Beauty and truth of physics: scientific contributions of Chen Ning Yang

Yu Shi

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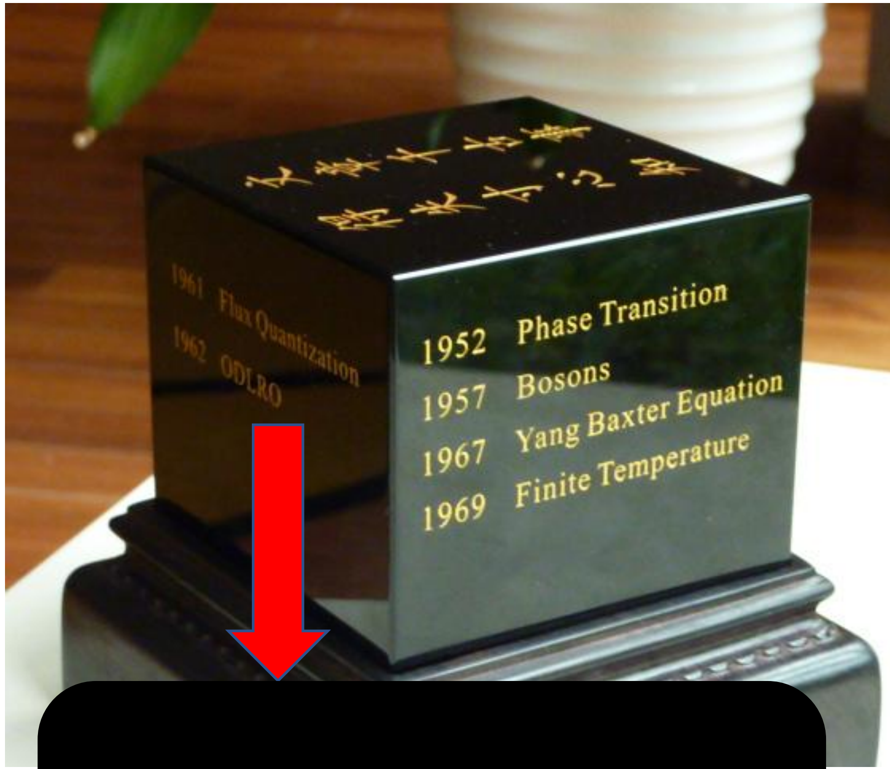
Special Session Honoring the Legacy of C. N, Yang and R. J. Baxter, International Workshop on Challenges in Integrability, Wu Han, 15/11/2025

Searching Shell and snail

- Yang at 6 searched for sea shells on the beach of Xiamen, setting eyesight on beautiful and elegant ones.
- Unique style and taste made him a preeminent architect of theoretical physics in the second half of 20th century.
- Found grains of sand a la William Blake.

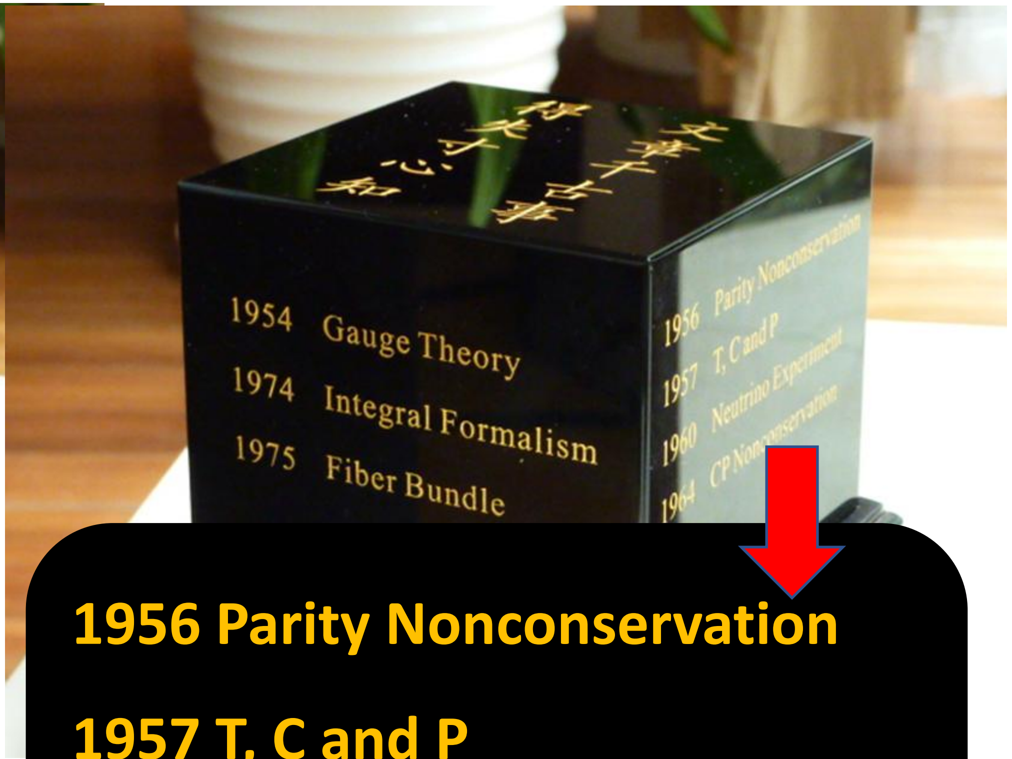
Auguries of Innocence, by William Blake
To see a World in a Grain of Sand
And a Heaven in a Wild Flower,
Hold Infinity in the palm of your hand
And Eternity in an hour.

13 of the contributions



1961 flux quantization

1962 ODLRO



1956 Parity Nonconservation

1957 T, C and P

1960 Neutrino Experiment

1964 CP Nonconservation

- Y. Shi, Beauty and Physics: 13 important contributions of Chen Ning Yang, Int. J. Mod. Phys. A 29 (17), 1475001 (2014).
- Y. Shi, Brief overview of C. N. Yang's 13 important contributions to physics, Int. J. Mod. Phys. A 30 (36), 1530069 (2015).

The 8th OCPA International Conference on Physics Education and Frontier Physics (23 - 27 June 2014)

School of Physical and Mathematical Sciences (SPMS), NTU

<u>Legend:</u>

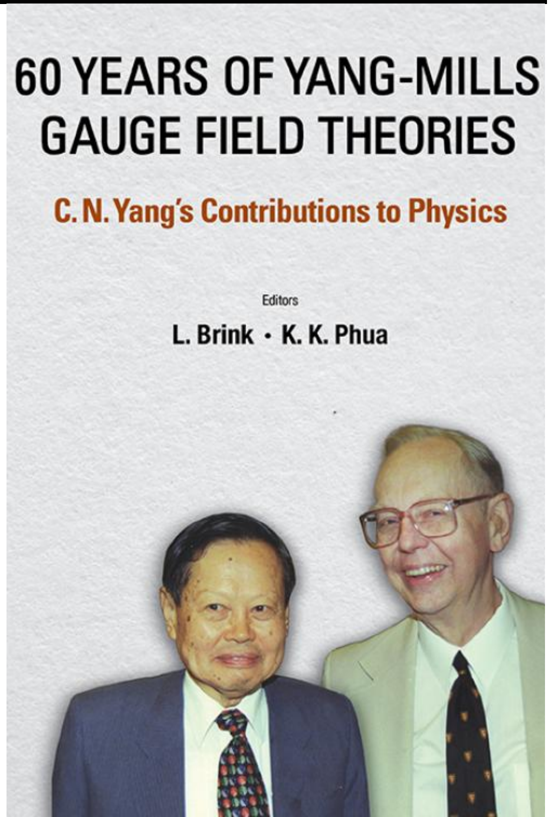
AMO	Atomic, Molecular and Optical Physics	**CP**	Computational and Mathematical Physics
AS	Accelerator, Astrophysics, Cosmology, Nuclear and Particle Physics	**IS**	Interdisciplinary Sciences
BC	Biophysics, Chemical Physics and Soft Condensed Matter	**PE**	Physics Education
CM	Condensed Matter and Materials Sciences	**PH**	Photonics and Photovoltaics

Day 1: 23 June 2014 (Monday)

	Venue: SPMS – LT 1
8.15am – 8.50am	Registration
8.50am – 9.00am	**Opening Ceremony** Welcome Address by Kok-Khoo Phua (Director, IAS, NTU) and Nai-Chang Yeh (President, OCPA) Opening address by Guaning Su (President Emeritus, NTU)
	Chair: Kok-Khoo Phua
9.00am – 9.30am	**Chen-Ning Yang** (Tsinghua University) Conceptual Origin of Maxwell Equation and Field Theory
9.30am – 10.00am	**Yu Shi** (Fudan University) Beauty and Physics: 13 Important Contributions of Chen-Ning Yang

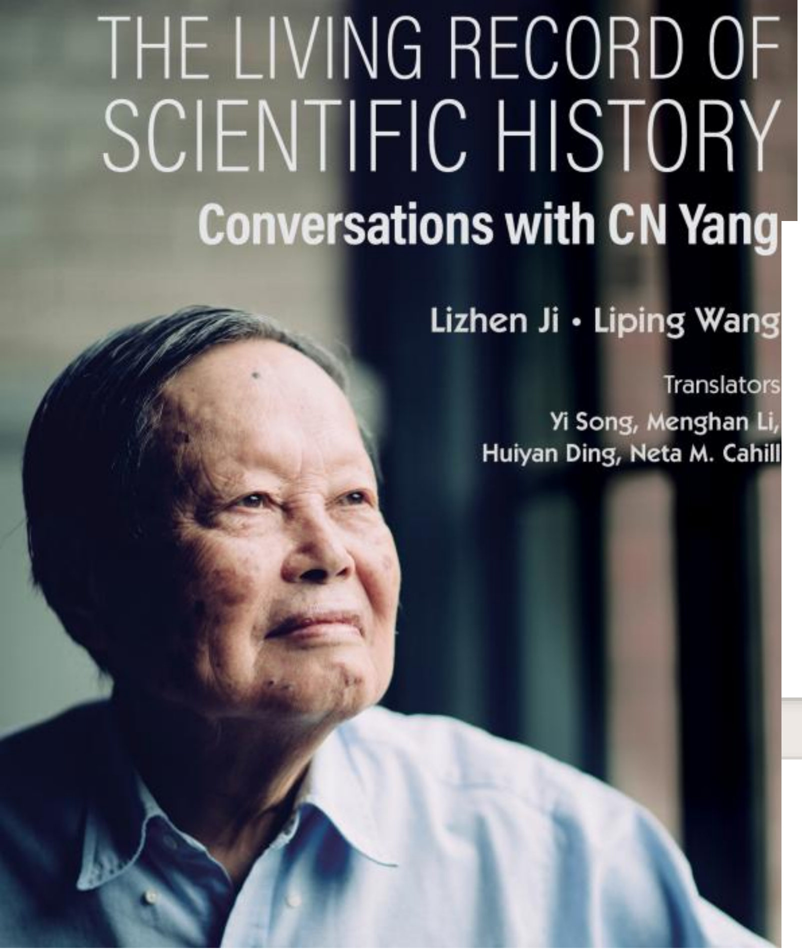
Yu SHI (Fudan University)

Brief Overview of Prof CN Yang's 13 Important Contributions to Physics



物理学的美与真：杨振宁的科学贡献

施郁



Yang Chen-Ning: You may ask Shi Yu from Fudan University to

review my 13 articles. He has written several articles specifically on these 13 articles already. Let's say he has another article, in which he lists the number of times the 13 articles have been cited so far. I'll email you what he has written regarding these 13 articles.

To Prof. C.N. Yang

Yu Shi

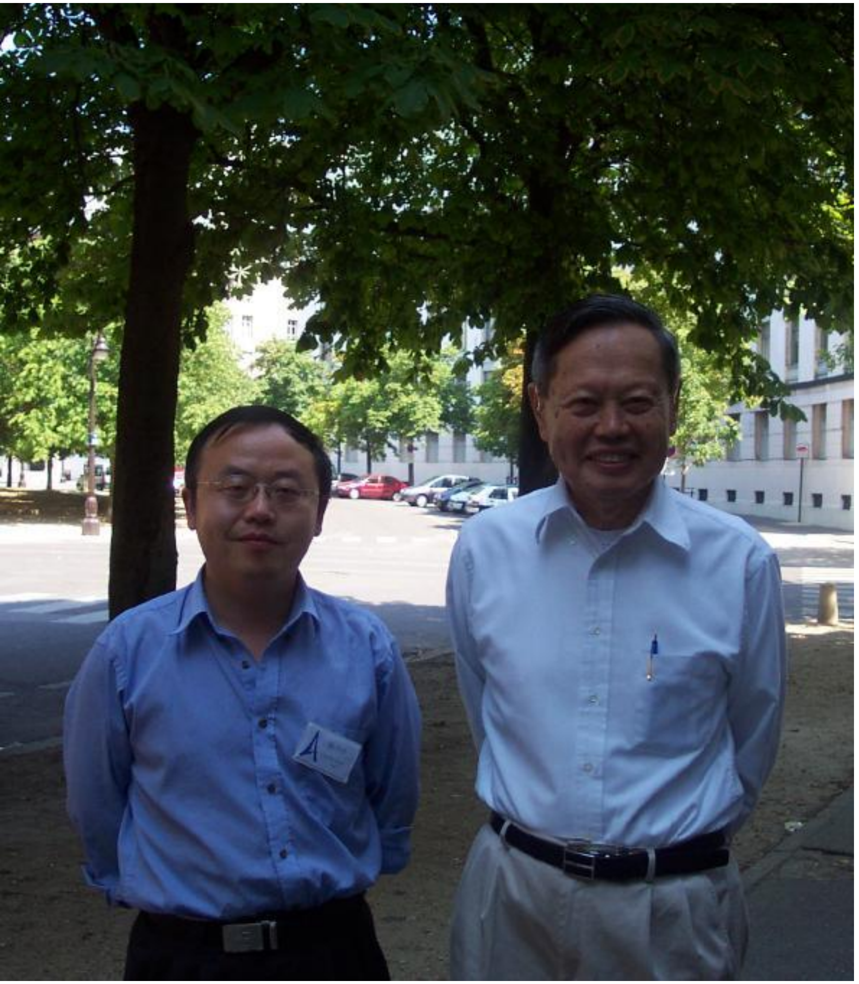
Symmetry begets force, beauty reveals truth.
Parity puzzle solved, conserved revolutionary.
Phase transition and bosons, Yang-Baxter equation.
Works stand eternal, gains and losses the heart knows.
With wisdom and smiles, you return to your root.

- Dyson 1999： **“Professor Yang is, after Einstein and Dirac, the preeminent stylist of the 20th century physics…He is a conserved revolutionary.”**

Yu Shi Yu, Beauty and Truth of Physics: The Scientific Contributions of Chen-Ning Yang

Outline

1. Melody and Taste
2. Growing Up with Particle Physics
3. Journey of Statistical Mechanics
4. Yang-Mills Theory
5. Characteristics of Yang's Work
6. Symmetry Dictates Interactions



- Yang at TH2002: "Three Melodies of 20th-Century Theoretical Physics: ". Quantisation, Symmetry, and Phase Factors
- Symmetry has been the central theme of Yang’s own research.
- Pursuit of beauty in physics has permeated Yang 's scientific career.

- Yang： “my taste in physics was largely formed during the six years from 1938 to 1944 when I was a student in Kunming. It was in those years that I learned to admire the work of Einstein, Dirac, and Fermi. They have, of course, very different styles. Nonetheless, they share the ability to extract the fundamentals of a physical concept, a theoretical structure, or a physical phenomenon and to zero in on the essentials.”

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Yang's characterization of the great physicists

- Einstein: deep and broad.
- Dirac: Cartesian purity
- Fermi: solidity, strength
- Heisenberg: deep insight
- Pauli: power

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- **Yang's style combines those of the three physicists.**
- **Also exhibits the ability to extract the , zero in on the essentials.**
- **Originality, elegance, power, physics.**

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Early education background

- Father K C Yang (general education, and gave him Dickson’s book Modern Algebraic Theories)
- T Y Wu (symmetry)
- J S Wang (statistical mechanics, advising Yang’s master thesis on quasi-chemical method, lectures on Mayer Theory)
- S T Ma (field theory)
- P L Hsu (matrix thory)

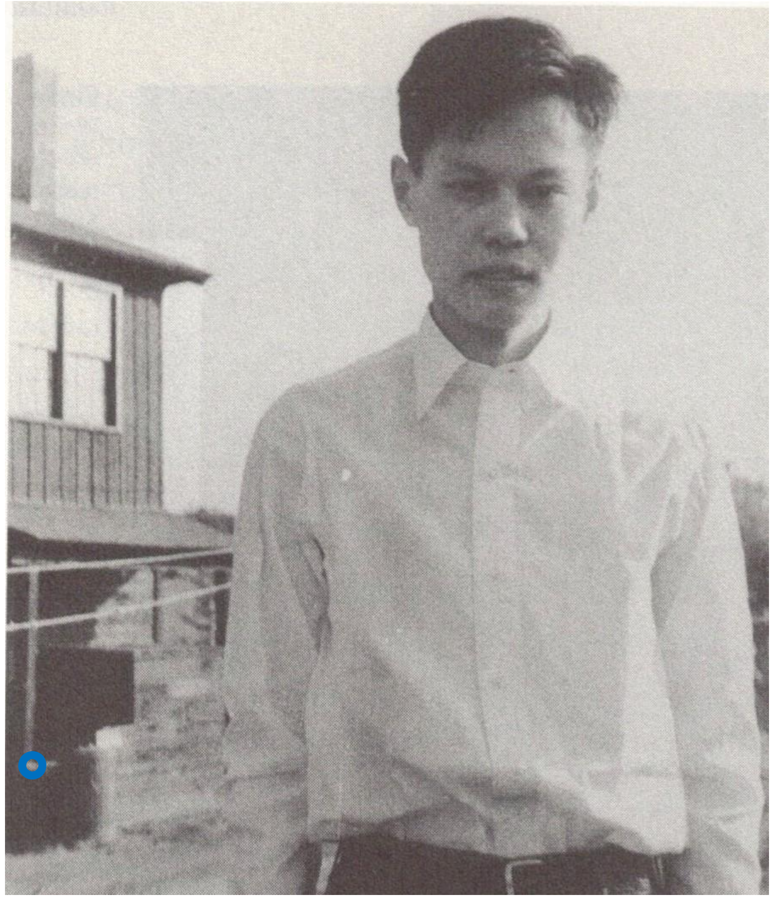
杨振宁对西南联大的新回忆

杨振宁 施郁

1949年，杨振宁、吴大猷和马仕俊

J. Steinberger

- "Before coming to Chicago, Frank Yang already knew all the that we covered in our courses from his studies under the most difficult circumstances, in a war-torn China devastated and brutalized by Japanese armies."



1947

- Beginning of the year: “felt disillusionment”.
- Studied 4 problems, leading to frustrations at the time, but later flourished.
- Still, published first paper in USA “On Quantized Space-time”.

Growing up with particle physics

- **After WWII, physicists returned to fundamental research, particle physics experienced a period of great development**
- **Symmetry analysis proved pivotal.**
- **This was precisely where Yang excelled.**

- **1948: angular distributions of nuclear reaction products, using symmetry analysis (PhD thesis, supervisor: Teller).**
- **1950: the selection rule governing the decay of a pion into two photons, from symmetry principles**
- **1950: phase factor of the parity operator for spin-1/2 particles in processes such as β decay (with Tiomno).**

1949 Fermi-Yang model: Are mesons elementary particles?

- First composite model of hadrons in history.
- Explored the possibility a pion is composed of nucleons and antinucleons (at the time, mesons were widely regarded as elementary particles). •
- Yang: “we did not really have any illusion that what we suggested may actually correspond to reality."
- Fermi: The question we raised is worthy publication.
- warrants publication.
- Steinberger later commented: "This is still the picture if we substitute quarks for nucleons."

Wilczek commented

1. Firmly set in the framework of relativistic QFT.
2. building up light particles on much heavier constituents was a liberating notion. It is an extreme extrapolation of the lowering of mass through binding energy. Nowadays we go much further along the same line, binding infinitely heavy quarks and gluons into the observed strongly interacting particles.
3. the possibility of deep similarity in the basic mechanism of the strong and the weak interaction, rooted in concepts later discovered by Yang-Mills theory is a central feature of the modern theory of matter, the so-called standard model.

Other two pre-parity works on particle physics

- 1949： **universality** of weak interactions: in μ-e decay and μ capture are in the same strength as β decay (with Lee and Rosenbluth).
- 1956: **G parity** (with Lee)。 Combines CP and isospin.

Parity Violation

1956: θ-τ Puzzle

- θ and τ possess different parities, yet equal mass and lifetime.
- If physical processes always conserve parity, then θ and τ are distinct particles. Nature must fine-tune their mass and lifetime to be perfectly identical. This is highly peculiar, unnatural and ugly.

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. Lee, *Columbia University, New York, New York*

AND

C. N. Yang,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

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- Extended the θ-τ problem to a general issue concerning **weak interactions**.
- Proposed the possibility that "parity is conserved in strong and electromagnetic interactions, but may not be conserved in weak interactions", thereby isolating decay processes dominated by weak interactions.
- Through calculations, it was discovered **that no previous experiments** had demonstrated whether parity is conserved in weak interactions.
- **Proposed** several key weak interaction experiments to test whether parity is conserved in weak interactions.

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- **For a long time, parity conservation had been considered intuitively attractive, natural, sacred and extreamly useful.**
- **The paper by Yang and Lee was met with universal disapproval and ridicule**
- **C. S. Wu group conducted the β decay experiment of polarized Co60 (Yang: "truly visionary").**
- **In early January the following year, Wu experiment demonstrated that parity was indeed not conserved in this process, causing a tremendous upheaval throughout the physics community.**

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The Nobel Prize in Physics 1957

Photo from the Nobel Foundation archive.

Chen Ning Yang

Prize share: 1/2

Photo from the Nobel Foundation archive.

Tsung-Dao (T.D.) Lee

Prize share: 1/2

The Nobel Prize in Physics 1957 was awarded jointly to Chen Ning Yang and Tsung-Dao (T.D.) Lee "for their penetrating investigation of the so-called parity laws which has led to important discoveries regarding the elementary particles."

- **Fastest Nobel Prize in history.**

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Weinberg commented

- Lee and Yang had reversed the tendency, beginning with Einstein, to take symmetries as given, nearly self-evident principles. The sense of excitement, of breakthrough, of accomplishment—in short, the "zing"—was evident to everyone

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1957: Follow-ups of parity nonconservation

January 7, 1957,
Relations among violations of P, C, T (with Lee, Oehme)

- Decisive impact on all theoretical analyses on CP violation later in 1964.

After 3 days： Two-component theory of neutrinos (with Lee)

- Parity non-conservation: Neutrinos and antineutrinos can be distinguished by their chiral properties, each possessing only two components.
- A thorough discussion of experimental issues was conducted, the cross-section for neutrino detection should be doubled.

- •1960: Neutrino discoverers Reines and Cowan: The cross section should indeed be doubled from their initial (1956) published value.
- 2002 Reines Nobel Lecture: "As we came to understand years later from Tsung-Dao Lee and Chen-Ning Yang, the cross section should be doubled due to parity non-conservation and neutrino chirality.”

- **With the establishment of parity non-conservation, researchers turned their attention to other weak decay experiments, including the decay of hyperons into pions and nucleons.**
- **Yang and Lee conducted analyses of these phenomena. Several key parameters they defined became standard terminology within the field.**

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Other contributions to phenomenology of particle physics

1960： theoretical analyses on high energy neutrino experiments (with Lee)

- First theoretical analysis on neutrino experiment, with great influence on subsequent neutrino experiments.
- Also discussed W boson, the mediator of weak interaction.

- **Steinberger 2005: "The physics interests of such experiments were tabulated in an accompanying paper by Lee and Yang that proved prophetic… These processes became topics of extensive experimentation in the following years, as neutrino beams and detectors became increasingly powerful."**

1960-1962：intermediate bosons (with Lee)

- Started phenomenological and logical analysis on intermediate bosons (with Lee)
- Veltman: “They had just started on a systematic investigation of vector bosons (the W and Z of weak interactions), and there is no telling how far they could have gone in developing the Standard Model.”

1964:
CP nonconservation (with T T Wu)

- Cronin remarked in 1993: "Among all of these theoretical papers of 1964, only two are still quoted today. One paper … by T. T. Wu and C. N. Yang … has served as a guide to experiments over the past 29 years".

- **Together with Lee-Oehme-Yang 1957, this paper defined the formalism and vocabulary of this subject that have been used up to present.**

- **Steinberger recalled in 2005 that it was the Wu-Yang paper that inspired him to measure the major parameters in neutral kaon decays.**

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Characteristic of Yang's work on phenomenology of particle physics

- **Did not speculate wildly**, rather, made analyses on experiment **using symmetry** principles, and considered possible future experiments.
- Reflect the **influence of Fermi**.

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A journey through statistical mechanics

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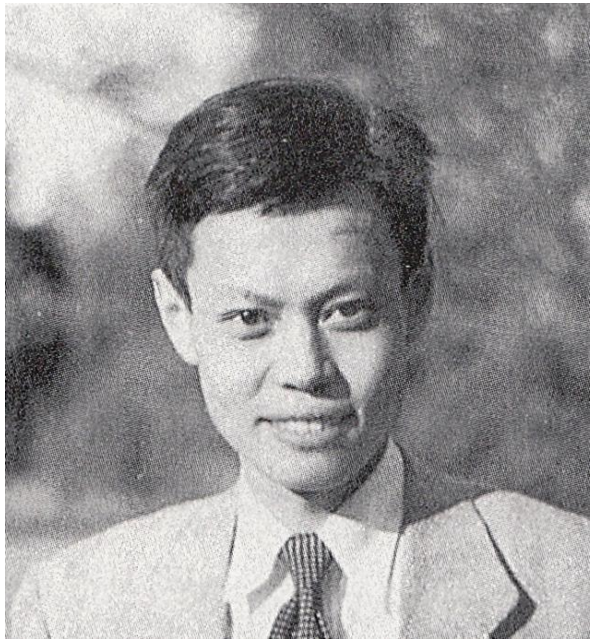
1952: Spontaneous magnetization of 2D Ising model



- Absolute tour de force;   longest calculation Yang did.
- "virtuoso exercise in the theory of Jacobian elliptic functions" (Dyson).
- Together with generalization by C. H. Chang under Yang's suggestion, gave the first indication of universality.

1952: Statistical theory of phase transition I、 II (with Lee)

- Extended Yang’s Ising work to lattice gas model, settled the debate “how can the gas molecules `know’ when they have to coagulate to form a liquid or solid.”
- Investigation of grand partition function for the lattice gas model, extended the fugacity to complex plane.

- **The distribution of zeros determines thermodynamic properties.**
- **Phase transition: in the thermodynamic limit, the zeros tend towards the positive real axis.**
- **Unit circle theorem: “minor gem”.**

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Bose Gas Problem Before and After the Nobel Prize

- **Physics Motivation**: superfluidity of liquid He4.
- Mathematically well defined: **dilute hard sphere** Bosons
- 1955, pseudopotential method (with Kerson Huang and others).
- 1956, after completing the θ-τ paper, developed the two-body collision method with Lee.

- **Subsequently obtained same result using the pseudopotential method with Huang and Lee.**
- **Following their Nobel Prize, Yang and Lee returned to the Bose gas, undertaking a series of subsequent studies.**
- **LHY correction was experimentally confirmed in cold atoms 50 years later!**

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1961：Quantized Magnetic Flux in Superconducting Cylinders (with Byers)

- Yang was visiting Stanford, where Fairbank and Deaver did an experiment showing flux quantization in units of hc/2e.
- Yang and Byers showed: flux quantization is a consequence of BCS pairing, without need of new principles of electromagnetism.

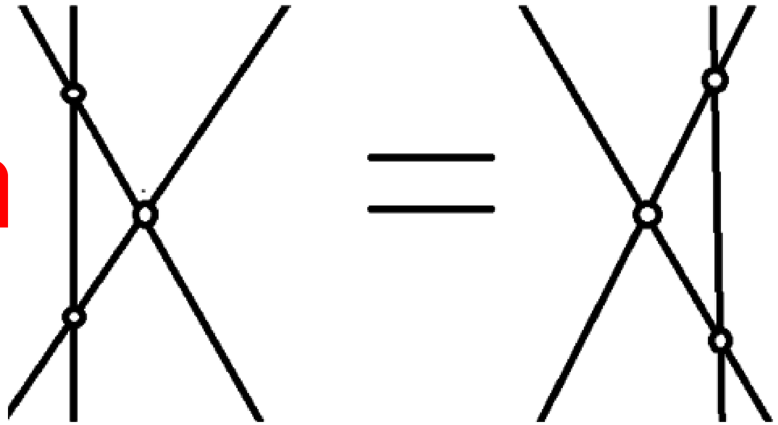
1962:　Off-diagonal long-range order

- 1962, defined ODLRO to **unify superfluidity and superconductivity**. A paper Yang has always been fond of. 。
- Leggett’s 2006 book <Quantum Liquids> on various quantum condensation phenomena: “I take from the start the viewpoint first enunciated explicitly by C. N. Yang …”

1966： revived Bethe Ansatz （wth C. P. Yang）

- Motivated by ODLRO search, which resurrected Yang’s interest in Bethe’s Hypothesis, leading to this paper.
- 1964-1966, phase transitions in lattice gases, and the quantum generalizations.
- Then studied anisotropic Heisenberg model, using Bethe’s hypothesis (ansatz).

1967: 1D with Repulsive Delta-Function Interaction



- Opened the doors to:
  - **Yang-Baxter equation**, a major area in math and physics.
  - **1D systems**, later became a basis for many 1D cold atom experiments.
- Following year, S matrix of this problem.

Thermodynamics of 1D Bosons with Repulsive Delta-Function Interaction (with C. P. Yang)

- First exact solution of a many-body model at **finite temperature**.
- Recently realized and confirmed in cold atom experiments.

- Some years later, finally identified a model featuring non-diagonal long-range order (excited state): the **Hubbard model** (significant for high-temperature superconductivity).
- **η-pairing**: pairing occurs between electrons with total momentum of π.

1990：SO4 in Hubbard model模型 (with S C Zhang)

- Simultaneously related to both superconductivity and magnetism.
- Building upon this foundation, Shoucheng Zhang later developed the SO_5 theory of high-temperature superconductivity.

1971 Remark on Statistical Mechanics and Brains

- "One of the great intellectual challenges for the next few decades is the question of brain organization, What is the basic mechanism for storage of memory? What are the processes that serve as the interphase between the basically chemical processes of the body and the very specific and non-statistical operations in the brain? Above all, how is concept formation achieved in the human brain?

(continued) I wonder whether the spirit of the physics thatwill be involved in these studies will not be akin to that which moved thefounders of the 'rational foundation of thermodynamics' ”

- **1982 Hopfield model**

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Characteristic of Yang's work on statistical mechanics

- Exact solution or analysis on general models based on physical reality, thus elegantly capturing the essence and core of the matter.
- Deep connection with physical phenomema, field theory and math.
- Symmetries also play roles.

Yang-Mills Theory

The 1947 Attempt

- Yang was profoundly impressed by the idea of deriving **charge conservation through gauge (phase) invariance**, and sought to extend it to non-Abelian gauge theories to describe particle interactions. However, he did not succeed.

March 47 (V)

Gauge Invariance and Interact.

I. The gauge invariance in Electromagnetic theory serves for two purp

(i) Give rise to the definition of a charge + current density.

(ii) Fix the interaction between an arbitrary field and the photon field, ~~in order~~ by the requirement of the gauge invariance.

By the second purpose, e.g. a real field cannot interact with the photon. (But notice that a complex field that does not undergo a ~~transfor~~ change under a Gauge transformat. will not interact with the photon either. e.g. The neutron field belong to this class)

II. We can easily formulate a // theory for Meson gauge transformat by requiring that $U_k \to U_k e^{i\alpha\varepsilon_k e}$, $U_k^* \to U_k^* e^{-i\alpha\varepsilon_k e}$ and get a "~~meson~~ charge & current" density when α is a constant, (ε_k = meson charge number of the particle U_k)

In order to fix the interact. between an arbitrary field & the meson field a Gauge transformat of the second kind is necessary. This can be done in the case of neutral mesons by writing

incorrect

?

$$\mathcal{L} = -\tfrac{1}{2} F_{\nu\mu}^* F_{\nu\mu} - K^2 U_\nu^* U_\nu + iA_\nu^* M_\nu \quad ?? \quad -\tfrac{1}{2k} B M_{\nu/\nu}$$

see IV. where A_ν is Stueckelberg's potential (cf. Pauli R.M.P.) & regarding A_ν & B as independent variables. Gauge transf: $A_\nu \to A_\nu + \frac{1}{k} f_{/\nu}$, $B \to B - kf$

For charged mesons, the theory perhaps works too, with α = an operator not commutable with T_3 (isotopic spin)

III. Notice that for photon field, $\varepsilon_k = 0$ or 1 because all particles are either uncharged or ...

(V6)

field $\frac{g}{e} = \varepsilon_k \cong 3$. if U_k = heavy particle field $\left(\frac{G^2}{\hbar c}\right)$

$\frac{g}{e} = \varepsilon_k \cong 10^{-8}$ if U_k = light particle field.

IV. The Gauge transformat described in II will not lead to any fixat. of the interact. because the field part of the $\mathcal{L}$ and the particle part are respectively invariant under the transformat, while in the e.m. case they ~~do~~ both vary.

This constitutes perhaps a fundamental difference between a field with $k=0$ & one with $k \neq 0$.

Masses of Particles

V. The present theory cannot give a satisfactory account of the masses of the different particles because of infinite difficulties and incompleteness of the theory. e.g. We cannot expect to be able to derive the mass ratio of the proton & the electron, or of the proton & the neutron on the present theory.

In a completely satisfactory theory there should be ~~three~~ only three universal constants: $\hbar$, c, G (gravitational const.) ~~and all other~~ which in natural units are all 1. The mass or compton wave length of the particles can be derived.

We see that a theory of such kind is necessarily a merge-together of the Q. theory & general relativity (in which the masses of the particles are defined)

* ... electron radius = range of meson force ... $= 137\,m$.

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1947.3.Note

1947 π^+ discovered in cosmic rays

- More and more particles were discovered.
- Non-abelian gauge theory:
  - C. N. Yang, R. Mills, Isotopic Spin Conservation and a Generalized Gauge Invariance, Phys. Rev. 95, 631 (1954); Yang’s APS talk.
  - C. N. Yang, R. Mills, Conservation of Isotopic Spin and Isotopic Gauge Invariance, Phys. Rev. 96, 191 (1954).

"the electric charge serves as a source of electromagnetic field; an important concept in this case is gauge invariance which is closely connected with

(1) the equation of motion of the electromagnetic field,

(2) the existence of a current density, and

(3) the possible interactions between a charged field and the electromagnetic field.

We have tried to generalize this concept of gauge invariance to apply to isotopic spin conservation."

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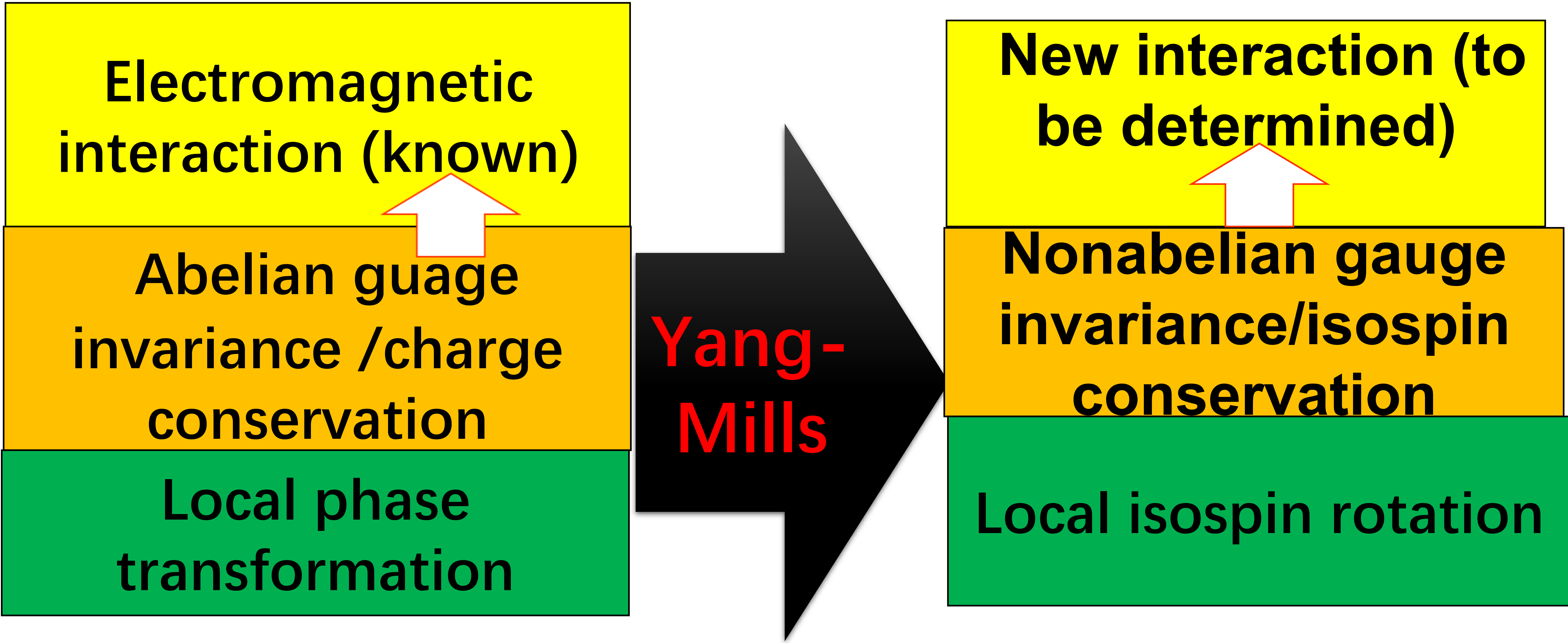
PHYSICAL REVIEW VOLUME 96, NUMBER 1 OCTOBER 1, 1954

Conservation of Isotopic Spin and Isotopic Gauge Invariance*

C. N. YANG † AND R. L. MILLS
Brookhaven National Laboratory, Upton, New York

(Received June 28, 1954)

It is pointed out that the usual principle of invariance under isotopic spin rotation is not consistant with the concept of localized fields. The possibility is explored of having invariance under local isotopic spin rotations. This leads to formulating a principle of isotopic gauge invariance and the existence of a **b** field which has the same relation to the isotopic spin that the electromagnetic field has to the electric charge. The **b** field satisfies nonlinear differential equations. The quanta of the **b** field are particles with spin unity, isotopic spin unity, and electric charge $\pm e$ or zero.



- **Knock the door of symmetry dictates interaction.**

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Standard Model of particle physics

- Through the concepts of spontaneous symmetry breaking and asymptotic freedom introduced in the 1960s and 1970s, Yang-Mills theory provides the framework and basis of today's Standard Model of particle physics.
- Electroweak unification theory: $U(1) \times SU(2)$ Yang-Mills theory.
- Quantum chromodynamics: $SU(3)$ Yang-Mills theory.

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- “In electrodynamics, by the requirement of electric charge conservation, it is argued that the mass of the photon vanishes. Corresponding arguments in the b field case do not exist even though the conservation of isotopic spin still holds. We have therefore not been able to conclude anything about the mass of the b quantum.”

- **Foresight.** It was later discovered that the mass of Yang-Mills gauge particles may indeed be non-zero, or it may be zero.

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- **Electroweak theory: While preserving the symmetry of physical laws, it permits spontaneous symmetry breaking, resulting in non-zero masses for weak interaction gauge particles.**
- **Quantum chromodynamics (strong interaction): Gauge symmetry remains unbroken, with gauge particles (gluons) possessing zero mass; however, there is asymptotic freedom, and quarks and gluons are confined.**

Return to gauge field

- **At the Rochester Conference in April 1957, Yang proposed that the intermediate vector field of the weak interaction might be a gauge field.**
- **However, he believed that gauge fields should not be carelessly reduced to phenomenological entities, arguing that such forced attempts undermined gauge invariance.**

- From the late 1960s onwards, Yang undertook a series of significant works on the mathematical structure of gauge fields and magnetic monopoles.
- The first of these was the **1967 paper on non-Abelian magnetic monopoles** (with TT Wu).

- In 1967–1968, Yang recognised the profound **geometric significance** of gauge fields and studied their integral forms, discovering the importance of **non-integrable phase factors.**
- This paper presents a third avenue for extending gauge theory: generalising non-integrable phase factors to non-integrable Lie group elements.

CONCEPT OF NONINTEGRABLE PHASE FACTORS AND...

TABLE I. Translation of terminology.

Gauge field terminology	Bundle terminology
gauge (or global gauge)	principal coordinate bundle
gauge type	principal fiber bundle
gauge potential b_μ^k	connection on a principal fiber bundle
S_{ba} (see Sec. V)	transition function
phase factor Φ_{QP}	parallel displacement
field strength $f_{\mu\nu}^k$	curvature
source [a] J_μ^K	?
electromagnetism	connection on a $U_1(1)$ bundle
isotopic spin gauge field	connection on a SU_2 bundle
Dirac's monopole quantization	classification of $U_1(1)$ bundle according to first Chern class
electromagnetism without monopole	connection on a trivial $U_1(1)$ bundle
electromagnetism with monopole	connection on a nontrivial $U_1(1)$ bundle

[a] I.e., electric source. This is the generalization (see Ref. 3) of the concept of electric charges and currents.

- **Translating concepts of gauge fields into those of fibre bundles**

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- **1975–1976: Stringless magnetic monopole (with Wu).**
- **The 1975 paper re-examined the Dirac magnetic monopole, using non-integrable phase factors, revealing profound topological and physical connections.**

- 1976: Further elaborated using fibre bundle concepts.
- 1978: Extended the Dirac magnetic monopole to SU_2 gauge fields in five-dimensional flat space or four-dimensional spherical space.

1977： Self-dual Yang-Mills field

- • Self-dual solutions (instantons) was first obtained by BPST in 1975.
- 1977, Yang discussed **the general self-dual conditions** for Yang-Mills fields, yielding the "**Yang equations**".
- Yang equations bear a close relationship to integrable systems.

Mathematics of YMTheory（1）

- **Atiyah** investigated the classification of instanton solutions and, with **Hitchin and Singer**, computed the dimension of the instanton moduli space.
- **Uhlenbeck** represented the Yang-Mills equations as an elliptic system, yielding a series of results (part of the 2019 Abel Prize work).
- **Taubes** studied the gluing of the boundary of the instanton moduli space to an self-dual four-dimensional manifold.

The Mathematics of YMTheory（2）

- **Donaldson** employed instanton moduli spaces to investigate the topology of four-dimensional differential manifolds, establishing Donaldson's theorem.
- Combined with Freedman's theorem, this demonstrates the existence of a four-dimensional differentiable manifold topologically equivalent to four-dimensional Euclidean space yet not diffeomorphic to it (Fields Medal, 1986).

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- **Witten employed supersymmetric Yang-Mills theory to investigate math problems such as low-dimensional topology.**

- **Atiyah: "After 1977, my interests turned towards gauge theory and the interplay between geometry and physics... Singer informed me that the Yang-Mills equations, through the influence of Yang, were beginning to permeate the mathematical community."**

Yu Shi Yu, Beauty and Truth of Physics: The Scientific Contributions of Chen-Ning Yang

Uhlenbeck:

- "The Yang-Mills equations were waiting to be discovered. But mathematicians cannot invent them themselves. Gauge field theory is an adopted child. Physicists Yang and Mills wrote down their equations in 1954."

- Yang pioneered **two convergence points** at the frontiers of physics and mathematics:
  - The **Yang-Baxter equation** (several Fields Medal works relate to this),
  - The mathematical structure of **Yang-Mills theory.**

Yu Shi Yu, Beauty and Truth of Physics: The Scientific Contributions of Chen-Ning Yang

- “Most of my physicist colleagues take a utilitarian view about mathematics. Perhaps because of my father's influence, I appreciate mathematics more. I appreciate the value judgment of the mathematicians, and I admire the beauty and power of mathematics: there are ingenuity and intricacy in tactical maneuvers, and breathtaking sweeps in strategic campaigns, And, of course, miracle of miracles, some concepts in mathematics turn out to provide the fundamental structures that govern the physical universe!”

Yu Shi Yu, Beauty and Truth of Physics: The Scientific Contributions of Chen-Ning Yang

Overall characteristics of Yang Chen-ning's Research Work

- Three elements summarised by Yang himself: perception, persistence, power.
- Particular attention paid to:
  - Physical phenomena and experimental facts.
  - Example: Encouraging Tonomura to verify the AB effect using a superconducting ring.
  - The fundamental structure of physics.
  - The formal beauty of physical theories.
- Making profound discoveries by starting from physical phenomena and his own physical ideas.

- **Down-to-earth**, embodying Fermi's spirit. For instance:
  - Two-component neutrino theory: refraining from speculation without experimental evidence, then conducting thorough investigation once evidence emerged (with Lee) .
  - • In 1964, disregarding impractical theoretical conjecture, he conducted a phenomenological analysis of CP non-conservation, focusing on analysis for future experiments and establishing a phenomenological framework (with Wu).

- "A piece ofliteraturels meant for the millennium. But its ups and downs are known Already in the author's beart.":
  - The significance of some work is swiftly recognised through experimental validation, such as phenomenological work in particle physics, particularly concerning parity non-conservation.
  - The importance of other work grows increasingly profound over time, such as Yang-Mills field theory and much of the work in statistical mechanics.

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Symmetry dictates interactions

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- In 1979, Yang summarised this principle.
- Einstein initiated : Generalised coordinate invariance → Gravitational laws (unknown)
- To extend general relativity, Weyl derived:
  - Abelian gauge invariance → Electromagnetic laws (known)
- Yang-Mills: Non-Abelian gauge invariance → New interaction laws (unknown)

- Thus, all four fundamental interactions are determined by symmetry.
- Yang-Mills theory represents the culmination, intertwining and evolving of the three melodies of 20th-century theoretical physics

Law of laws

- “Symmetry dictates interaction” :
  - summrises the conceptual development of theoretical physics in the last half century;
  - will continue to the guide the development of theoretical physics

- "Symmetry dictates interactions" renders fundamental laws inevitable and determinable, thereby eliminating arbitrariness.
- At the deepest level of law, beauty becomes truth.
- Ascertain truth through beauty (naturally, this must be verified experimentally).
- This constitutes a profound contribution to human thought.

Yang appreciated guage invariance

- (A)the beauty and power of gauge invariance, and
- (B) the mass problem of the gauge Bosons.
- For Mills and me, the central drive derives from (A). Gauge invariance……As to item (B), Mills and I explored various possibilities and concluded at the end of our1954 paper: “We have therefore not been able to conclude anything about the mass oftheb quantum." That is, we regarded (B) as a problem for the future.

A fortune of physics

- Deriving interactions from non-Abelian gauge invariance constitutes the core concept of Yang-Mills theory, which proved correct; the issue of gauge particle masses was subsequently resolved.
- The success of the Standard Model in particle physics vindicates Yang's aesthetic vision, judgement and courage.
- This represents a fortunate turn in the history of physics.

Analog with and lesson from Copernicus's Heliocentric Theory

- Core concept: All planets revolve around the Sun.
- Its merit lies in its aesthetic appeal, while did not align better with observations than Ptolemy's geocentric model, nor does it depart from Ptolemy's methods (eccentricity, epicycles).
- Observational advantages only became apparent with Galileo's telescope-based discoveries.
- Theoretical flaws (perfect circles) were resolved later by Kepler's elliptical orbits

- **The core principle (like gauge invariance) proved correct, with other issues (like mass problem) later addressed.**
- **Circular symmetry (extended to spherical symmetry) transitioned to deeper gravitational laws.**
- **Orbital shapes exhibit spontaneous symmetry breaking (analogous to Yang-Mills theory).**

“Symmetry dictates Interaction” resonates with the ideal of IAS:

Beauty is truth,
truth beauty

John Keats

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