

International Workshop on “Challenges in Integrability”

9–21 November 2025, Wuhan, China



- We thank all speakers for their excellent talks, which presented novel insights into diverse areas of physics via mathematical models
- Gratitude to all participants—your engagement made this workshop successful
- We also acknowledge our service team and collaborative partner **Qi-Zhen Company**
- Sincere thanks to the Organizing Committee for their meticulous organization
- and to all sponsors for their generous financial support



We thank Professor Changrim Ahn and other international experts for their votes, and acknowledge **Modern Mathematical Physics** for supporting the Best Poster Award.

International Workshop on "Challenges in Integrability"



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Thank Miss Xin Zeng for preparing freshment



Welcome speech delivered by the Director



Topics at “Challenges in Integrability”

1. Bethe ansatz for spin chains, condensed matter, ultracold atoms, etc.

Batchelor, A. Foerster, J. Links, J.-P. Cao, C. Paletta, Y.-Z. Zhang, Y. Miao

S. Chen, H. Katsura, Y.F. Jiang

2. Open boundaries, boundary CFT, QFT, Logarithmic CFT, AdS/CFT, Kondo Physics, Bosonization

R. Nepomechie, N. Andrei, E. Vemier, V. Meden, B. Chen, J.-B. Wu, W.-L. Yang, G.-L. Li,

M. Yamazaki, W. Gu, J.E. Bourguin, D. Ridout, Y.-D. Wan, X.Y. Zhang

3. Quantum entanglement, Integrable models out of equilibrium and other related topics

B. Pozsgay, C.-S. Tian, M. Oshikawa, M. Kormos, J. Jacobsen, J.-D. Wu, R. D. Zhu

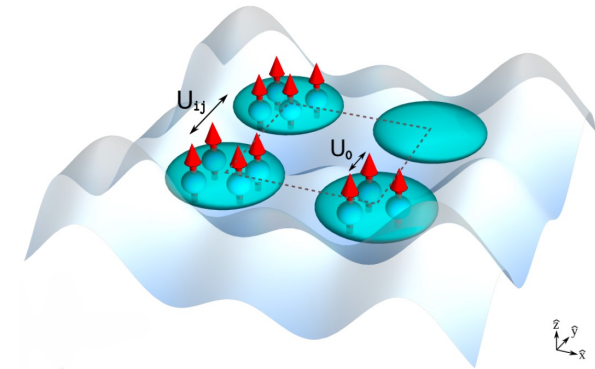
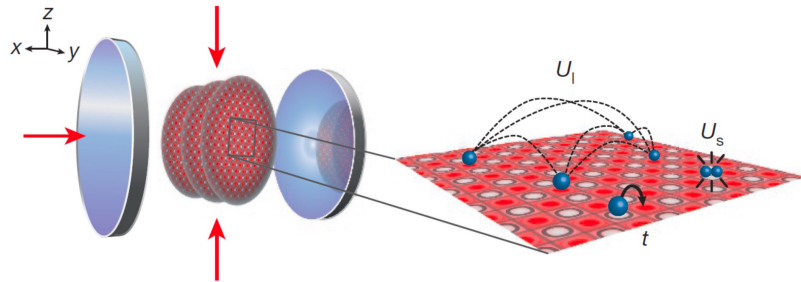
4. Special Session (H-Q. Lin, M. Batchelor, Y. Shi, H. Zhai, Z.C. Gu, V. Korepin, P. Wiegmann)

1. Bethe ansatz for spin chains, condensed matter, ultracold atoms, etc.

Integrable models of bosons constructed from the L-operator (J. Links, J.-P. Cao, S. Chen)

$$H = \sum_{j=1}^L U_j N_j^2 + \sum_{j=1}^L \sum_{k \neq j}^L \frac{U_{jk}}{2} N_j N_k - \sum_{j=1}^L \sum_{k \neq j}^L J_{jk} b_j^\dagger b_k$$

Which offers many cases of integrability!

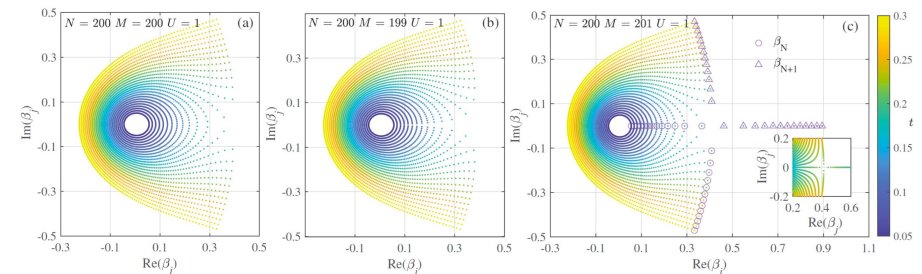
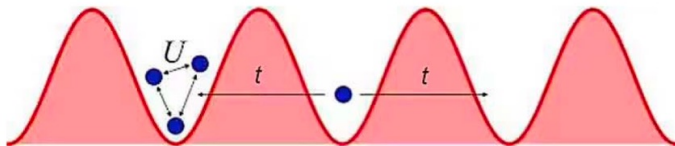


Are such integrable models experimentally relevant?

What is unique feature from integrability?

Bosonic Hubbard model with hopping to one side, non-hermitian model (J.-P. Cao, S. Chen)

$$H = -t \sum_{j=1}^{N-1} (b_j^\dagger b_{j+1} + \epsilon b_N^\dagger b_1) + \frac{U}{2} \sum_{j=1}^N n_j(n_j - 1)$$



Multi-well Hamiltonian and entanglement of quantum states (Links, Foerster)

Two Wells

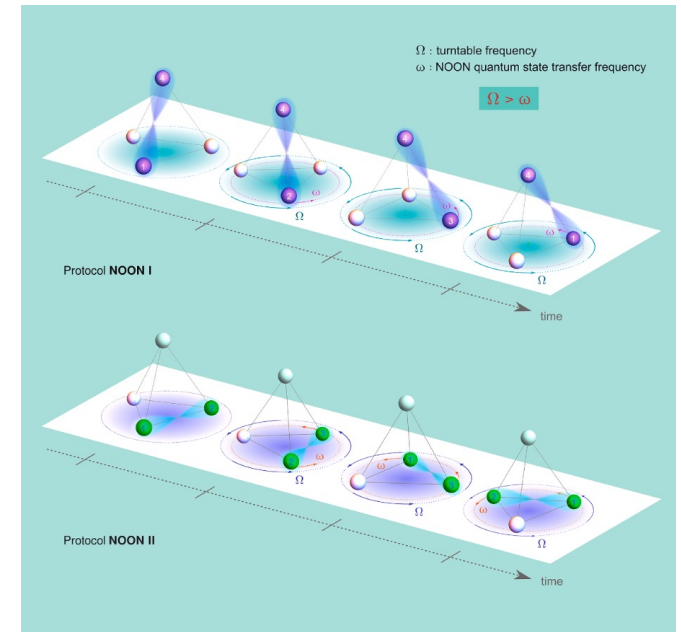
$$H = \frac{K}{8}(N_1 - N_2)^2 - \frac{\Delta\mu}{2}(N_1 - N_2) - \frac{\mathcal{E}_J}{2}(a_1^\dagger a_2 + a_2^\dagger a_1)$$

Three Wells

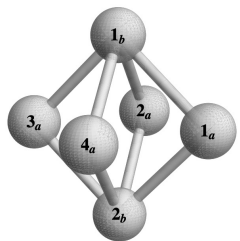
$$H = U(N_1 + N_3 - N_2)^2 + \Delta\mu(N_1 + N_3 - N_2) + t_1(a_1^\dagger a_2 + a_1 a_2^\dagger) + t_3(a_2^\dagger a_3 + a_2 a_3^\dagger)$$

Four Wells

$$H_{n,m} = U(N_A - N_B)^2 + \Delta\mu(N_A - N_B) + \sum_{i=1}^n \sum_{j=1}^m t_{i,j}(a_i b_j^\dagger + a_i^\dagger b_j)$$



Supercontrol of quantum states



What are more can be expected from quantum integrability than the numerical results?

Are such quantum states suseful in quantum technology?

Integrable models can be picked up from old literature (M. Batchelor)

Z(N) spin chains

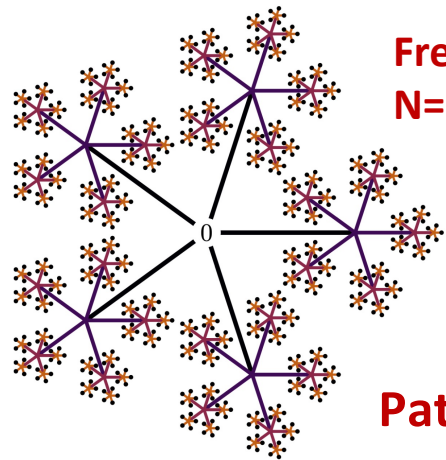
$$H = - \sum_{j=1}^L \sum_{n=1}^{N-1} a_n \left(\lambda \tau_j^n + \sigma_j^n \sigma_{j+1}^{N-n} \right)$$

$$\tau_j = 1 \otimes 1 \otimes \cdots \otimes 1 \otimes \tau \otimes 1 \otimes \cdots \otimes 1$$

$$\sigma_j = 1 \otimes 1 \otimes \cdots \otimes 1 \otimes \sigma \otimes 1 \otimes \cdots \otimes 1$$

$$(\tau)_{\ell m} = \delta_{\ell, m+1} \pmod{N}$$

$$\sigma = \text{diag} \left(1, \omega, \omega^2, \dots, \omega^{N-1} \right)$$



Free parafermion eigenstates
N=5

Patterns in nature are integrable?

Temperley-Lieb model

$$e_{2j-1} = \frac{1}{\sqrt{N}} \sum_{n=1}^N \tau_j^n$$

$$e_{2j} = \frac{1}{\sqrt{N}} \sum_{n=1}^N \left(\sigma_j \sigma_{j+1}^\dagger \right)^n$$

Superintegrable chiral Potts chain

$$H_{\text{CP}} = - \sum_{j=1}^L \sum_{n=1}^{N-1} \left(\lambda \alpha_n \tau_j^n + \bar{\alpha}_n \left(\sigma_j \sigma_{j+1}^\dagger \right)^n \right)$$

$$\alpha_n = \frac{e^{i(2n-N)\phi/N}}{\sin(\pi n/N)}, \quad \bar{\alpha}_n = \frac{e^{i(2n-N)\bar{\phi}/N}}{\sin(\pi n/N)}$$

- Quasi-Integrable models constructed from polynomial algebra (Y-Z Zhang)

What new physics is expected in such kind of models with different underlying algebraic structure?

$$[P_0, P_{\pm}] = \pm P_{\pm},$$

$$[P_+, P_-] = \phi^{(2k)}(P_0, C) - \phi^{(2k)}(P_0 - 1, C)$$

$$\phi^{(2k)}(P_0, C) = - \prod_{i=1}^{\kappa} [C - (kP_0 + k - i + 1)(kP_0 + k - i)] + \prod_{i=1}^{\kappa} [C - i(i - 1)]$$

$$\mathcal{H} = c_+ P_+ + c_- P_- + \sum_{s=1}^k c_s (k P_0)^s + c_*$$

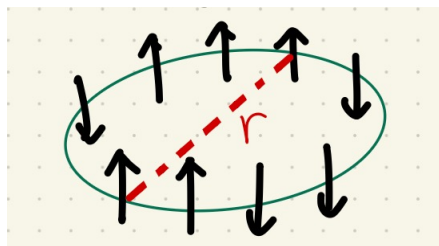
Spin operator:

$$P_{\pm} = J_{\pm}^k, \quad P_0 = \frac{1}{k} J_0$$

- Haldane-Shastry spin chain (Y. Miao)

Triangular function, yes!
Elliptic function?

$$H_{HS} = \sum_{i < j}^N \frac{I - P_{ij}}{4 \sin^2 \frac{i-j}{N}}$$



Spin Calogero-Sutherland model

$$H_{CS} = -\frac{1}{2} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} + \beta \sum_{i < j} \frac{\beta + P_{ij}}{4 \sin^2(\frac{x_i - x_j}{2})}$$

2. Open boundaries, boundary CFT, Logarithmic CFT, AdS/CFT, Kondo Physics, Bosonization

(R. Nepomechie, N. Andrei, E. Vemier, V. Meden, B. Chen, J.-B. Wu, W.-L. Yang, G.-L. Li, M. Yamazaki, W. Gu, J.E. Bourguine, D. Ridout, Y.-D. Wan, X.Y. Zhang)

Quantum group symmetry D_{n+1}^2 models and Non-compact CFT

R. Nepomechie

Unified form of the k-matrix:

$$K^R(u, \varepsilon, p) = \begin{pmatrix} k_-(u) \mathbb{I}_{p \times p} & & & & \\ & g(u) \mathbb{I}_{(n-p) \times (n-p)} & & & \\ & & k_1(u) & k_2(u) & \\ & & k_2(u) & k_1(u) & \\ & & & & g(u) \mathbb{I}_{(n-p) \times (n-p)} & \\ & & & & & k_+(u) \mathbb{I}_{p \times p} \end{pmatrix},$$

D_2^2 : Reminscence of my experience in the time before 2000 in Sao Carlos, Brazil

Finite size spectrum:

$$E(N) \sim N e_\infty + f_\infty + \frac{\pi v_F}{N} X_{eff}$$

Continuous components for $\epsilon = 1$:

$$X_{eff} = -\frac{c}{24} + h_{S,s} = -\frac{1}{12} + \frac{\gamma}{4\pi} \left(2S + 1 - \frac{\pi}{\gamma} \right)^2 + \frac{\gamma}{\pi - 2\gamma} s^2$$

$$N \rightarrow \infty$$

Non-compact boundary CFT

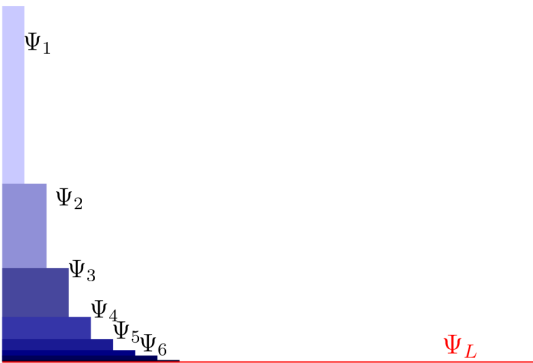
$$X_{eff} = [\text{number}] + n^2 \frac{f(\eta)}{(\log N)^2}, \quad \text{For } 0 < \eta < i\pi/4 \quad \begin{array}{l} \text{Scaling dimension} \\ \text{Continuous with } \gamma! \end{array}$$

The ground state has a non-zero spin s , the symmetry of the model spontaneously broken, related a black hole sigma model.

Open Boundaries: a choice of boundaries becomes a symmetry
(Eric Venier, ...)

Strong zero mode: $[\Psi, H] = 0, \quad \Psi^n \propto 1$

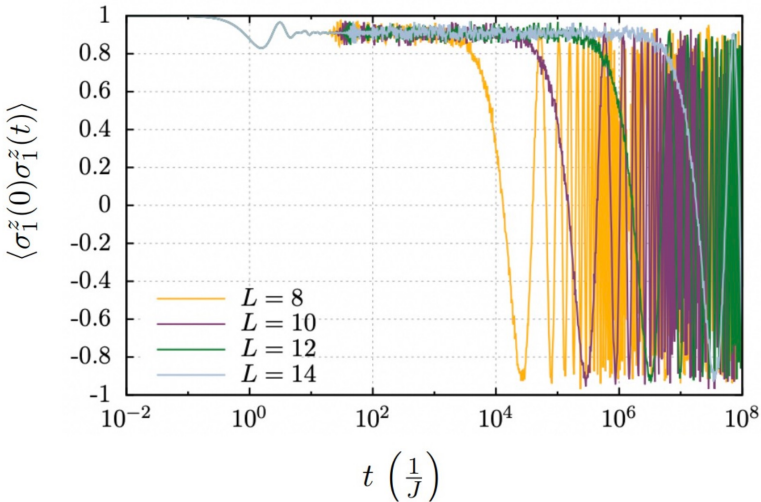
$$\Psi = \sigma_1^z + \frac{h}{J} \sigma_1^x \sigma_2^z + \left(\frac{h}{J}\right)^2 \sigma_1^x \sigma_2^x \sigma_3^z + \dots$$



$$H = \sum_{j=1}^{L-1} h_{j,j+1} + \hbar \sigma_L^z$$

$$\Psi = \sum_{r=1}^{L-1} \Psi_r + \frac{1}{\hbar} \Psi_L$$

$$[H, \Psi] = 0$$



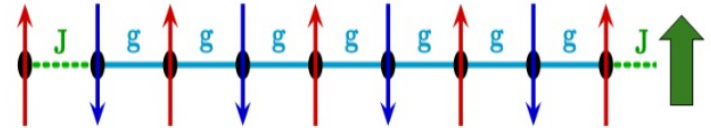
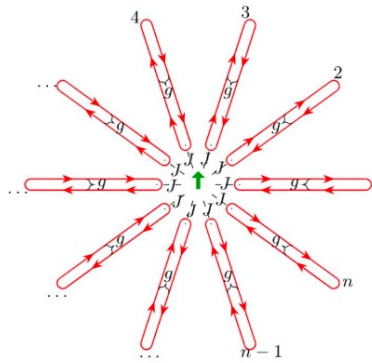
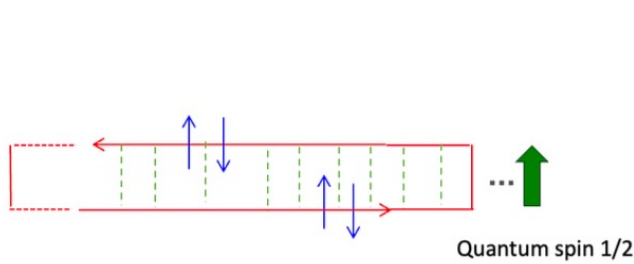
Long coherent time

$$T'(i\pi/2) = \sum_{j=1}^L$$

Interesting extension of the OBC:

What is important physics under the thermodynmaic limit?

Open Boundaries: Kondo impurities coupled to the edges (Andrei, ...)

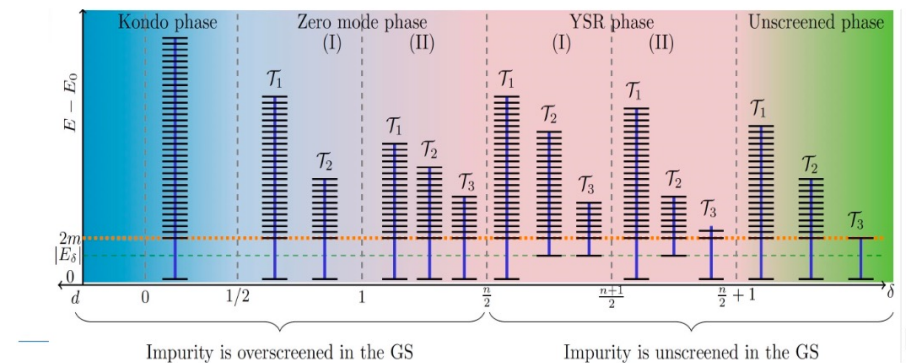
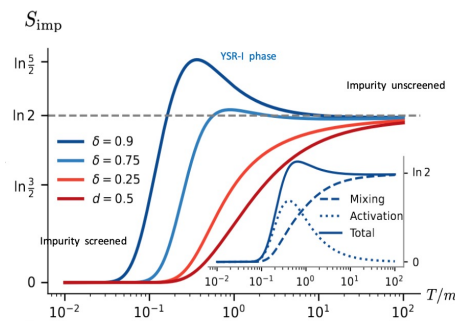


Impurity at the edge of spin chain

1D SC wire with an impurity

Impurity coupled to many SC wires

$$H = \underbrace{-i \int_{-L}^0 dx (\psi_+^\dagger \partial_x \psi_+ - \psi_-^\dagger \partial_x \psi_-)}_{\text{1D spin-singlet superconductor}} + 2g \underbrace{\int_{-L}^0 dx \psi_+^\dagger \tilde{\sigma} \psi_+ \cdot \psi_-^\dagger \tilde{\sigma} \psi_-}_{\mathcal{O}_{\text{SSS}}(x) = \psi_-^\dagger(x) \sigma^y \psi_+^\dagger(x)} + \underbrace{J \psi_-^\dagger(0) \tilde{\sigma} \psi_+(0) \cdot \tilde{S}}_{\text{Quantum spin 1/2}}$$



Schreened, Overscreened , Unscreened

Why do we study Kondo impurities?

The off-diagonal Bethe ansatz method of quantum integrable systems

$$H = \sum_{k=1}^{N-1} \left(\sigma_k^x \sigma_{k+1}^x + \sigma_k^y \sigma_{k+1}^y + \sigma_k^z \sigma_{k+1}^z \right) + \frac{\eta}{p} \sigma_1^z + \frac{\eta}{q} (\sigma_N^z + \xi \sigma_N^x)$$

Essence of Integrability (T-Q relation)

$$t(u) = \text{tr}(K^+(u) \mathcal{T}(u)) = \text{tr}(K^+(u) T(u) K^-(u) T^{-1}(-u))$$

- Spin- 1/2 Heisenberg chain with the periodic boundary condition
- Spin- 1/2 Heisenberg chain with arbitrary boundary fields
- Open spin chains with general BC associated with the other algebras
- Super-symmetric t-J model with unparallel boundary fields
- Hubbard model with unparallel boundary fields
- Spin chain associated with the G_2 exceptional Lie algebra symmetry
- W.L. Yang, G.-L. Li, A. Klumper, ...

Question: What can we observe from boubnaries more than finite spectra?

T Q-relations → finite NLIE for open integrable models

A. Klumper

TQ relation and auxiliary functions

$$q_1(x) := Q_1 \left(\frac{i}{2} x - \frac{1}{2} \right) \quad q_2(x) := Q_2 \left(\frac{i}{2} x - \frac{1}{2} \right)$$

$$t(x) = T \left(\frac{i}{2} x - \frac{1}{2} \right) = \underbrace{\Phi_1(x) \frac{q_1(x+2i)}{q_2(x)}}_{\lambda_1(x)} + \underbrace{\Phi_2(x) \frac{1}{q_1(x)q_2(x)}}_{\lambda_2(x)} + \underbrace{\Phi_3(x) \frac{q_2(x-2i)}{q_1(x)}}_{\lambda_3(x)}$$

We – H. Frahm, AK, D. Wagner and X. Zhang (2025) – find that the following auxiliary functions have useful properties:

$$a := \frac{\lambda_2(x) + \lambda_3(x)}{\lambda_1(x)},$$

$$A := 1 + a = \frac{\lambda_1(x) + \lambda_2(x) + \lambda_3(x)}{\lambda_1(x)},$$

$$\bar{a} := \frac{\lambda_1(x) + \lambda_2(x)}{\lambda_3(x)},$$

$$\bar{A} := 1 + \bar{a} = \frac{\lambda_1(x) + \lambda_2(x) + \lambda_3(x)}{\lambda_3(x)},$$

$$c := \frac{\lambda_2(x) [\lambda_1(x) + \lambda_2(x) + \lambda_3(x)]}{\lambda_1(x)\lambda_3(x)},$$

$$C := 1 + c = \frac{[\lambda_1(x) + \lambda_2(x)] [\lambda_2(x) + \lambda_3(x)]}{\lambda_1(x)\lambda_3(x)},$$

Question: What can we observe from boundaries more than finite spectra?

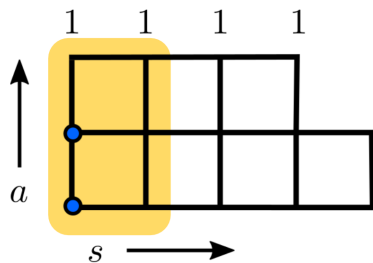
Theorem

The solution of TQ -relation for **both** $Q(u)$ and $P(u)$ being polynomials give the complete spectrum of the XXX spin chain.

Root patterns/structure in T-Q relation and BA Equations

Yun-Feng Jiang

From rational Q -system to BAE



Focus on the first column

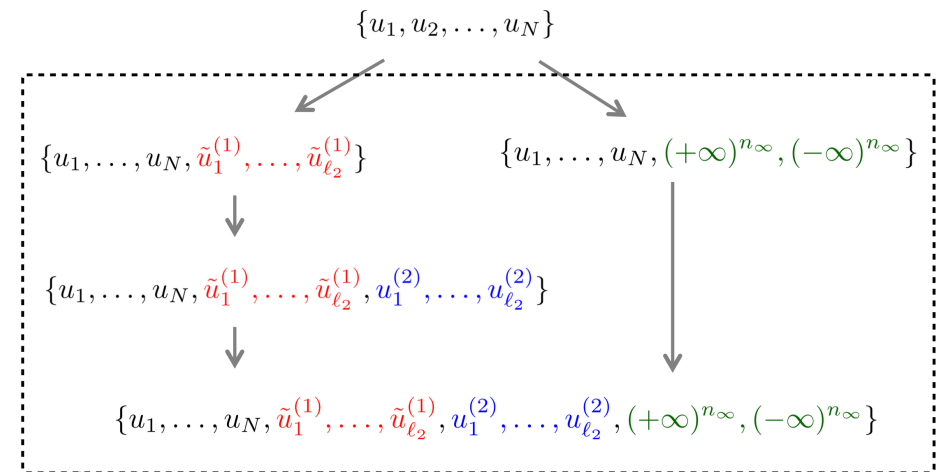
$$Q_{1,1} = Q_{1,0}^+ - Q_{1,0}^-$$

$$Q_{1,0}Q_{0,1} = Q_{1,1}^-Q_{0,0}^+ - Q_{1,1}^+Q_{0,0}^-$$

$$Q_{1,1}^-(u_k)Q_{0,0}^+(u_k) = Q_{1,1}^+(u_k)Q_{0,0}^-(u_k)$$

$$\begin{aligned} Q_{1,1}^+(u_k) &= Q_{1,0}^{++}(u_k), \\ Q_{1,1}^-(u_k) &= -Q_{1,0}^{--}(u_k) \end{aligned}$$

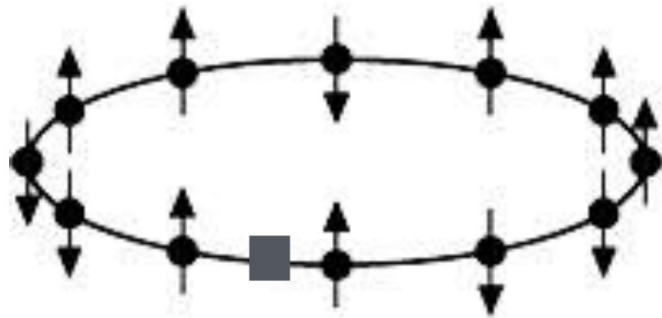
Structure of Hilbert space



Can we use such Hilbert space structure to determine the entanglement structure?

Do it!

Modified Bethe ansatz for twisted BC



Mysterious twist decomposition

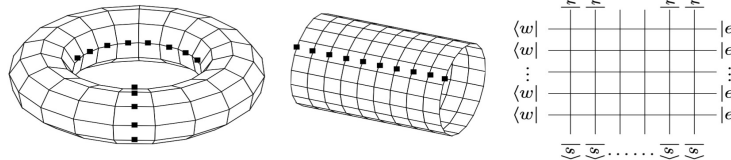
$$K = LDM = \begin{pmatrix} \tilde{\kappa} & \kappa^+ \\ \kappa^- & \kappa \end{pmatrix}$$

Modified transfer matrix

$$t(z) = \text{Tr}(D\bar{T}(z)) = (\tilde{\kappa} - \rho_1)\bar{A}(z) + (\kappa - \rho_2)\bar{D}(z)$$

Twisted one dimensional spin chains

Eigenvalue



More interesting applications

$$\Lambda(z, \bar{u}) = (\tilde{\kappa} - \rho_1)\lambda_1(z)f(\bar{u}, z) + (\kappa - \rho_2)\lambda_2(z)f(z, \bar{u}) + (\rho_1 + \rho_2)F(z)g(z, \bar{u}),$$

$$E(u_i, \bar{u}_i) = (\kappa - \rho_2)\lambda_2(u_i)f(u_i, \bar{u}_i) - (\tilde{\kappa} - \rho_1)\lambda_1(u_i)f(\bar{u}_i, u_i) + (\rho_1 + \rho_2)F(u_i)g(u_i, \bar{u}_i)$$

Scholar product of Bethe vector

$$S_K(\bar{u}; \bar{v}) = \langle 0 | \bar{C}(\bar{v}) \bar{B}(\bar{u}) | 0 \rangle$$

Applicable to the Hubbard model liked models??

Quantum statistics can be factorized!

1. Two identical particles exchange statistics

$$\psi(r_1, r_2) = e^{i\alpha} \psi(r_2, r_1)$$

Fermions: antisymmetric $\alpha = \pi$

Bosons: symmetric $\alpha = 0$

Anyons: arbitrary $\alpha \neq 0$

3. Wave function undergoes a matrix transformation

$$\psi^I(\{x_i\}_{i=1}^n) |_{x_j \leftrightarrow x_{j+1}} = \sum_J (R_j)^I_J \psi^J(\{x_i\}_{i=1}^n)$$

$$\begin{array}{c} \text{Diagram 1: Two lines crossing, labeled } j, j+1 \text{ on the left and } j, j+1 \text{ on the right.} \\ R_j^2 = 1, \end{array} \quad \begin{array}{c} \text{Diagram 2: Two lines crossing, labeled } j-1, j \text{ on the left and } j, j+1 \text{ on the right.} \\ R_{j-1} R_j R_{j-1} = R_j R_{j-1} R_j \end{array}$$

(Wang, Hazzard, Nature 2025)

2. Vortices: expression of flux quanta leading to a reduction of Coulomb energy and forming a Composite particle, i.e. **electrons+flux**

Fractional QHE(2D, low T, high H)

Laughlin, Störmer, Tsui, ($q = e/3$)
1998 Nobel Prize

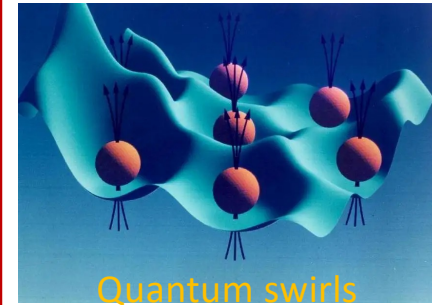
2D Electron Gas

Quantized motion of electrons:

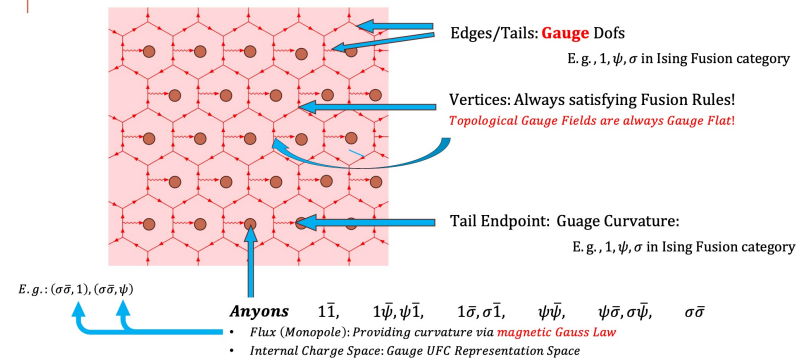
Electron-electron interaction+

flux=composite particle:

charge, flux quanta, statistics (Laughlin)



ENLARGED HU-GEER-WU STRING-NET MODEL ArXiv: 2506.05319



New way of anyon

Yi-Dun Wan

Question: Lattices allow the condensation of topological anyons?

- **Non-Abelian Hall effect (J E Bourguine)**

Quasi hole as anyon

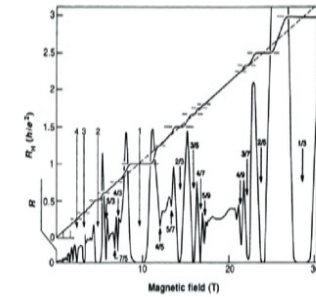
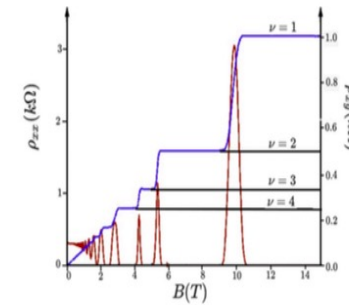
$$\psi(\zeta_i, \mathbf{x}) = \prod_{i < j} (\zeta_i - \zeta_j)^{1/k} \times \prod_i \prod_a (\zeta_i - x_a) \times \prod_{a < b} (x_a - x_b)^k e^{-\frac{eB}{4\hbar} \sum_a |x_a|^2}$$

Abelian FQHE: U(1) Chern-Simons level k ($\nu = 1/k$)

Non-Abelian FQHE: U(1) Chern-Simons level k ($\nu = p/(k + p)$)

New quantum model for non-Abelian FQHE: **High spin & A_2 Calogero model, QHE matrix model**

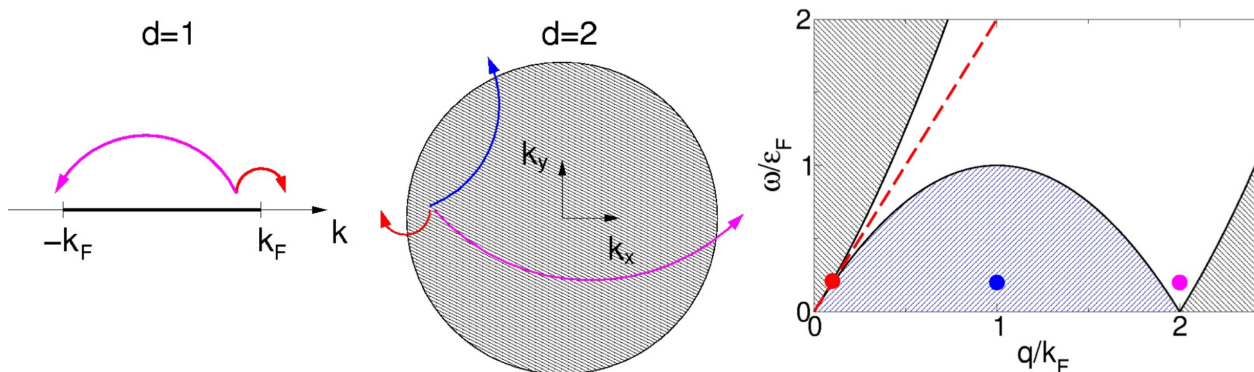
Plateaux are labeled by the **filling factor ν** , it takes either integer or fractional values.



Integer QHE

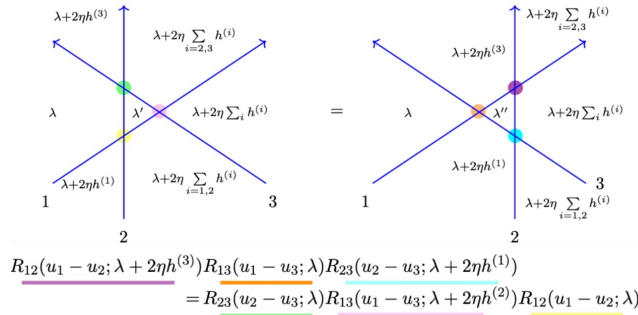
Fractional QHE

- **Tomonaga-Luttinger liquid theory:** old topic but needs to reconfirm (Volker Meden)



Open question: Spectral Function with momentum dependent potential to be confirmed!

Dynamical Yang-Baxter equation (X.-Y. Zhang)



$$R_{12}(u_1 - u_2; \lambda + 2\eta h^{(3)}) R_{13}(u_1 - u_3; \lambda) R_{23}(u_2 - u_3; \lambda + 2\eta h^{(1)}) = R_{23}(u_2 - u_3; \lambda) R_{13}(u_1 - u_3; \lambda + 2\eta h^{(2)}) R_{12}(u_1 - u_2; \lambda)$$

The R-matrix of elliptic models, depending on the choice of basis, may obey a dynamical YBE, while that of rational/trigonometric models always obey the usual YBE.

Expectation: the spectral problem of $\mathcal{N} = 2$ SCFTs $\iff$ dynamical elliptic models.

RG from IR to UV (C Ahn)

$T\bar{T}$ -integrability [Smirnov-Zamolodchikov (2017)]

- Define an IQFT, with a known S -matrix $S_0(\theta)$, by

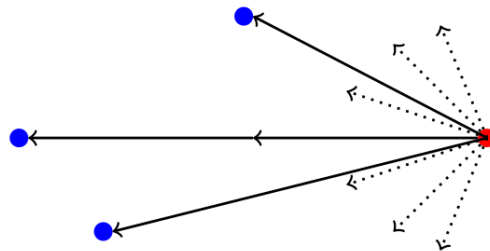
$$\text{IQFT} = \text{CFT} + \lambda \Phi_{\text{rel}}$$

- Deformations of this IQFT by $T\bar{T}$ fields preserve quantum integrability

$$\text{New IQFTs} = \text{IQFT} + \sum_{s=1}^{\infty} \alpha_s [T\bar{T}]_s$$

- Particle spectrum remain identical
- The S -matrix gets a CDD (scalar) correction

$$S(\theta) = S_{\text{cdd}}(\theta) S_0(\theta), \quad S_{\text{cdd}}(\theta) = \prod_{s=1}^{\infty} e^{2i\alpha_s M^{2s} \sinh(s\theta)}$$



Majorana Yang-Baxter Equation

H. Katsura

Sachdev-Ye-Kitaev model

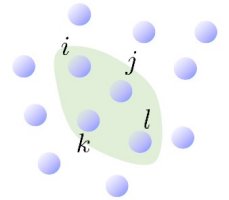
$$H_{\text{SYK}} = \sum_{1 \leq i < j < k < l \leq N} J_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_l$$

$$H_4 = \sum_{k, \ell=1}^{N/2} J_{k\ell} \sigma_k \sigma_\ell + \text{const.} \quad \text{Ising variable}$$

Complex SYK model

$$H_{\text{c4}} = \sum_{1 \leq j < i \leq N} \sum_{1 \leq k < l \leq N} c_i^\dagger c_j^\dagger c_k c_l$$

Pairing models



The main theme also includes: **CFT, QFT and string theory**

(R. Nepomechie, N. Andrei, E. Vemier, V. Meden, B. Chen, J.-B. Wu, M. Yamazaki, W. Gu, J.E. Bourguine, D. Ridout, Y.-D. Wan, X.Y. Zhang)

- Traditional research on integrability in higher dimensional gauge theories
- TTbar deformation and 4D nonlinear electrodynamics
- Aspects Light-Cone Quantization
- Integrable Structures Emerging from Compact BF Theory
- Matrix model descriptions of the quantum Hall effect
- Aspects of discrete light-cone quantizations and 4D Chern-Simons theory
- Bosonic ghost correlators: a case study
- Symmetry-resolved entanglement in 1+1D conformal field theories with non-invertible symmetries
- Exact wave function for relativistic Toda chains from 5d SQFTs
- Hidden symmetry of 4F gauge theory
- ...

3. Quantum entanglementm, Integrable models out of equilibrium and other related topics

M. Oshikawa, M. Kormos, J. Jacobsen, J.-D. Wu, C. Matsui, T. Quella

Drude weight: Kohn formula and bulk

$$\sigma'_s(k=0, \omega) = 2\pi D_s \delta(\omega) + \sigma_s^{\text{reg}}(\omega)$$

$$\lim_{t \rightarrow \infty} \frac{1}{t} \int_0^t dt' \langle J(t') J(0) \rangle$$

Drude weight can be obtained from real-time equilibrium current-current correlation function

Linear & nonlinear Drude weight from Kohn formula

$$D^{\alpha\beta} = \frac{L}{2} \left\langle \frac{\partial^2 E}{\partial \phi^\alpha \partial \phi^\beta} \right\rangle \Big|_{\phi^\alpha, \phi^\beta=0}$$

$$D^{(l)} = \frac{L^l}{2Z} \sum_i e^{-\beta E_i} \frac{\partial^{l+1} E_i}{\partial \phi^{l+1}} \Big|_{\phi_{c,s}=0}$$

Oshikawa instantaneous & Kohn formulas

f-sum rules (instantaneous response = ω -integral)

$$\frac{\sigma^{(n)}(0, 0, \dots, 0)}{2^n} = \langle \Psi_0 | \frac{\partial^{n+1} \mathcal{H}(\mathcal{A}_x)}{\partial \mathcal{A}_x^{n+1}} \Big|_{\mathcal{A}_x=0} | \Psi_0 \rangle$$

energy gain by **sudden** flux insertion

“Kohn formulas” for non-linear Drude weights
(long-time response = $1/\omega$ pole)

$$D^{(n)} = \frac{1}{V} \frac{\partial^{n+1} \mathcal{E}_0}{\partial \mathcal{A}_x^{n+1}}(\mathcal{A}_x) \Big|_{\mathcal{A}_x=0}$$

energy gain by **adiabatic** flux insertion

Bulk Drude weight and conductivities

$$I_{(N)}^{L,\theta}(\Omega) \equiv \frac{1}{\pi^N} \int_{-\Omega}^{\Omega} \frac{d\omega_1}{2\pi} \cdots \int_{-\Omega}^{\Omega} \frac{d\omega_N}{2\pi} \sigma_{(N)}^{L,\theta}(\omega_1, \dots, \omega_N)$$

$$\mathcal{D}_{(N)}^{\text{bulk}} \equiv \lim_{\Omega \rightarrow +0} \lim_{L \rightarrow \infty} I_{(N)}^{L,\theta}(\Omega),$$

Symmetry-Protected Topological Scar Subspaces



Chihiro Matsui

“Challenges in Integrability” Workshop
@Wuhan, Nov 9–21

Chihiro Matsui (The University of Tokyo)

Entanglement scaling in 1+1D CFT

T Quella

Symmetry and entanglement

From bipartition to symmetry resolved entanglement entropy

Renyi and von Neumann entropy

$$S_n = \frac{1}{1-n} \log \text{tr}_A(\rho_A^n)$$

$$S_{EE} = -\text{tr}_A(\rho_A \log \rho_A) = \lim_{n \rightarrow 1} S^{(n)}$$

projectors

$$\mathcal{Z}_n(r) = \text{tr} \left[\Pi_r \rho_A^n \right]$$

$$S_n(r) = \frac{1}{1-n} \log \frac{\mathcal{Z}_n(r)}{\mathcal{Z}_1(r)^n}$$

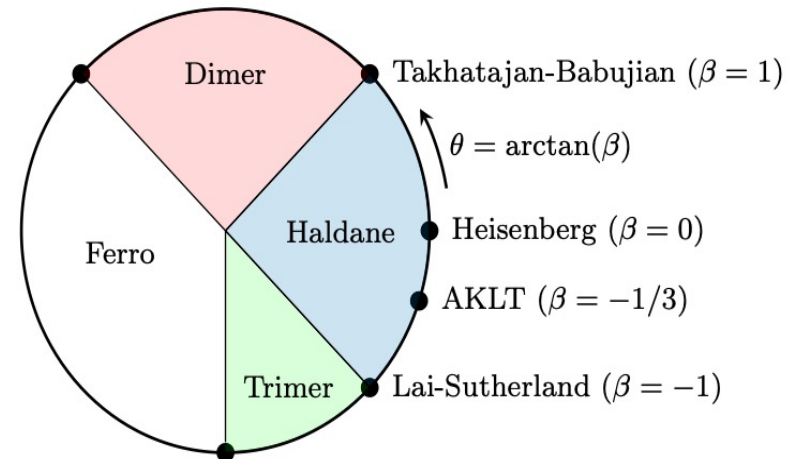
Spin-1 Affleck-Kennedy-Lieb-Tasaki model

$$H = J \sum_j \left(\vec{S}_j \cdot \vec{S}_{j+1} + \frac{1}{3} (\vec{S}_j \cdot \vec{S}_{j+1})^2 \right)$$

1D Spin-1 model

$$H = J \sum_{i=1}^N [S_i \cdot S_{i+1} - \beta (S_i \cdot S_{i+1})^2]$$

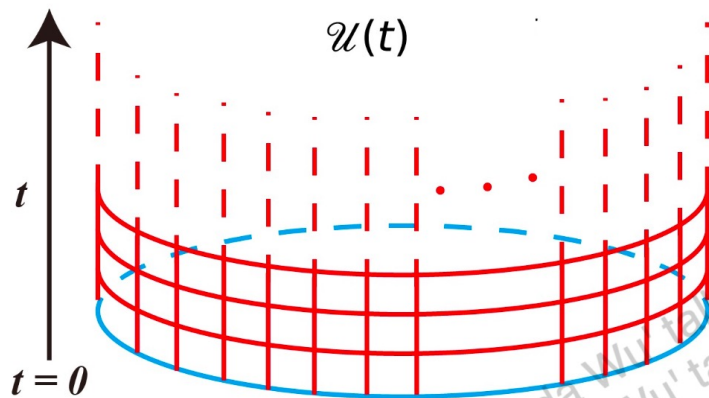
Scar states in 1D spin-1 model



Quantum geometry from integrable quantum field theory: a good start in new application

J. D Wu

Quenching from finite size QIFT



Berry connection

The initial state $|\psi_n\rangle_{\text{ini},L} = |A_{a_1}(I_1) \dots A_{a_N}(I_N)\rangle_L$
 $|\phi_n(t)\rangle_L = U_a(t)|\phi_n\rangle_{\text{ini},L}$

$$H_{\text{adia}} = iU_a^{-1}\dot{U}_a, \quad H_{\text{adia}} = \sum_i^{N_O} \int_0^L dx f_i(t) O_i(x)$$

The Berry connection matrix element:

$$\begin{aligned} \gamma_{nm} &= i\langle\phi_n(t)|\partial_t|\phi_m(t)\rangle \\ &= \langle\phi_n(0)|iU_a^{-1}\dot{U}_a|\phi_m(0)\rangle = \langle\phi_n(t)|H_{\text{adia}}|\phi_m(t)\rangle \end{aligned}$$

$$\begin{aligned} \gamma_{nm} &= L \sum_i f_i(t) {}_L\langle\phi_n|O_i|\phi_m\rangle_L \\ &= L \sum_i f_i(t) {}_L\langle A_1(I'_1) \dots A_N(I'_N)|O_i|A_1(I_1) \dots A_M(I_M)\rangle_L \end{aligned}$$

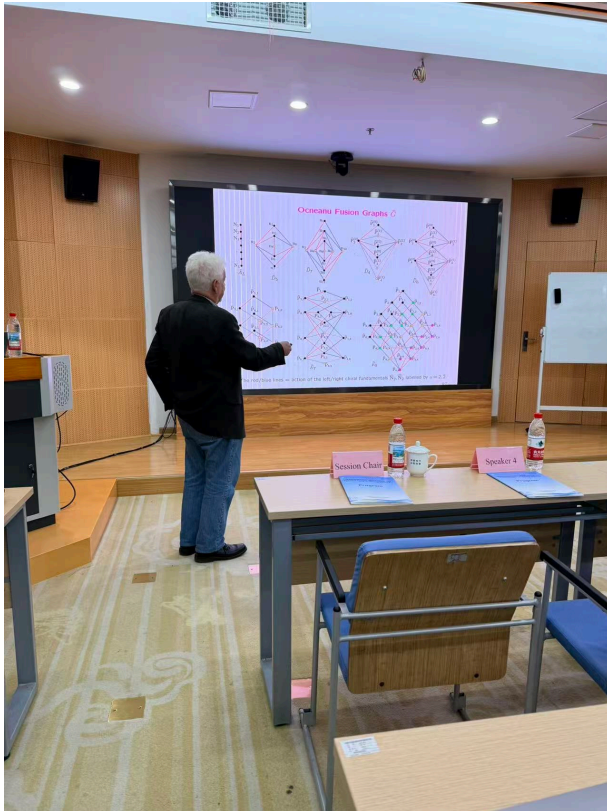
Beyond integrable QFT?

2D lattice models with Ocneanu Algebra

P. Pearce

Critical Unitary A-D-E RSOS Lattice models

Conformal weights, central charges, partition functions, integrable structure,

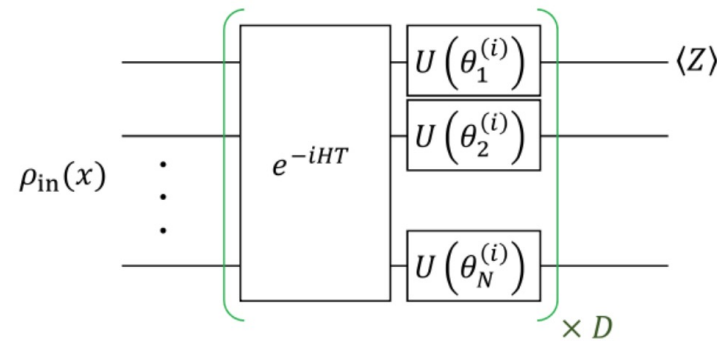


Variational BA: applicable beyond Integrability

R-D Zhu

Variational quantum algorithm

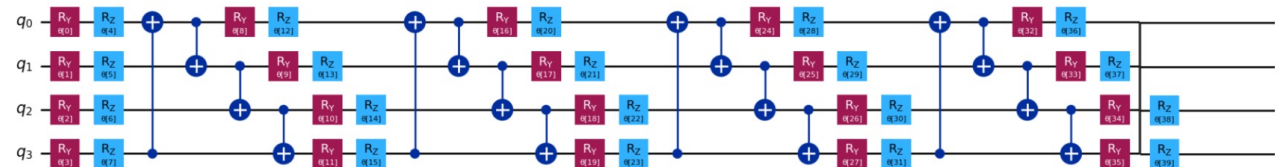
Quantum circuit learning



Simulations

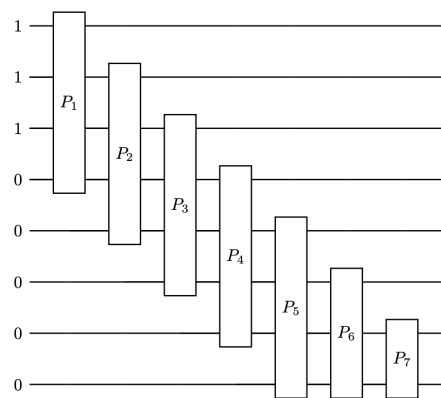
$$H = -J \sum_{i=1}^L Z_i Z_{i+1}$$

$$H = \sum_{i=1}^L \vec{S}_i \cdot \vec{S}_{i+1} + \beta \sum_{i=1}^L (\vec{S}_i \cdot \vec{S}_{i+1})^2$$

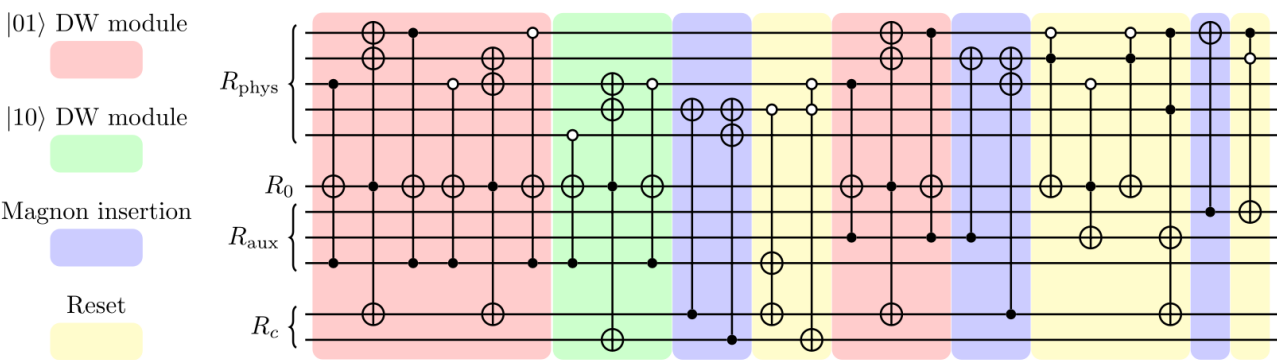


Quantum circuit models of quantum computation: new development of Integrability (Pozsgay)

Algebraic Bethe ansatz gates



Prepare an eigenstate of the XX model



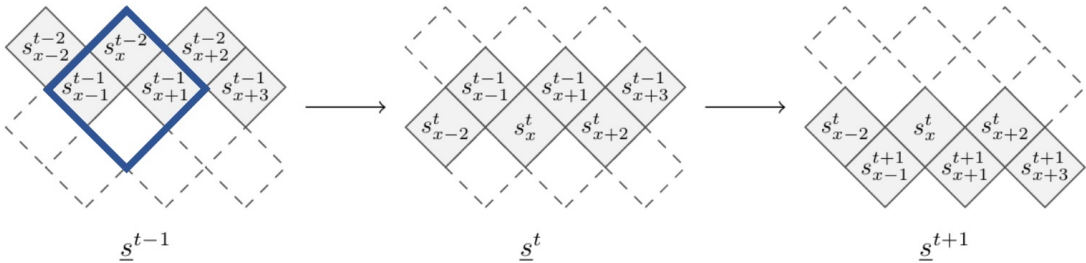
Will the combination of tensor network + quantum transfer matrix + Bethe ansatz be applied in future?

Model: Cellular automata

[Bobenko *et al.* (1993)]

[YB integrability proven by
T. Gombor, B. Pozsgay]

Rule 54 model (Paletta)



Exactly
solvable
model:
Super-
integrable

4. Special Session



Speakers:

H-Q. Lin

M. Batchelor

Y. Shi

H. Zhai

Z.C. Gu

V. Korepin

P. Wiegmann

Gratitude to the speakers for sharing the profound wisdom, scientific attainments and noble virtues of two great masters. Their intellectual brilliance, ideological depth, and noble sentiments are a priceless treasure of humanity.

In-depth communications with the two great scientists

C N Yang, Nobel Prize Laureate, US National Medal of Science, the Franklin Medal, the Onsager Prize, etc. the King Faisal International Prize for Science

Rodney Baxter, Fellow of the Royal Societies of Australia and the United Kingdom, recipient of the Onsager Prize, the Poincaré Prize in Mathematics, the 2013 Royal Medal, etc



With Professor Chen Ning Yang



With professor Rodney Baxter

“True Feelings and Profound Insights Forge a Literary Work”— Fan Zeng



真情妙悟铸文章

巨擘从来诗作魄

太极平衡律是纲

天衣剪掇丛无缝

迁流物类总成场

递变时空皆有数

宇宙浑茫即大荒

纷繁造化赋玄黄

Professors C N Yang and Rodney Baxter's profound wisdom, the beauty of their scientific pursuits, and their noble virtues as mentors will endure forever.



**Thank you for your participation and persistence!
Welcome to Wuhan, China again!**