Integrable Boundary States in SABJM Theory

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References

- *Three-point functions in ABJM and Bethe ansatz*, Peihe Yang, Yunfeng Jiang, Shota Komatsu, **JW**, JHEP, **01** (2022) 002.
- *Wilson-loop one-point functions in ABJM theory*, Peihe Yang, Yunfeng Jiang, **JW**, JHEP, **09** (2023) 047.
- *Three-point functions in Aharnoy-Bergman-Jafferis-Maldacena theory and integrable boundary states*, Peihe Yang, **JW**, JHEP, **02** (2025) 030.
- *Chiral Integrable Boundary States in the $SU(4)$ Alternating Spin Chain*, Yang Liu, **JW**, accepted by Science China: Physics, Mechanics $\&$ Astronomy.
- Work in progress, Nan Bai, Yang Liu, Mao-Zhong Shao, **JW**.

Outline

- Introduction
- Integrable boundary states in XXX spin chain
- Integrable boundary states in ABJM theory
- Chiral integrable boundary states
- Conclusion and outlook

Introduction

Characters of $2d$ integrable field theories

- Particle number is conserved during scattering.
- Particles merely exchange their momenta.
- The multi-particle S-matrix factorizes. This leads to the Yang-Baxter equations.
- There exist infinitely many conserved charges.

Integrability in higher dimensional gauge theories

- Integrable structure was found in four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory in the planar limit ('t Hooft limit).
- This does NOT simply mean infinity many conserved charges of S-matrix.

- Gauge invariant composite operators mix due to loop corrections.
- Such mixing is described by the dilatation operator.
- In the planar limit, the dilatation operator obtained in perturbation theory corresponds to integrable spin-chain Hamiltonians.

Integrability in higher dimensional gauge theories

- 4d $\mathcal{N} = 4$ super Yang-Mills theory is dual to type IIB superstring theory on $AdS_5 \times S^5$ background.
- The worldsheet theory of this string is a $1 + 1$d integrable theory.
- These results at both weak coupling and strong coupling lead people to believe that the integrable structure should exist at arbitrary coupling in the planar limit.
- Integrable structure leads to many highly non-trivial non-perturbative results. (Reviews: [Beisert et al., 10][Yunfeng Jiang, *Physics*, 25 (in Chinese)])
- Similar integrable structure was also found in 3d $\mathcal{N} = 6$ super-Chern-Simons theory (ABJM theory)[Bergman, Jafferis, Maldacena, 08].

Integrable boundary states

- In the last decade, integrable boundary states play an important role in the study of quantum quench dynamics and correlation functions in gauge theories. [de Leeuw, Kristjansen, Zarembo, 15][Piroli, Pozsgay, Vernier, 17]
- Thsese integrable states appear in both $\mathcal{N} = 4$ SYM and ABJM theory.
- The focus of this talk is on the latter.

Integrable boundary states in XXX spin chain

XXX spin chain

- The Hamiltonian of $XXX_{1/2}$ spin chain ([Heisenberg, 28]) is

$$H = J \sum_{j=1}^{L} \left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + S_j^y S_{j+1}^y \right) . \tag{1}$$

- Here J is the coupling constant and $\mathbf{S}_j$ is the spin operator on the j-th site.
- We consider a closed chain with periodic boundary condition,

$$\mathbf{S}_{L+1} = \mathbf{S}_1 . \tag{2}$$

Conserved charges

- This Hamiltonian is integrable and has infinite number of commuting conserved charges, $Q_1 =: U, Q_2 = H, Q_3, \cdots$,

$$[Q_i, Q_j] = 0\,. \tag{3}$$

- Here U is the shift operator,

$$U|i_1, \cdots, i_L\rangle = |i_L, i_1, \cdots, i_{L-1}\rangle\,. \tag{4}$$

- These charges can be packaged inside the transfer matrix,

$$\tau(u) = Q_1 \exp\left(\sum_{n=1}^{\infty} \frac{u^n}{n!} Q_{n+1}\right)\,. \tag{5}$$

- The transfer matrix can be obtained using algebraic Bethe ansatz [Faddeev School, 1970's].

Bethe roots

- The eigenvalues E and eigenstates $|\psi\rangle$ of the above Hamiltonian can be solved by (coordinate/algebraic) Bethe ansatz. [Bethe, 31][Faddeev School]
- They can be parametrized by a set of Bethe roots $\mathbf{u} = \{u_1, \cdots, u_M\}$ which satisfies the following Bethe ansatz equations,

$$1 = \left(\frac{u_j + \frac{\mathrm{i}}{2}}{u_j - \frac{\mathrm{i}}{2}} \right)^L \prod_{\substack{k=1 \\ k \neq j}}^{M} \frac{u_j - u_k - \mathrm{i}}{u_j - u_k + \mathrm{i}}, \tag{6}$$

Integrable boundary states in XXX spin chain

- The integrable boundary state $|\mathcal{B}\rangle$ in the Hilbert space of XXX spin chain is defined by the integrable condition ([Piroli, Pozsgay, Vernier, 17])

$$\tau(u)|\mathcal{B}\rangle = \Pi\tau(u)\Pi|\mathcal{B}\rangle\,. \tag{7}$$

- Here Π is the reflection operator

$$\Pi|i_1,\cdots,i_L\rangle = |i_L, i_{L-1},\cdots,i_1\rangle\,. \tag{8}$$

Overlaps

- We are interested in the overlap between a state $|\Psi\rangle$ and an eigenstate (Bethe state) $|\mathbf{u}\rangle$.
- When $|\Psi\rangle$ is a integrable state $|\mathcal{B}\rangle$, the overlap $\langle\mathcal{B}|\mathbf{u}\rangle$ has some special properties.
- A necessary condition for $\langle\mathcal{B}|\mathbf{u}\rangle \neq 0$ is that $\mathbf{u}$ satisfies the pairing condition $\mathbf{u} = -\mathbf{u}$ as a set.
- When the pairing condition is satisfied, we have

$$\langle\mathcal{B}|\mathbf{u}\rangle = f_{\mathcal{B}}(\mathbf{u})\,\det(G_+(\mathbf{u}))\,, \tag{9}$$

where G_+ is a Gaudin matrix, independent of $|\mathcal{B}\rangle$.

Integrable boundary states in ABJM theory

ABJM theory

- ABJM theory is a three-dimensional $(3d)$ $\mathcal{N} = 6\, U(N) \times U(N)$ Chern-Simons-matter theory.
- The fields in the theory include

Fields	representation of $U(N) \times U(N)$	representation of $SU(4)$ R-symmetry group
A_μ	$(\mathbf{adjoint}, \mathbf{1})$	$\mathbf{1}$
$\hat{A}_\mu$	$(\mathbf{1}, \mathbf{adjoint})$	$\mathbf{1}$
Y^I	$(\square, \bar{\square})$	$\mathbf{4}$
Ψ_I	$(\square, \bar{\square})$	$\bar{\mathbf{4}}$

Integrability of ABJM theory

- We consider the single-trace operators like

$$\mathcal{O}_C = C^{J_1 \cdots J_L}_{I_1 \cdots I_L} \mathrm{Tr}(Y^{I_1} Y^\dagger_{J_1} \cdots Y^{I_L} Y^\dagger_{J_L}) \,. \tag{10}$$

- These operators mix among themselves under renormalization in the large N limit.
- Operator mixing is controlled by the dilatation operator which is related to an integrable spin chain.
- We focus on the operator in the scalar sector and limited to planar two-loop order.

Integrability of ABJM theory

- We focus on the operator in the scalar sector and limited to planar two-loop order.
- The operator $\mathcal{O}_C$ can be mapped to a state

$$|C\rangle := C^{J_1 \cdots J_L}_{I_1 \cdots I_L} |I_1 \bar{J}_1 \cdots I_L \bar{J}_L\rangle \,, \tag{11}$$

  on an alternating closed $SU(4)$ spin chain with length $2L$.
- The Hilbert space of this chain is $\mathbf{C}^{8L} = \otimes_{i=1}^{2L} \mathbf{C}^4$.
- The odd site of the chain is in the $\mathbf{4}$ representation of $SU(4)$, while the even site is in the $\bar{\mathbf{4}}$ representation.

The spin chain Hamiltonian

- The planar two-loop anomalous dimension matrix can be map to the following Hamiltonian on the above chain ([Minahan, Zarembo, 08][Bak, Rey, 08]),

$$H = \frac{\lambda^2}{2} \sum_{l=1}^{2L} \left(2 - 2\mathrm{P}_{l,l+2} + \mathrm{P}_{l,l+2}\mathrm{K}_{l,l+1} + \mathrm{K}_{l,l+1}\mathrm{P}_{l,l+2} \right) , \tag{12}$$

- where P_{ab} and K_{ab} are permutation and trace operators acting on the a-th and b-th sites. We denote the set of orthonormal basis of the Hilbert space at each site by $|i\rangle$, $i = 1, \cdots, 4$. These two operators act as

$$\mathrm{P}_{ab}|i\rangle_a \otimes |j\rangle_b = |j\rangle_b \otimes |i\rangle_a, \qquad \mathrm{K}_{ab}|i\rangle_a \otimes |j\rangle_b = \delta_{ij} \sum_{k=1}^{4} |k\rangle_a \otimes |k\rangle_b . \tag{13}$$

Integrability

- The algebraic Bethe ansatz approach for this ABJM spin chain starts with the following set of R-matrices,

$$\begin{aligned} \mathcal{R}_{12}^{\bullet\bullet}(u) = \mathcal{R}_{12}^{\circ\circ}(u) = u + P_{12} \equiv \mathcal{R}_{12}(u)\,, \\ \mathcal{R}_{12}^{\bullet\circ}(u) = \mathcal{R}_{12}^{\circ\bullet}(u) = -u - 2 + K_{12} \equiv \overline{\mathcal{R}}_{12}(u)\,, \end{aligned} \tag{14}$$

- Here $\bullet/\circ$ denotes the states in the $4/\bar{4}$ representation of $SU(4)_R$.
- These R-matrices satisfy the following set of Yang-Baxter equation,

$$\begin{aligned} &\mathcal{R}_{12}^{\star_1\star_2}(u-v)\mathcal{R}_{13}^{\star_1\star_3}(u)\mathcal{R}_{23}^{\star_2\star_3}(v) \\ &= \mathcal{R}_{23}^{\star_2\star_3}(v)\mathcal{R}_{13}^{\star_1\star_3}(u)\mathcal{R}_{12}^{\star_1\star_2}(u-v)\,. \end{aligned} \tag{15}$$

where $\star_i, i = 1,2,3$ take values in $\{\bullet,\circ\}$.

Integrability

- Based on these R-matrices, one can construct two transfer matrices $\tau(u)$ and $\bar{\tau}(u)$, satisfying

$$[\tau(u), \tau(v)] = [\tau(u), \bar{\tau}(v)] = [\bar{\tau}(u), \bar{\tau}(v)] = 0\,. \tag{16}$$

- They are generating functions of commuting conserved charges, among whom there is the Hamiltonian.
- This proves the integrability of two-loop ABJM spin chain. [Minahan, Zarembo, 08][Bak, Rey, 08]

Bethe roots

- Eigenstates of $\mathbb{H}$ can be constructed using algebraic Bethe ansatz and the states are parameterized by three set of Bethe roots,

$$u_1, \cdots, u_{K_{\mathbf{u}}}, \tag{17}$$

$$v_1, \cdots, v_{K_{\mathbf{v}}}, \tag{18}$$

$$w_1, \cdots, w_{K_{\mathbf{w}}}. \tag{19}$$

Bethe ansatz equations

- These Bethe roots should satisfy the following Bethe ansatz equations,

$$1 = \left(\frac{u_j + \frac{i}{2}}{u_j - \frac{i}{2}} \right)^L \prod_{\substack{k=1 \\ k \neq j}}^{K_{\mathrm{u}}} S\left(u_j, u_k\right) \prod_{k=1}^{K_{\mathrm{w}}} \tilde{S}\left(u_j, w_k\right), \tag{20}$$

$$1 = \prod_{\substack{k=1 \\ k \neq j}}^{K_{\mathrm{w}}} S\left(w_j, w_k\right) \prod_{k=1}^{K_{\mathrm{u}}} \tilde{S}\left(w_j, u_k\right) \prod_{k=1}^{K_{\mathrm{v}}} \tilde{S}\left(w_j, v_k\right), \tag{21}$$

$$1 = \left(\frac{v_j + \frac{i}{2}}{v_j - \frac{i}{2}} \right)^L \prod_{\substack{k=1 \\ k \neq j}}^{K_{\mathrm{v}}} S\left(v_j, v_k\right) \prod_{k=1}^{K_{\mathrm{w}}} \tilde{S}\left(v_j, w_k\right), \tag{22}$$

Bethe ansatz equations

- In the previous page, the S-matrices $S(u,v)$ and $\tilde{S}(u,v)$ are given by

$$S(u,v) \equiv \frac{u-v-i}{u-v+i}, \quad \tilde{S}(u,v) \equiv \frac{u-v+\frac{i}{2}}{u-v-\frac{i}{2}}. \tag{23}$$

- The cyclicity property of the single trace operator is equivalent to the zero momentum condition

$$1 = \prod_{j=1}^{K_{\mathbf{u}}} \frac{u_j + \frac{i}{2}}{u_j - \frac{i}{2}} \prod_{j=1}^{K_{\mathbf{v}}} \frac{v_j + \frac{i}{2}}{v_j - \frac{i}{2}}. \tag{24}$$

- The eigenvalues of $\tau(u), \bar{\tau}(u), \mathbb{H}$ on the Bethe state $|\mathbf{u}, \mathbf{v}, \mathbf{w}\rangle$ can be expressed in terms of the Bethe roots, $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

Correlation functions

- We consider $\mathcal{O}_C$ with $|C\rangle$ being an eigen-vector of H, and compute various correlation functions involving one such $\mathcal{O}_C$.
- Such $|C\rangle$ can be parametrized by three sets of Bethe roots $\mathbf{u}, \mathbf{v}, \mathbf{w}$ satisfying ABJM Bethe ansatz equations.
- In many cases, the correlator can be written as overlap of the Bethe state $|\mathbf{u}, \mathbf{v}, \mathbf{w}\rangle$ and a boundary state $|\mathcal{B}\rangle$ depending on the concrete correlator.
- We find in many cases, such $|\mathcal{B}\rangle$ is integrable.

Some integrable states in supersymmetric gauge theories

Correlation functions	$\mathcal{N}=4$ super Yang-Mills	ABJM theory
Giant graviton 3-point functions $\langle \mathcal{D}^\circ \mathcal{D}^\circ \mathcal{O} \rangle$	[Jiang, Komatsu, Vescovi, 19]	[Yang, Jiang, Komatsu, **JW**, 21]
Single-trace 3-point functions $\langle \mathcal{O}^\circ \mathcal{O}^\circ \mathcal{O} \rangle$	Unexplored	[**JW**, Yang, 24]
Wilson-loop one-point functions $\langle W[C] \mathcal{O} \rangle$	[Jiang, Komastu, Vescovi, to appear]	[Jiang, **JW**, Yang, 2023]

Bosonic $1/6$-BPS circular WLs

- We consider the Wilson loop (WL) along $x^\mu(\tau) = (R\cos\tau, R\sin\tau, 0), \tau \in [0, 2\pi]$.
- There are two-kinds of bosonic $1/6$-BPS WLs,

$$W^B_{1/6} = \mathrm{Tr}\mathcal{P}\,\exp\left(-i\oint d\tau \mathcal{A}^B_{1/6}(\tau)\right)\,, \tag{25}$$

$$\hat{W}^B_{1/6} = \mathrm{Tr}\mathcal{P}\,\exp\left(-i\oint d\tau \hat{\mathcal{A}}^B_{1/6}(\tau)\right)\,, \tag{26}$$

$$\mathcal{A}^B_{1/6} = A_\mu \dot{x}^\mu + \frac{2\pi}{k} R_I{}^J Y^I Y^\dagger_J |\dot{x}|\,, \tag{27}$$

$$\hat{\mathcal{A}}^B_{1/6} = \hat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k} R_I{}^J Y^\dagger_J Y^I |\dot{x}|\,, \tag{28}$$

with $R^I{}_J = \mathrm{diag}(i, i, -i, -i)$.
[Drukker, Plefka, Young, 08][Chen, **JW**, 08]
[Rey, Suyama, Yamaguchi, 08]

Wilson-loop one-point functions

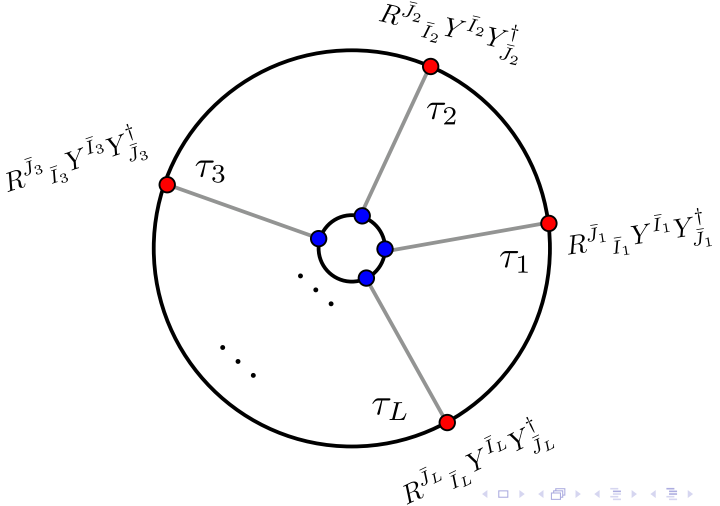
- We computed the tree-level correlation function of a BPS WL and a local operator.
- We take $W(\mathcal{C})^B_{1/6}$ as an example. At tree-level, the correlator $\langle W(\mathcal{C})^B_{1/6}\mathcal{O}_C(0)\rangle$ only gets contributions from

$$\int_0^{2\pi} d\tau_1 \int_0^{\tau_1} d\tau_2 \cdots \int_0^{\tau_{L-1}} d\tau_L \left(\frac{2\pi}{k}\right)^L$$
$$\langle \mathrm{tr}(R^{\tilde{J}_1}{}_{\tilde{I}_1} Y^{\tilde{I}_1}(x_1) Y^\dagger_{\tilde{J}_1}(x_1) \cdots R^{\tilde{J}_L}{}_{\tilde{I}_L} Y^{\tilde{I}_L}(x_L) Y^\dagger_{\tilde{J}_L}(x_L))$$
$$C^{J_1\cdots J_L}_{I_1\cdots I_L} \mathrm{tr}(Y^{I_1}(0) Y^\dagger_{J_1}(0) \cdots Y^{I_L}(0) Y^\dagger_{J_L}(0))\rangle\,, \tag{29}$$

- Here $x_i = (R\cos\tau_i, R\sin\tau_i, 0),\ i = 1, \cdots, L$.

Wick contractions

- The tree-level correlator $\langle W(\mathcal{C})^{B}_{1/6}\mathcal{O}_C(0)\rangle$ in the large N limit can be computed using the planar Wick contractions.



The overlap

- Then the above correlation function can be expressed as

$$\langle W(\mathcal{C})^B_{1/6}\mathcal{O}_C(0)\rangle = \frac{\lambda^{2L}k^L}{(L-1)!(2R)^{2L}}\langle\mathcal{B}^B_{1/6}|\mathcal{O}_C\rangle\,. \qquad (30)$$

- Here $|\mathcal{O}_C\rangle$ is the spin chain state corresponding to the operator $\mathcal{O}_C$.
- The boundary state $\langle\mathcal{B}^B_{1/6}|$ is defined as

$$\begin{aligned}\langle\mathcal{B}^B_{1/6}| \quad &\equiv R^{I_1}{}_{J_1}R^{I_2}{}_{J_2}\cdots R^{I_L}{}_{J_L}\langle I_1, J_1, \cdots, I_L, J_L| \\ &= \left(R^I{}_J\langle I, J|\right)^{\otimes L}\,. \end{aligned} \qquad (31)$$

WL one-point function

- We define the WL one-point function as

$$\langle\!\langle \mathcal{O}_C \rangle\!\rangle_{W(\mathcal{C})} = \frac{\langle W(\mathcal{C}) \mathcal{O}_C \rangle}{\sqrt{\mathcal{N}_{\mathcal{O}_C}}} \,. \tag{32}$$

- The normalization factor $\mathcal{N}_{\mathcal{O}}$ is defined using the following two-point function,

$$\langle \mathcal{O}(x) \mathcal{O}^\dagger(y) \rangle = \frac{\mathcal{N}_{\mathcal{O}}}{|x-y|^{2\Delta_{\mathcal{O}}}} \,, \tag{33}$$

where $\Delta_{\mathcal{O}}$ is the conformal dimension of $\mathcal{O}$.

- At tree level and the planar limit, we have

$$\mathcal{N}_{\mathcal{O}} = \left(\frac{N}{4\pi}\right)^{2L} L \langle \mathcal{O} | \mathcal{O} \rangle \,. \tag{34}$$

One-point function and overlap

- For $W^B_{1/6}$, we have

$$\langle\langle\mathcal{O}\rangle\rangle_{W(\mathcal{C})^B_{1/6}} = \frac{\pi^L\lambda^L}{R^{2L}(L-1)!\sqrt{L}}\frac{\langle\mathcal{B}^B_{1/6}|\mathcal{O}_C\rangle}{\sqrt{\langle\mathcal{O}_C|\mathcal{O}_C\rangle}}\,. \tag{35}$$

- The computation of the Wilson loop one-point function thus amounts to the calculation of

$$\frac{\langle\mathcal{B}^B_{1/6}|\mathcal{O}\rangle}{\sqrt{\langle\mathcal{O}|\mathcal{O}\rangle}}\,. \tag{36}$$

Integrable boundary states from WL

- We proved that $\langle \mathcal{B}^R_{1/6}|$ is an achiral integrable boundary state satisfying the following twisted integrable condition,

$$\Pi\tau(u)\Pi|\mathcal{B}^R_{1/6}\rangle = \bar{\tau}(u)|\mathcal{B}^R_{1/6}\rangle\,. \tag{37}$$

- This leads to the pairing condition $\mathbf{u} = -\mathbf{v}, \mathbf{w} = -\mathbf{w}$ for the overlap $\langle \mathcal{B}^R_{1/6}|\mathbf{u}, \mathbf{w}, \mathbf{v}\rangle$ being nonzero.
- The following two facts are important ingredients of the proof,
  1. The constant reflection matrix $\mathcal{K}(u) := R^\star$ and the R-matrix $\mathcal{R}(u)$ satisfying the boundary Yang-Baxter equation

  $$\mathcal{R}_{12}(u-v)\mathcal{K}_1(u)\mathcal{R}_{12}(u+v)\mathcal{K}_2(v) = \mathcal{K}_2(v)\mathcal{R}_{12}(u+v)\mathcal{K}_1(u)\mathcal{R}_{12}(u-v).$$

  2. The R-matrices $\mathcal{R}(u)$ and $\overline{\mathcal{R}}(u)$ satisfy the following crossing symmetry relations,

  $$\mathcal{R}_{12}(u)^{t_1} = \overline{\mathcal{R}}_{12}(-u-2), \qquad \overline{\mathcal{R}}_{12}(u)^{t_1} = \mathcal{R}_{12}(-u-2)\,, \tag{38}$$

Overlaps

- We got the following formula for the normalized overlap between $\langle \mathcal{B}^R_{1/6}|$ and a Bethe state $|\mathbf{u},\mathbf{v},\mathbf{w}\rangle$ ([Jiang, **JW**, Yang, 23]),

$$\frac{|\langle \mathcal{B}^R_{1/6}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle|^2}{\langle \mathbf{u},\mathbf{v},\mathbf{w}|\mathbf{u},\mathbf{v},\mathbf{w}\rangle} = \prod_{i=1}^{K_{\mathbf{w}}/2} \frac{w_i^2}{w_i^2+1/4} \times \frac{\det G^+}{\det G^-}. \tag{39}$$

- Here the Bethe roots satisfy above the pairing condition, G^+ and G^- are Gaudin matrices depending on $\mathbf{u},\mathbf{v},\mathbf{w}$. The definition of these matrices can be found in [Yang, Jiang, Komatsu, **JW**, 21].

Boundary states from more WLs

- We also show that the boundary states from $1/2$-BPS, $1/3$-BPS WLs are integrable and compute the norm of the overlaps between them and the Bethe state.
- We give strong evidence for the result that the generic fermionic $1/6$-BPS WLs do not provide integrable boundary states.

Some observations

- Here are some observations on integrable states in AdS/CFT correspondence (These are just some empirical rules not strict rules).
- Many states are from operators with supersymmetries or many global symmetries.
- Many are from operators with less or even no continuous parameters.
- Many are dual to supersymmetric D-branes. But there are also cases dual to probe strings or fluctuations of background fields. (Motivation of work on single-trace three-point functions [**JW**, Yang, 24])

Chiral Integrable States

Two kinds of integrable boundary states

- Previously mentioned correlation functions in ABJM theory all lead to achiral integrable boundary states $|\mathcal{B}\rangle$ satisfying the twisted integrable condition,

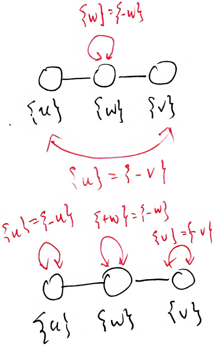
$$\Pi\tau(u)\Pi|\mathcal{B}\rangle = \bar{\tau}(u)|\mathcal{B}\rangle\,. \tag{40}$$

  with pairing condition $\mathbf{u} = -\mathbf{v}, \mathbf{w} = -\mathbf{w}.$ (See also [Bai, Shao, 24])
- How about chiral integrable states $|B\rangle$ which satisfy the untwisted integrable condition

$$\Pi\tau(u)\Pi|B\rangle = \tau(u)|B\rangle \tag{41}$$

  The corresponding pairing condition is $\mathbf{u} = -\mathbf{u}, \mathbf{v} = -\mathbf{v}, \mathbf{w} = -\mathbf{w}.$

Two kinds of pair conditions



Chiral integrable states among basis states

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- Among the basis states, we ([Liu, **JW**, 2025]) have strong evidence that the only following states

$$|A\bar{B}A\bar{D}\cdots A\bar{B}A\bar{D}\rangle\,, \tag{42}$$
$$|A\bar{B}C\bar{B}\cdots A\bar{B}C\bar{B}\rangle\,, \tag{43}$$
$$|A\bar{B}C\bar{D}\cdots A\bar{B}C\bar{D}\rangle\,, \tag{44}$$

in the $SU(2)\times SU(2)$ sector are chiral integrable.

Chiral integrable states among matrix product states

- More complicated chiral integrable states are found among MPS, by using solutions to BYBE. [Bai, Liu, Shao ,**JW**].
- Currently we are studying the overlap between these new integrable states and Bethe states.

Conclusion and outlook

Summary

- We have performed systematic research on integrable states in ABJM theory.
- This deepens the understanding on integrable structure of this theory and its string theory dual.
- This makes us be able to compute some nontrivial correlation functions.

Challenges

- We should generalize our results to all-order ones, first in the asymptotic sense (in the limit of $L \to \infty$).
- Then we should try to take into account the finite size effect using thermodynamic Bethe ansatz (TBA) or quantum spectral curve (QSC) method.

Thanks for your Attention !