# Exact solution of the Bose-Hubbard model with unidirectional hopping

Junpeng Cao

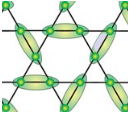
Institute of Physics, Chinese Academy of Sciences

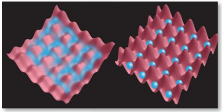
Collaborators: Mingchen Zheng, Yi Qiao, Yupeng Wang, Shu Chen

# Contents

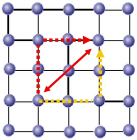
- Research object: one-dimensional quantum many-body systems.

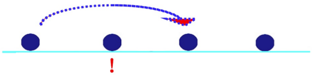
- The strong correlation can induce many interesting phenomena such as fractional elementary excitations, various phases of quantum liquids, Mott physics, collective modes, and critical behaviors.





- The correlation effects in low dimensional systems are more prominent.





• Due to the strong correlation, many traditional methods such as mean field and perturbation are invalid.

• No universal quantum many-body theory. Exact solution is a good method.

$$\begin{array}{ccccc} \text{Actual physical problem} & \Rightarrow & \text{Quantum integrable model} & \Rightarrow & \text{Quantitative results} \\ \Uparrow & & & & \Downarrow \\ \text{Physical mechanism} & \Leftarrow & \text{Experiment} & \Leftarrow & \text{Universal class theory} \end{array}$$

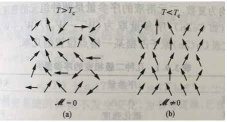
• Integrability: number of degree of freedom = number of conserved quantities

• Exact solution can provide the benchmark for many new phenomena and physical concepts, and check the correction of numerical methods and numerical results.

Examples: 2D Ising model (thermodynamic phase transition), 1D Hubbard model (Mott insulator), Heisenberg model (spinon, fractional charge), Hydrogen atom (quantum mechanics).

• Quantum integrable system is an important branch of condensed matter physics, statistical physics, theoretical and mathematical physics.

Examples:




- 1920, Wilhelm Lenz,

$$H = J\sum_{j=1}^{N}\sigma_j\sigma_{j+1}$$

1. two kinds of states in nature: ordered and disordered.

paramagnetic $\Leftrightarrow$ ferromagnetic states

normal $\Leftrightarrow$ superfluid or superconductive states

2. the transition between two states.

3. 1920, Ising, 1D

4. 1944, Onsager, 2D

5. Open a new field: Ising model has many applications in many fields such as spread of viruses, artificial intelligence, activation and deactivation of brain nerve cells, weather forecast, forest fire, social sciences, finance.

- 1928, Heisenberg model

$$H = J \sum_{j=1}^{N} \vec{\sigma}_j \cdot \vec{\sigma}_{j+1}$$

1. spin exchanging interaction
2. quantum magnetism: AFM, FM
3. 1931, Bethe, coordinate Bethe ansatz
4. $\sim$1980, Faddeev, quantum inverse scattering method
5. spinon, fractional excitations
6. anisotropic couplings, magnetic ordered states, quantum phase transitions

$$H = -\sum_{j=1}^{N} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y + \Delta \sigma_j^z \sigma_{j+1}^z)$$

$|\Delta| < 1$, gapless phase; $|\Delta| > 1$, gapped phase; $|\Delta| = 1$, critical points.
$\Delta = -1$, first order phase transition; $\Delta = 1$, KT phase transition

- Particles with $\delta$-function interaction

$$H = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + c \sum_{j<l} \delta(x_j - x_l)$$

Bosons

E. H. Lieb and W. Liniger, Phys. Rev. 130, 1605 (1963);

E. H. Lieb, Phys. Rev. 130, 1616 (1963);

Fermions

M. Gaudin, Phys. Lett. A 24, 55 (1967);

C. N. Yang, Phys. Rev. Lett. 19, 1312 (1967); **Yang-Baxter equation**

C. N. Yang, Phys. Rev. 168, 1920 (1968); **factorizableof scattering matrix**

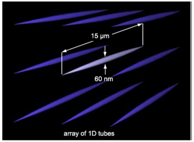
B. Sutherland, Phys. Rev. Lett. 20, 98 (1968).

Bose-fermi mixtures

C. K. Lai, C. N. Yang, Phys. Rev. A 3, 393 (1971);

C. K. Lai, J. Math. Phys. 15, 954 (1974).

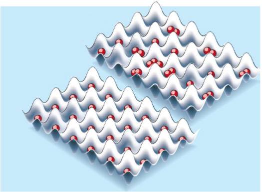
Many applications in the cold atom physics



- Fermi-Hubbard Model: strongly correlated electronic systems

$$H = -t\sum_{j=1}^{N}\sum_{\sigma=\uparrow,\downarrow}(c^{\dagger}_{j,\sigma}c_{j+1,\sigma} + h.c.) + U\sum_{j=1}^{N} n_{j,\uparrow}n_{j,\downarrow}$$

1. 1968, Lieb and Wu, Phys. Rev. Lett. 20, 1445
2. $su(2)\times su(2)/Z_2 = so(4)$ symmetry, particle-hole symmetry, $u(1)$ symmetry
3. $\eta$-pair
4. Without double occupancy: $t-J$ model. $J=\pm 2t$: integrable
5. In the limit of strong coupling, the model reduces to the XXX spin chain
6. Critical Metal-Insulator transition: $U_c = 0$



• Exact solution

Eigenstates

$$|\Psi\rangle = \sum_{j=1}^{M} \sum_{\alpha_j=\uparrow,\downarrow} \sum_{x_j=1}^{N} \Psi^{\{\alpha\}}(x_1, \cdots, x_M) c^\dagger_{x_1,\alpha_1} \cdots c^\dagger_{x_M,\alpha_M} |0\rangle$$

Eigenvalue equation

$$-t \sum_{j=1}^{M} [\Psi^{\{\alpha\}}(\cdots, x_j+1, \cdots) + \Psi^{\{\alpha\}}(\cdots, x_j-1, \cdots)]$$

$$+U \sum_{i<j}^{M} \delta_{x_i,x_j} \Psi^{\{\alpha\}}(x_1, \cdots, x_M) = E\, \Psi^{\{\alpha\}}(x_1, \cdots, x_M)$$

Wave function

$$\Psi^{\{\alpha\}}(x_1, \cdots, x_M) = \sum_{p,q} A_p^{\{\alpha\}}(q) \exp\left[ i \sum_{j=1}^{M} k_{p_j} x_{q_j} \right] \theta(x_{q_1} \le x_{q_2} \le \cdots \le x_{q_M})$$

▶ For all $x_j \neq x_l$ cases, we obtain the eigenvalue

$$E = -2t \sum_{j=1}^{M} \cos k_j$$

▶ For the case of two electrons occupying the same site, from the continuity of wave function

$$A_p(q) + A_{p'}(q) = A_p(q') + A_{p'}(q')$$

and the eigen-equation

$$\begin{aligned} &-t\Big[A_p(q')e^{ik_{p_{j+1}}} + A_{p'}(q')e^{ik_{p_j}} + A_p(q)e^{-ik_{p_j}} + A_{p'}(q)e^{-ik_{p_{j+1}}} \\ &+A_p(q)e^{ik_{p_{j+1}}} + A_{p'}(q)e^{ik_{p_j}} + A_p(q')e^{-ik_{p_j}} + A_{p'}(q')e^{-ik_{p_{j+1}}}\Big] + U[A_p(q) + A_{p'}(q)] \\ &= -2t[\cos k_{p_j} + \cos k_{p_{j+1}}][A_p(q) + A_{p'}(q)] \end{aligned}$$

we obtain the two-body scattering matrix

$$S_{j,l}(k_j, k_l) = \frac{\sin k_j - \sin k_l - i\frac{U}{2t}\mathbf{P_{j,l}}}{\sin k_j - \sin k_l - i\frac{U}{2t}}$$

We consider following scattering process: The 1-st particle moves from left to the right by scattering with all the other particles.

Initial state:

$$\Psi_{in} \sim e^{i\sum_{j=1}^{N} k_j x_j}, \quad x_1 \ll x_2 \ll \cdots \ll x_N$$

Final states:

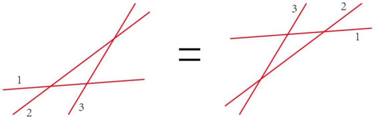
$$\Psi_{out} \sim S_{12\cdots N} e^{i\sum_{j=1}^{N} k_j x_j}, \quad x_2 \ll x_3 \ll \cdots \ll x_N \ll x_1$$

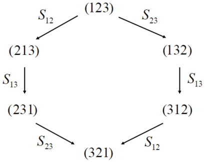
1 Assume that the many-body scattering can be factorized as the product of two-body scattering

$$S_{12\cdots N} = S_{1N} S_{1N-1} \cdots S_{12}$$

2 Yang-Baxter equation

$$S_{12} S_{13} S_{23} = S_{23} S_{13} S_{12}$$



From the periodic boundary condition

$$\Psi(\cdots, x_j + N, \cdots) = \Psi(\cdots, x_j, \cdots)$$

we obtain

$$\Psi_{in} = S_{j,j+1}(k_j, k_{j+1}) \cdots S_{j,M}(k_j, k_M) S_{j,1}(k_j, k_1) \cdots S_{j,j-1}(k_j, k_{j-1}) e^{ik_j N} \Psi_{in}$$

which gives the Bethe ansatz equation to determine the values of quasi-momentum.

$$e^{ik_j N} = \prod_{l \neq j}^{M} S_{jl}^{-1}, \quad j = 1, \cdots, M$$

However, they are the operator equations.

### ▼ Nested Bethe ansatz

The two-body scattering matrix is the one for XXX spin chain. Thus we reduce the eigenvalue problem to that of a inhomogeneous *XXX* spin chain model. The related transfer matrix is

$$S_{j,j+1}(k_j, k_{j+1}) \cdots S_{j,M}(k_j, k_M) S_{j,1}(k_j, k_1) \cdots S_{j,j-1}(k_j, k_{j-1}) = \frac{\tau(\sin k_j)}{\eta \prod_{l \neq j}^{M} (\sin k_j - \sin k_l + \eta)}$$

- Using the algebraic Bethe ansatz, we obtain the Bethe ansatz equations in spin sectior

$$\prod_{j=1}^{M} \frac{\mu_\alpha - \sin k_j - \frac{\eta}{2}}{\mu_\alpha - \sin k_j + \frac{\eta}{2}} = -\prod_{\beta=1}^{\bar{M}} \frac{\mu_\alpha - \mu_\beta - \eta}{\mu_\alpha - \mu_\beta + \eta}, \qquad \alpha = 1, \cdots, \bar{M}$$

Then the BAEs in charge sector can be written as

$$e^{ik_j N} = \prod_{\alpha=1}^{\bar{M}} \frac{\sin k_j - \mu_\alpha - \frac{\eta}{2}}{\sin k_j - \mu_\alpha + \frac{\eta}{2}}, \quad j = 1, \cdots M$$

These equations completely determine the spectrum of the Hamiltonian.

- Thermodynamic Bethe ansatz

Ground state energy, elementary excitation, Mott gap at half filling, free energy at finite temperature

- Bose-Hubbard model

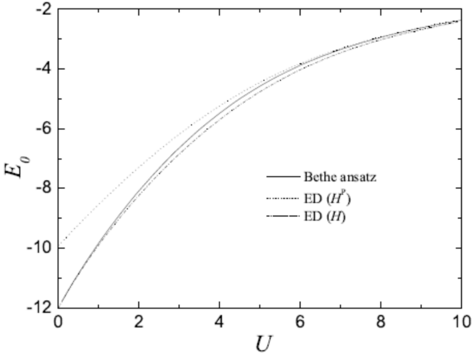
$$H = -t\sum_{j=1}^{N}(c_j^\dagger c_{j+1} + h.c.) + U\sum_{j=1}^{N} n_j(n_j - 1)$$

has many applications in the cold atomic systems.

Continue case: Integrable

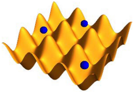
$$H = -\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2} + c\sum_{i<j}^{N}\delta(x_i - x_j)$$

Haldane, PLA 80, 281 (1980);
Haldane, PLA 81, 545 (1981);
Choy, PLA 80, 49 (1980);
Choy, Haldane, PLA 90, 83 (1982).



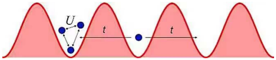
## Mean-field phase diagram

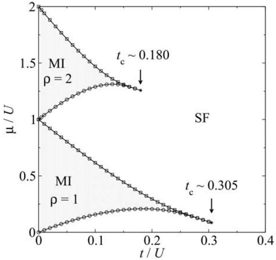
The one dimensional **Bose-Hubbard model** Hamiltonian is



$$H_{BH} = -t\left[\sum_{j=1}^{N}(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j)\right] + \frac{U}{2}\sum_j n_j(n_j-1) - \mu\sum_j n_j$$

$t$ quantifies the hopping strength, and $U$ is the strength of on-site interaction.




There exists a **superfluid-Mott phase transition** at zero temperature.

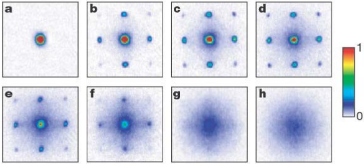
$|\Psi\rangle_{t/U=0} \propto \prod_i |n_i\rangle$

$|\Psi\rangle_{t/U=\infty} \propto \left(\sum_i \hat{b}_i^\dagger\right)^N |0\rangle$

**Mott** insulator phase $\quad t/U = \boldsymbol{t_c} \quad$ **Superfluid** phase

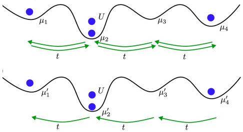
- Integer filling
- Incompressible
- Incoherent
- Gapped

- Compressible
- Coherent
- Gapless




**The Bose-Hubbard model is non-integrable and the integrable bosonic lattice model is rare!**

• Integrable non-Hermitian Bose-Hubbard model with unidirectional hopping




$$H = -t \sum_{j=1}^{N-1} (b_j^\dagger b_{j+1} + \epsilon b_N^\dagger b_1) + \frac{U}{2} \sum_{j=1}^{N} n_j(n_j - 1)$$

Filling: The number of total particles is $M = \sum_{j=1}^{N} n_j$, where $n_j = b_j^\dagger b_j$ is the number of bosons on site $j$.

This model is an unstable non-Hermitian model ($N > 2$) which all particles only hop to one side.

Significantly, when $N = 2$ and $\epsilon = 1$, the model is Hermitian and reduces to the well-know integrable Bose-Hubbard dimer.

## • Integrability

The model is constructed by the spin-boson Lax operator

$$L_{0j}(u) = \begin{pmatrix} u - n_j & gb_j \\ gb_j^\dagger & -g^2 \end{pmatrix}_0, \quad R_{0\bar{0}}(u) = \begin{pmatrix} u-1 & 0 & 0 & 0 \\ 0 & u & -1 & 0 \\ 0 & -1 & u & 0 \\ 0 & 0 & 0 & u-1 \end{pmatrix}$$

where $g$ is a constant. The L operator satisfies the Yang-Baxter equation

$$R_{0\bar{0}}(u-v)\hat{L}_{01}(u)\hat{L}_{\bar{0}2}(v) = \hat{L}_{\bar{0}2}(v)\hat{L}_{01}(u)R_{0\bar{0}}(u-v)$$

Monodromy matrix

$$T_0(u) = L_{01}(u)L_{02}(u)\cdots L_{0N}(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix}_0$$

In order to characterize the twisted boundary coupling, we define a diagonal matrix $K$

$$K_0 = \begin{pmatrix} 1 & 0 \\ 0 & \epsilon \end{pmatrix}_0$$

the transfer matrix is

$$t(u) = tr_0[T_0(u)K_0] = A(u) + \epsilon D(u)$$

$$[t(u), t(v)] = 0$$

$$t(u) = u^N + C_1 u^{N-1} + C_2 u^{N-2} + \cdots,$$

$$C_1 = -\sum_{j=1}^{N} n_j,$$

$$C_2 = \frac{1}{2}\sum_{i\neq j}^{N} n_i n_j + g^2\left(\sum_{j=1}^{N-1} b_j^\dagger b_{j+1} + \epsilon b_N^\dagger b_1\right).$$

the non-Hermitian Hamiltonian

$$H = -\frac{t}{g^2}\left(C_2 - \frac{1}{2}(C_1^2 + C_1)\right)$$

$$= -t\sum_{j=1}^{N-1}(b_j^\dagger b_{j+1} + \epsilon b_N^\dagger b_1) + \frac{t}{2g^2}\sum_{j}^{N} n_j(n_j - 1)$$

- Exact solution: Algebraic Bethe ansatz (Quantum inverse scattering method)

$$[H, t(u)] = 0$$

Diagonalize the transfer matrix instead of Hamiltonian

Reference state

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_1 \otimes \cdots \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}_N$$

$$\begin{aligned} A(u)|0\rangle &= u^N|0\rangle, \\ D(u)|0\rangle &= \epsilon\big(-\frac{t}{U}\big)^N|0\rangle, \\ C(u)|0\rangle &= 0. \end{aligned}$$

Assumed eigen-state

$$|\mu_1, \cdots, \mu_M\rangle = B(\mu_1)\cdots B(\mu_M)|0\rangle$$

Eigen-equation

$$\begin{aligned} t(u)|\mu_1, \cdots, \mu_M\rangle &= [A(u) + D(u)]|\mu_1, \cdots, \mu_M\rangle \\ &= [A(u) + D(u)]B(\mu_1)\cdots B(\mu_M)|0\rangle \end{aligned}$$

From the Yang-Baxter relations

$$R_{0\bar{0}}(u-v)\,\hat{T}_0(u)\,\hat{T}_{\bar{0}}(v) = \hat{T}_{\bar{0}}(v)\,\hat{T}_0(u)R_{0\bar{0}}(u-v)$$

we obtain following commutation relations

$$\begin{aligned}
&[A(\lambda), A(\mu)] = [D(\lambda), D(\mu)] = 0, \\
&A(\lambda)B(\mu) = \frac{\lambda-\mu-1}{\lambda-\mu}B(\mu)A(\lambda) + \frac{1}{\lambda-\mu}B(\lambda)A(\mu), \\
&D(\lambda)B(\mu) = \frac{\lambda-\mu+1}{\lambda-\mu}B(\mu)D(\lambda) - \frac{1}{\lambda-\mu}B(\lambda)D(\mu), \\
&[B(\lambda), B(\mu)] = [C(\lambda), C(\mu)] = 0, \\
&[B(\lambda), C(\mu)] = \frac{1}{\lambda-\mu}(D(\mu)A(\lambda) - D(\lambda)A(\mu)).
\end{aligned}$$

$$t(u)|\mu_1,\cdots\mu_M\rangle = \Lambda(u;\mu_1,\cdots\mu_M)|\mu_1,\cdots,\mu_M\rangle$$
$$+\sum_{j=1}^{M}\Lambda_j(u;\mu_1,\cdots\mu_M)B(\mu_1)\cdots B(\mu_{j-1})B(u)B(\mu_{j+1})\cdots B(\mu_M)|0\rangle.$$

The eigenvalues of the transfer matrix

$$\Lambda(u) = u^N\prod_{j=1}^{M}\frac{u-\mu_j-1}{u-\mu_j} + \epsilon(-g^2)^N\prod_{j=1}^{M}\frac{u-\mu_j+1}{u-\mu_j}$$

The unwanted terms give the Bethe ansatz equations

$$\frac{\mu_j^N}{\epsilon(-g^2)^N} = \prod_{j\neq l}^{M}\frac{\mu_j-\mu_l+1}{\mu_j-\mu_l-1} \qquad\qquad \epsilon\neq 0$$

Expanding $\Lambda(u)$ and comparing with the polynomial of $t(u)$, we get

$$C_1 = -M, \qquad C_2 = -\sum_{j=1}\mu_j - \frac{M(M-1)}{2}$$

So the eigenvalue of Hamiltonian

$$E = -\frac{t}{g^2}(C_2 - \frac{1}{2}(C_1^2 + C_1)) = \frac{t}{g^2}\sum_j \mu_j$$

Define $U = t/g^2$ and $\beta_j = U\mu_j$, the BAEs read

$$\beta_j^N = \epsilon(-t)^N \prod_{l \neq j}^{M} \frac{\beta_j - \beta_l + U}{\beta_j - \beta_l - U} \tag{1}$$

and the eigenvalue of Hamiltonian is

$$E = \sum_{j=1}^{M} \beta_j.$$

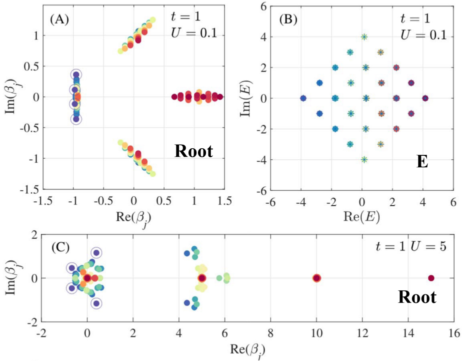
Taking the logarithm of (1), we have

$$N \ln(\frac{\beta_j}{-t}) = i \sum_{l \neq j}^{M} \Theta(\frac{-iU}{\beta_j - \beta_l}) + i2\pi I_j.$$

Here $\Theta(z) \equiv 2\arctan(z)$, and $I_j$ is the quantum number.

In the fermion Bethe ansatz equations, the quantum number $I_j$ of each particle shall not be equal. Here the $\{I_j | j = 1, 2, \ldots, M\}$ could be equal.

- Numerical check




The Bethe roots pattern

The Hamiltonian is non-Hermitian, thus the energies are complex.

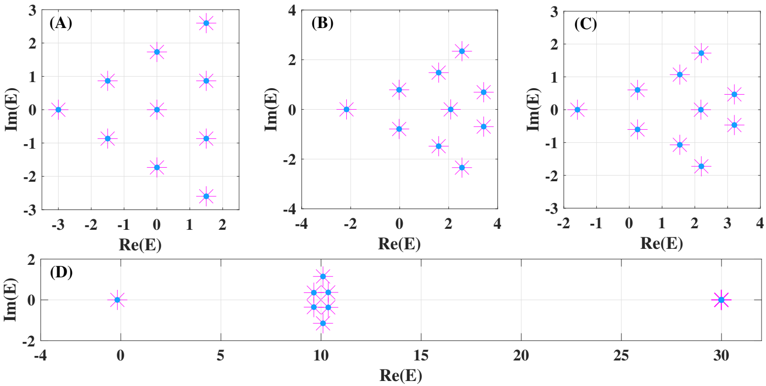



Figure 1: The numerical results of the eigenvalues of NHBH model. The blue points are obtained by directly solving the BAEs (1) and the data marked by purple asterisk are results from exact diagonalization with $U = 0.001$ $\epsilon = 1$ (A), $U = 1$ $\epsilon = 1$ (B), $U = 1$ $\epsilon = 0.5$ (C) and $U = 10$ $\epsilon = 1$ (D). Common parameters: $N = 3$, $M = 3$, $t = 1$.

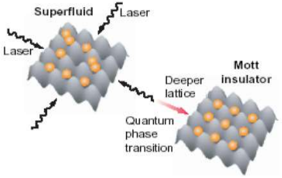
We define the ground state as the eigenstate whose eigenvalue has the lowest real part. To the present model, the ground state energy is real.

• Filling: $N = M$, $\rho = M/N$.

△ If $U$ is small, the system is in the gapless superfluid phase.

△ If $U$ is large, the system is in the gapped Mott-insulator phase. That is every site is occupied by one boson due to the strong repulsive interaction.

△ There is a Mott-superfluid transition for ceratin value of interaction.



• Now, we determine the value of phase transition point $t_c$.

As usual, we define the chemical potential as

$$\mu_+ = E_{M=N+1} - E_{M=N},$$
$$\mu_- = E_{M=N} - E_{M=N-1}.$$

In the thermodynamic limit

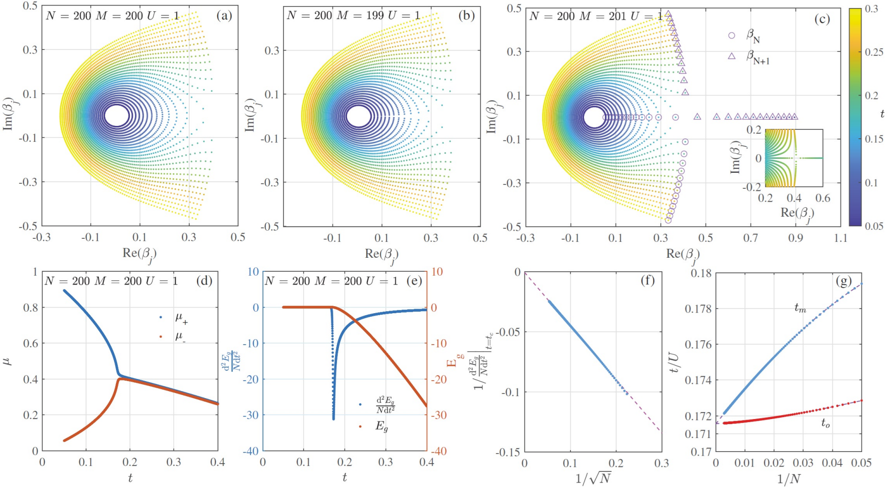
$$\mu_+ = Re(\beta_{N+1}), \qquad \mu_- = Re(\beta_N)$$

1. When $t > t_c$, $\Delta\mu = \mu_+ - \mu_- = Re(\beta_{N+1}) - Re(\beta_N) = 0$.

2. When $t < t_c$, $\Delta\mu = Re(\beta_{N+1}) - Re(\beta_N) > 0$.

3. $t_c$ is the position where the gap opens.

4. The Mott-superfluid phase transition point can be determined from $u/t_c$ in the thermodynamic limit.

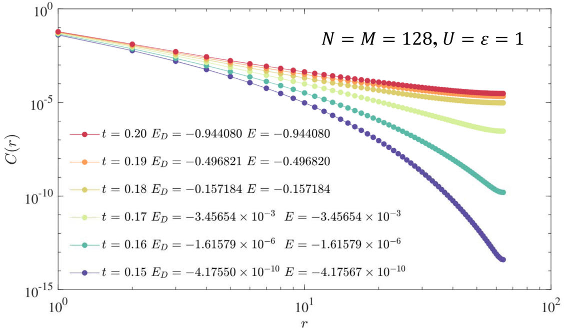



- From a scaling, in the thermodynamic limit, the values of $t_o$ and $t_m$ are the same.

- The value of the superfluid-Mott transition critical point is $t_c = 0.17155 \pm 10^{-5}$.

Check again: Density matrix renormalization group

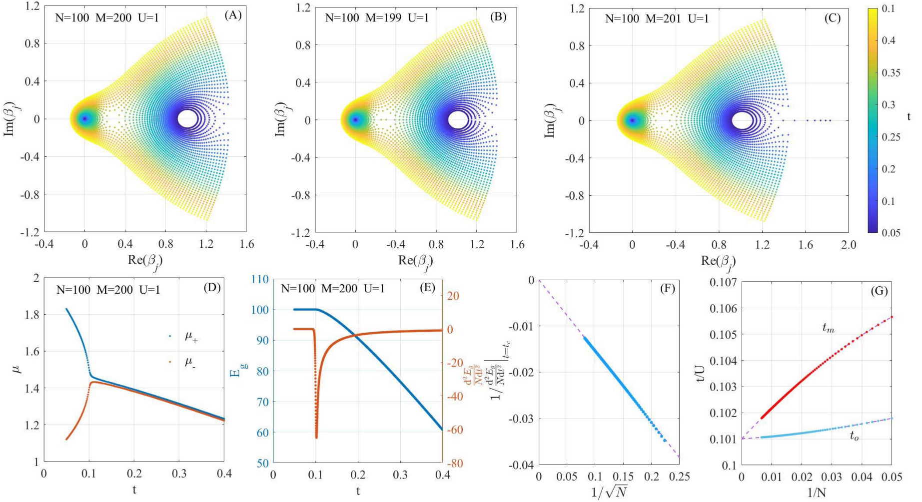
Define the density-density correlation function: $C(r) = \langle n_r n_0 \rangle - \langle n_r \rangle \langle n_0 \rangle$.



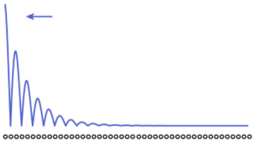

The density-density correlation function by DMRG algorithm.

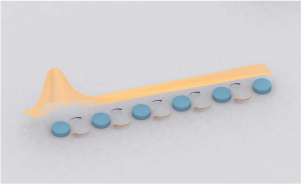
1. Power-law decay in Superfluid phase: $C(r) \sim \frac{1}{r^{\alpha}}$.
2. Exponential-law decay Mott insulator phase: $C(r) \sim e^{-r/\xi}$.
3. On either side of $t = 0.17$, the decay behavior of the density-density correlation function is completely different.

- Superfluid-Mott like phase transition with $2N = M$.



**Non-Hermitian Skin effect (NHSE)** refers to the localization of eigenstates at the boundary in a non-Hermitian system.



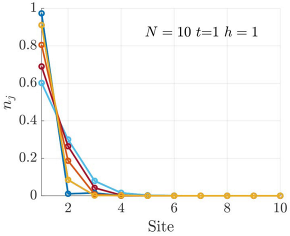


Considering a non-reciprocal free particle lattice model with open boundary condition

$$H = t\sum_{j=1}^{N-1}\left(e^{h}b_j^{\dagger}b_{j+1} + e^{-h}b_{j+1}^{\dagger}b_j\right)$$

where $h$ is a real parameter that describes the non-Hermiticity.

**NHSE is sensitive to boundary condition**




**Many-body non-Hermitian physics study is constrained by numerical methods!**

- The non-Hermitian skin effect for the interacting system

The wave function is

$$\Psi = \sum_{x_1,x_2,\ldots,x_M=1}^{N} \psi(x_1,\ldots,x_M) b^\dagger_{x_1} b^\dagger_{x_2} \ldots b^\dagger_{x_M} |0\rangle$$

$$\psi(x_1,\ldots,x_M) = (-1)^M \sum_{p,q} A_p(q) \prod_{j=1}^{M} \beta_{p_j}^{x_{q_j}} \theta(x_{q_1} \leq x_{q_2} \leq \cdots \leq x_{q_M})$$

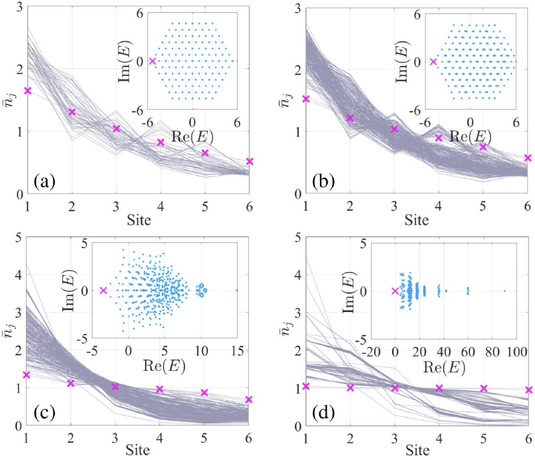
The relation between $A_p$ is

$$\frac{A_{p_1\ldots p_{i+1}p_i\ldots p_M}}{A_{p_1\ldots p_i p_{i+1}\ldots p_M}} = S(k_{p_i}, k_{p_{i+1}})$$

and the scattering matrix of the system is

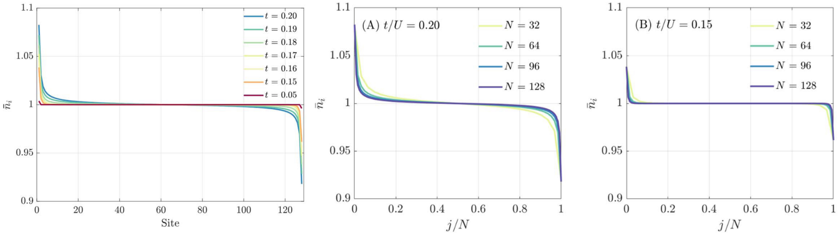
$$S(k_{p_i}, k_{p_{i+1}}) = \frac{\beta_{p_i+1} - \beta_{p_i} + U}{\beta_{p_i+1} - \beta_{p_i} - U}.$$

The solution of BAEs (1) can completely determine the wave function.




1. For the case that $M = N$, there is only one state with eigenvalue close to 0 which is $|1, 1, \ldots, 1\rangle$, so that the NHSE will disappear. The particle number of the ground state is almost evenly distributed.
2. When $N \neq M$, all state will have NHSE when $U \to \infty$.

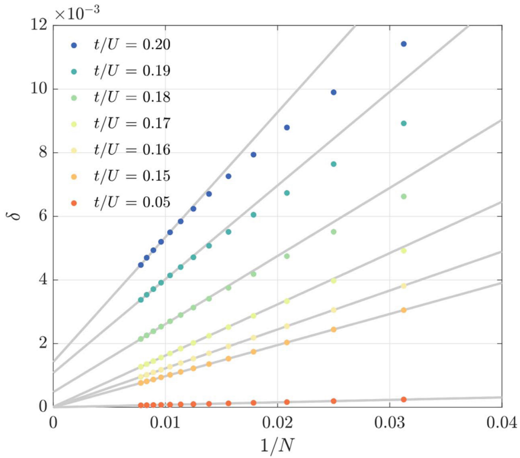
The boson distribution of the ground state from DMRG algorithm with $\rho = 1$.




The boson distribution approaches a uniform distribution with the increasing of interaction.

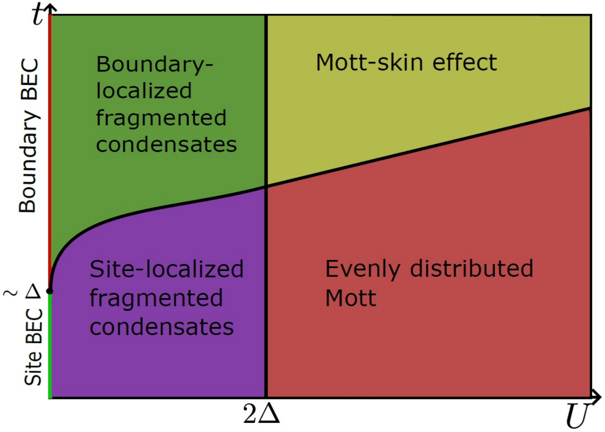
The behavior of the boson distribution varies with increasing size.

Define $\delta = \frac{1}{N}\sum_{j=1}^{N}|n_j - \rho|$ to quantify an average deviation of the uniform distribution.




We see that the skin effect of a Mott state is completely suppressed in the thermodynamical limit!

The story of this model is not finished.




Reference: C. Ekman and E. J. Bergholtz, Liouvillian skin effects and fragmented condensates in an integrable dissipative Bose-Hubbard model, Phys. Rev. Research 6, L032067 (2024).

# Summary

• Construction and diagonalization of a one-dimensional non-Hermitian Bose-Hubbard model with unidirectional hopping.

• We reveal the presence of a superfluid-Mott phase transition in the ground state and determine the critical point.

• We discover the presence of non-Hermitian skin effects even when interactions are present. However, these effects are completely suppressed in the Mott insulator state at the thermodynamic limit.

## Concluding remarks and perspective

• Non-Hermite physics is a hot topic recently. Most results are based on the single particle systems. By using the quantum inverse scattering method, we can construct some non-Hermite integrable models. Then we can study the correlation effects in the non-Hermite systems exactly.

• Besides the quantum inverse scattering, one can also use the fusion technique to construct new quantum integrable spin chain or the strongly correlated electronic model. The Mott insulator to superfluid transition at finite critical interaction is very interesting.

**Thank you for your attention!**