{0(0,0)}$
$(\mathbf{1},\mathbf{3})_0$	$\lvert 0,\pm1,0\rangle, \lvert 0,0,0\rangle$	$\mathfrak{B}_1$
$(\mathbf{3},\mathbf{1})_0$	$\lvert \pm1,0,0\rangle, \lvert 0,0,0\rangle$	$\hat{\mathfrak{B}}_1$
$(\mathbf{2},\mathbf{2})_1$	$\lvert \pm\frac{1}{2},\pm\frac{1}{2},+1\rangle$	$\mathfrak{D}^{(\pm\frac{1}{2})}_{\frac{1}{2}(0,0)}$
$(\mathbf{2},\mathbf{2})_{-1}$	$\lvert \pm\frac{1}{2},\pm\frac{1}{2},-1\rangle$	$\bar{\mathfrak{D}}^{(\pm\frac{1}{2})}_{-\frac{1}{2}(0,0)}$

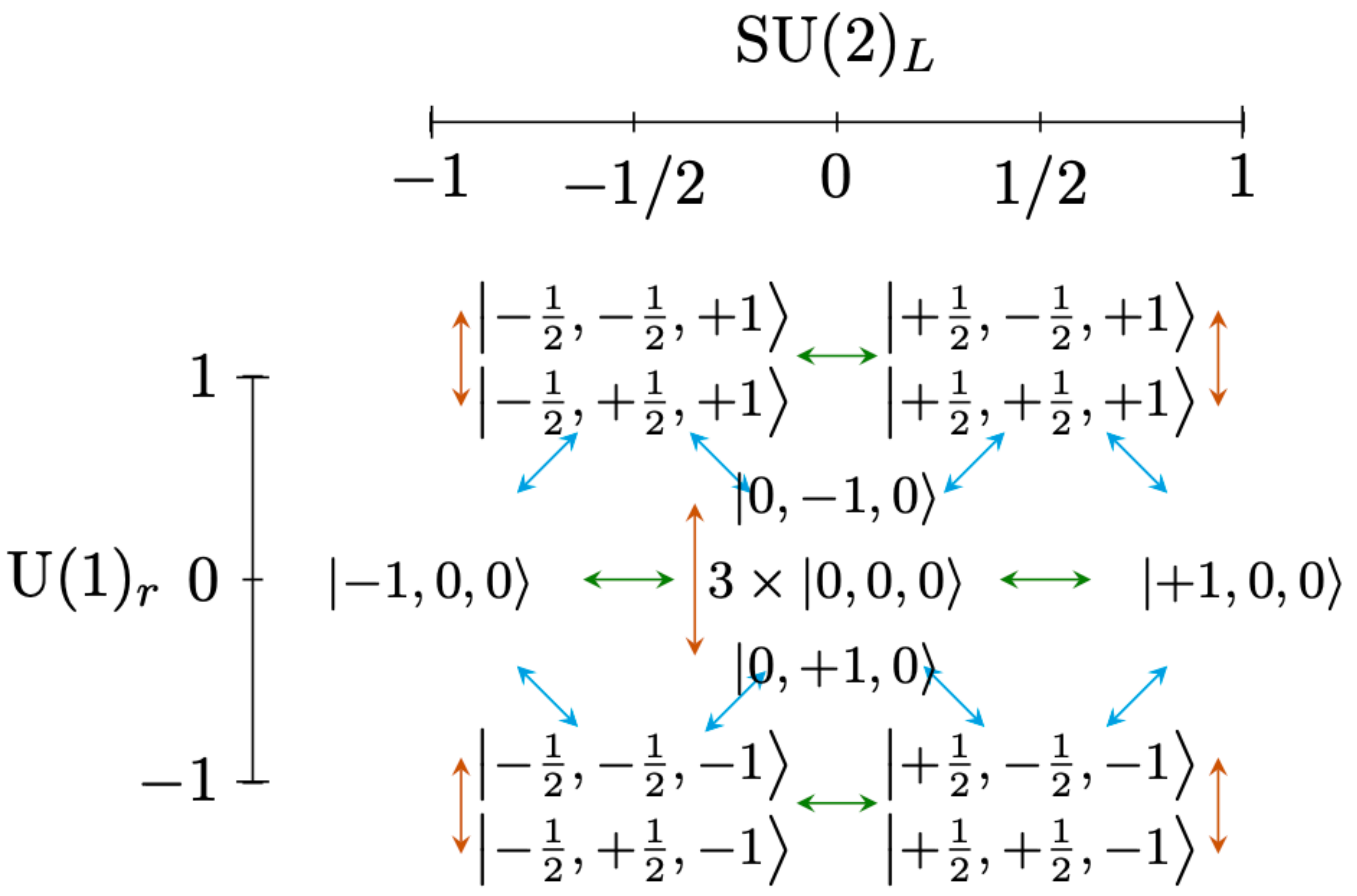
$$|(\mathbf{1},\mathbf{1})_0\rangle = \kappa^{\frac{(-1)^i}{2}}\left(\bar{Z}_i Z_i - Z_i \bar{Z}_i\right)$$

$$|(\mathbf{1},\mathbf{3})_0\rangle = \kappa^{\frac{(-1)^i}{2}}\left(X_i Y_{i+1} - Y_i X_{i-1}\right)$$

$$|(\mathbf{3},\mathbf{1})_0\rangle = \kappa^{\frac{(-1)^i}{2}}\left(X_i \bar{Y}_{i+1} - \bar{Y}_i X_{i+1}\right)$$

$$|(\mathbf{2},\mathbf{2})_1\rangle = X_i \bar{Z}_{i+1} - \kappa^{(-1)^i} \bar{Z}_i X_i$$

$$|(\mathbf{2},\mathbf{2})_{-1}\rangle = X_i Z_{i+1} - \kappa^{(-1)^i} Z_i X_i,$$



3- and 4-sites: Invariance of the Lagrangian

We find that opening up $\mathscr{L}_{\text{orbifold}}$ for generic κ leads to the open states

$$|K_1\rangle_\kappa = X_{12}\bar{X}_{21} + \bar{X}_{12}X_{21} + Y_{12}\bar{Y}_{21} + \bar{Y}_{12}Y_{21} + Z_1\bar{Z}_1 + \bar{Z}_1Z_1$$

$$|\mathscr{W}_1\rangle_\kappa = \frac{1}{\sqrt{\kappa}}\left(X_{12}Y_{21} - Y_{12}X_{21}\right)Z_1 + \frac{1}{\sqrt{\kappa}}Z_1\left(X_{12}Y_{21} - Y_{12}X_{21}\right) + \sqrt{\kappa}\left(Y_{12}Z_2X_{21} - X_{12}Z_2Y_{21}\right)$$

and their $\mathbb{Z}_2$ conjugates.

They are singlets for $H_{\kappa,\mathrm{SO}(6)}$ and $H_{\kappa,\mathrm{SU}(3)_{XYZ}}$.

A much more tedious computation confirms that the Lagrangian in components is invariant.

For the BPS states in the XZ sector: $\mathscr{F}^{(L)} = \kappa^{s/2} \otimes \kappa^{s/2} \otimes \cdots \otimes \kappa^{s/2}$

The coproduct: $\Delta^{(L)}(R^a_b) = \sum_{\ell=1}^{L} \left(I \otimes \cdots \otimes I \otimes \overset{\ell}{R^a_b} \otimes K^a_b \otimes \cdots \otimes K^a_b \right)$

$$K^a_b = \begin{cases} \gamma\kappa^{-s}, & \text{if } R^a_b \text{ is broken} \\ I, & \text{if } R^a_b \text{ is unbroken} \end{cases}$$

$$X_{12}X_{21}X_{12}X_{21}$$

$$\Delta^{(4)}(\mathcal{R}^3{}_2) \qquad \Delta^{(4)}(\mathcal{R}^2{}_3)$$

$$X_{12}Z_2X_{21}X_{12} + X_{12}X_{21}X_{12}Z_2 + \kappa\left(X_{12}X_{21}Z_1X_{12} + Z_1X_{12}X_{21}X_{12}\right)$$

$$\Delta^{(4)}(\mathcal{R}^3{}_2) \qquad \Delta^{(4)}(\mathcal{R}^2{}_3)$$

$$\tfrac{1}{\kappa}X_{12}Z_2Z_2X_{21} + Z_1X_{12}Z_2X_{21} + X_{12}Z_2X_{21}Z_1 + \kappa Z_1Z_1X_{12}X_{21} + \kappa X_{12}X_{21}Z_1Z_1 + \kappa Z_1X_{12}X_{21}Z_1$$

$$\Delta^{(4)}(\mathcal{R}^3{}_2) \qquad \Delta^{(4)}(\mathcal{R}^2{}_3)$$

$$\tfrac{1}{\kappa}X_{12}Z_2Z_2Z_2 + Z_1X_{12}Z_2Z_2 + \kappa Z_1Z_1X_{12}Z_2 + \kappa^2 Z_1Z_1Z_1X_{12}$$

$$\Delta^{(4)}(\mathcal{R}^3{}_2) \qquad \Delta^{(4)}(\mathcal{R}^2{}_3)$$

$$\kappa^2 Z_1Z_1Z_1Z_1$$

The action of the broken generators correctly relates states in the BPS multiplet:

$$\mathrm{tr}_1(X_{12}X_{21}X_{12}X_{21}), \quad \mathrm{tr}_1(Z_1Z_1Z_1Z_1), \quad \mathrm{tr}_2(Z_2Z_2Z_2Z_2)$$

$$\mathrm{tr}_1\left(\kappa\, X_{12}X_{21}Z_1Z_1 + X_{12}Z_2X_{21}Z_1 + \kappa^{-1}\, X_{12}Z_2Z_2X_{21}\right)$$

Summary

- We provide a new perspective on the symmetries of 4d quiver SCFTs in the planar limit: the naively broken R-symmetry generators can be recovered by extending our notion of symmetry to the framework of quantum Lie groupoid. This remains true even when marginally deforming away from the orbifold point.
- We probe the symmetry mainly via the one-loop anomalous dimension.
- We explicitly check that the planar Lagrangian is invariant under the twisted version of the $\mathfrak{su}(4)_R$ algebroid.
- We discuss implications of this hidden symmetry for the spectrum of the theory.

Future directions

- A large number of other observables, such as all-loop anomalous dimension, higher-point correlation functions, scattering amplitudes, Wilson loops, etc, should be analyzed in order to provide further information of the symmetry. [Working in progress with Zhengwen Liu]

- For ordinary global symmetries, there are standard procedures to derive constraints, e.g. Ward identities, anomaly matching conditions, etc.

🤔 Can we find similar results from our quantum symmetry? Any applications in bootstrap program?

- It would be interesting to combine our quantum symmetry with other types of generalized symmetries.


THANK YOU
FOR YOUR
ATTENTION