Eigenstate preparation in integrable spin chains: A review

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Wuhan, November 11, 2025

ELTE Eötvös Loránd University, Faculty of Science

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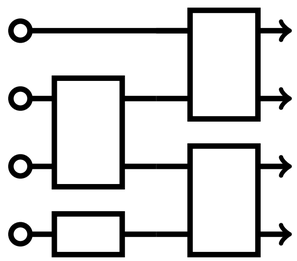

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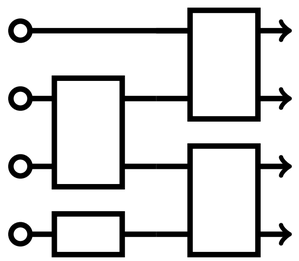

Quantum complexity of a state =
Number of gates needed to prepare it

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Diagonalized via

$\Psi_k = \sum_j C_{j,k} \chi_k$

$H = \sum_j \varepsilon_j \Psi_j^\dagger \Psi_j$

Eigenstate preparation in the XY model

PHYSICAL REVIEW A **79**, 032316 (2009)

Quantum circuits for strongly correlated quantum systems

Frank Verstraete,[1] J. Ignacio Cirac,[2] and José I. Latorre[3]
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[2]*Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Str. 1, D-85748 Garching, Germany*
[3]*Department of ECM, Universitat de Barcelona, Martí i Franquès 1, 08028 Barcelona, Spain*
(Received 2 June 2008; published 16 March 2009)

We present an approach to gain detailed control on the quantum simulation of strongly correlated quantum many-body systems by constructing the explicit finite quantum circuits that diagonalize their dynamics. As a particularly simple instance, the full dynamics of a one-dimensional Quantum Ising model in a transverse field with four spins is shown to be reproduced using a quantum circuit of only six local gates. This opens up the possibility of experimentally producing strongly correlated states, their time evolution at zero time, and even thermal superpositions at zero temperature. Our method also allows one to uncover the exact circuits corresponding to models that exhibit topological order and to stabilizer states.

DOI: 10.1103/PhysRevA.79.032316 PACS number(s): 03.67.Lx, 05.10.Cc, 05.50.+q

I. INTRODUCTION

Recent advances in quantum information have led to novel ways of looking at strongly correlated quantum many-body systems. On the one hand, a great deal of theoretical work has been done identifying the basic structure of entanglement in low-energy states of many-body Hamiltonians. This has led, for example, to new interpretations of renormalization-group ideas in terms of variational methods in classes of quantum states with some very special local dynamics using a finite set of quantum gates. There is no approximation in our circuit and its computational complexity is explicitly polynomial.

Our work can also be interpreted in terms of an extension of the renormalization-group ideas [7]. There, one is interested in obtaining a simple effective Hamiltonian which describes the low-energy physics of a given problem. This is done by a series of transformations that involve getting rid of high-energy modes. In our case, we find a unitary transformation which takes the whole Hamiltonian into a simple

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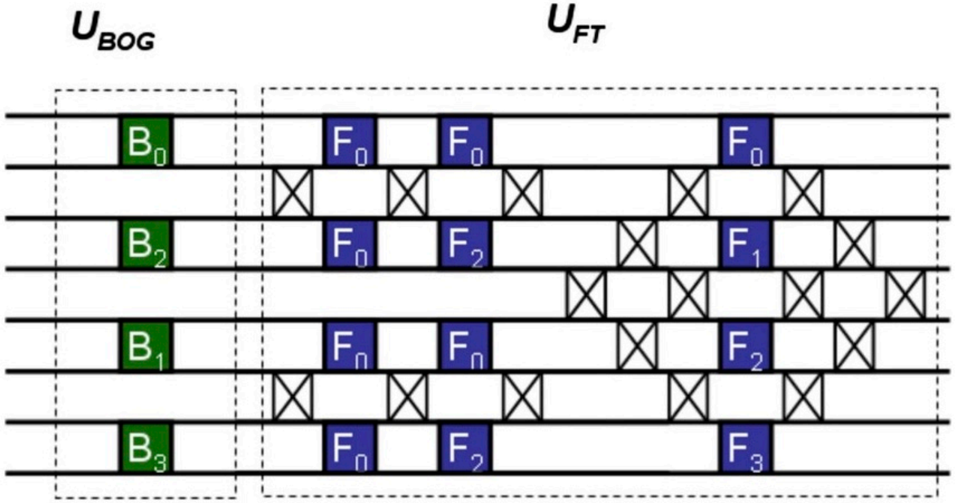

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Matchgates: exponentials of bilinears in the Majoranas!

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Bilinears $\Gamma_{jk} = i[\chi_j, \chi_k]/2$ form the algebra $so(2L)$

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Alternatively:

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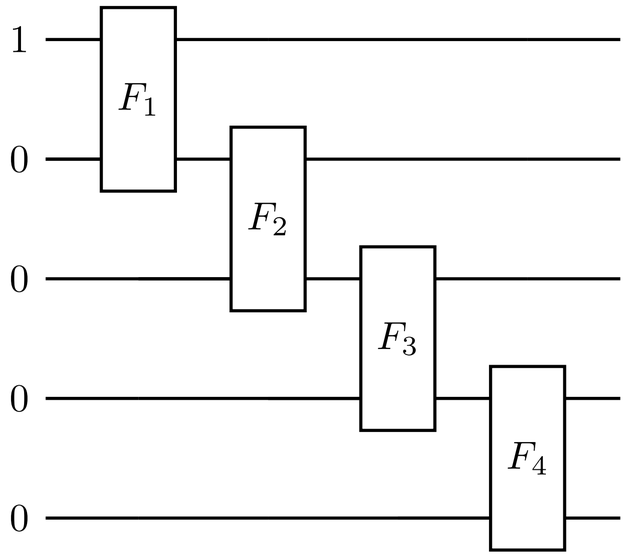

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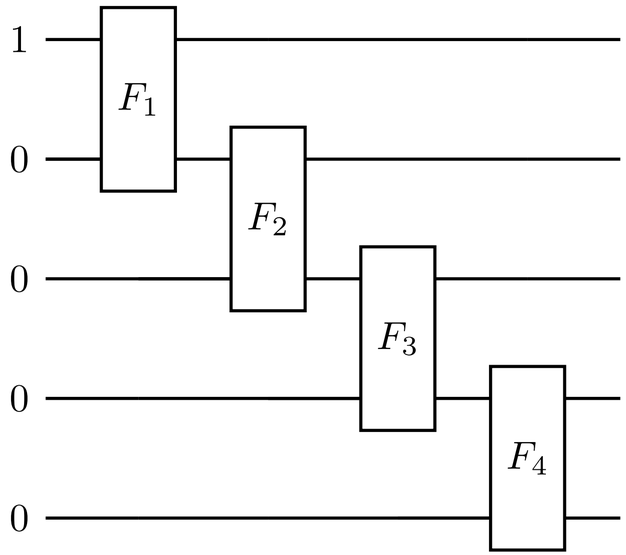

$$c_j = \frac{1}{\sqrt{L-j+1}}, \qquad d_j = e^{ip}\sqrt{\frac{L-j}{L-j+1}}$$

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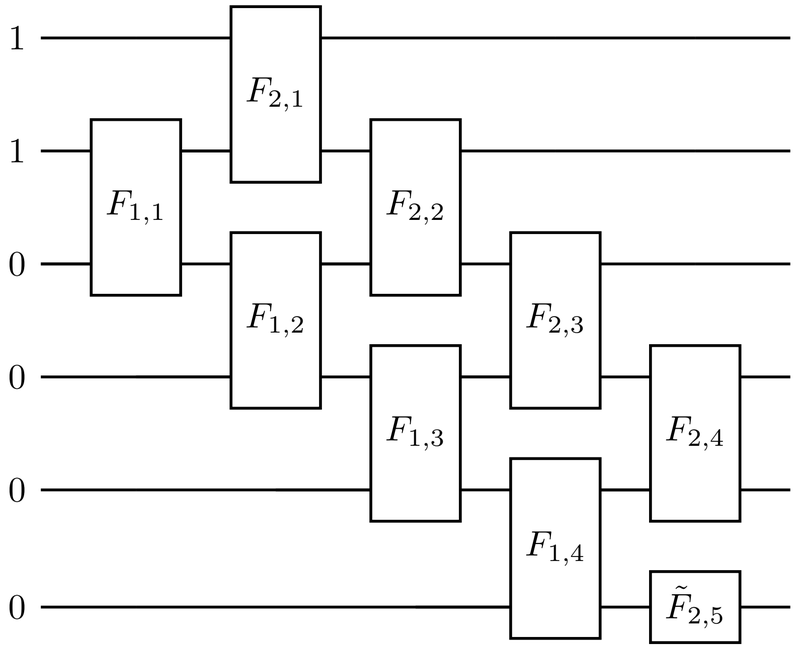

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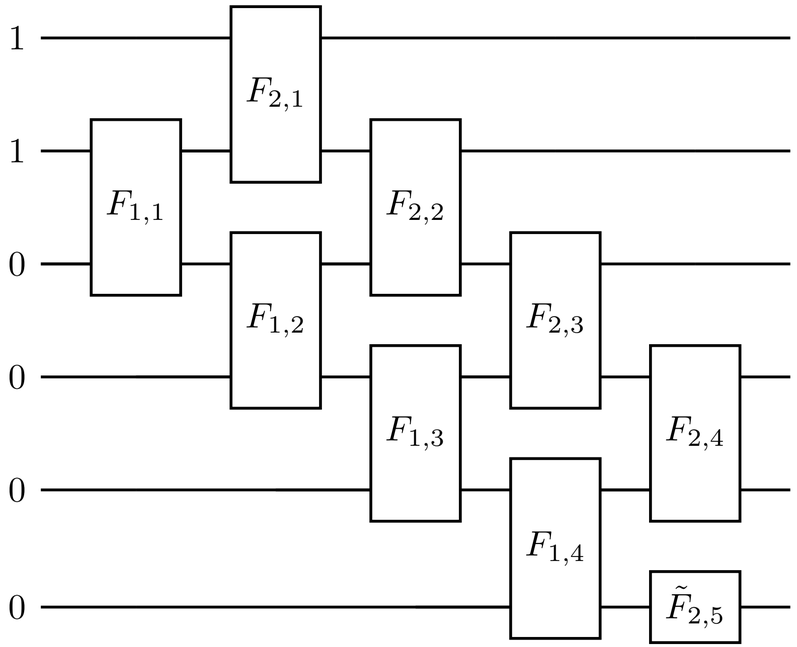

Sopena, M. H. Gordon, D. García-Martín, G. Sierra, E. López (2022)

R. Ruiz, A. Sopena, M. H. Gordon, G. Sierra, E. López (2023)

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Probabilistic method

Preparing Bethe Ansatz Eigenstates on a Quantum Computer

John S. Van Dyke,[1] George S. Barron,[1] Nicholas J. Mayhall,[2] Edwin Barnes,[1] and Sophia E. Economou[1]

[1] *Department of Physics, Virginia Tech, Blacksburg, VA 24061*
[2] *Department of Chemistry, Virginia Tech, Blacksburg, VA 24061*

(Dated: November 25, 2021)

Several quantum many-body models in one dimension possess exact solutions via the Bethe ansatz method, which has been highly successful for understanding their behavior. Nevertheless, there remain physical properties of such models for which analytic results are unavailable, and which are also not well-described by approximate numerical methods. Preparing Bethe ansatz eigenstates directly on a quantum computer would allow straightforward extraction of these quantities via measurement. We present a quantum algorithm for preparing Bethe ansatz eigenstates of the spin-1/2 XXZ spin chain that correspond to real-valued solutions of the Bethe equations. The algorithm is polynomial in the number of T gates and circuit depth, with modest constant prefactors. Although the algorithm is probabilistic, with a success rate that decreases with increasing eigenstate energy, we employ amplitude amplification to boost the success probability. The resource requirements for our approach are lower than other state-of-the-art quantum simulation algorithms for small error-corrected devices, and thus may offer an alternative and computationally less-demanding demonstration of quantum advantage for physically relevant problems.

Quantum computers hold the promise of transformative applications in a variety of fields including cryptanalysis [1], quantum chemistry [2, 3], materials science [4, 5], and potentially combinatorial optimization [6, 7]. To realize the full potential of quantum computing, large-scale, fault-tolerant devices will ultimately be necessary. As these do not yet exist, much recent work has studied possible near-term applications in the present era of noisy, intermediate-scale quantum computers (NISQ) [8, 9]. In quantum computer as a computationally less-demanding route to the demonstration of quantum advantage for problems relevant to physics, including quantum magnetism [19, 20], ultracold atoms [21], and unconventional superconductivity [22]. BA methods allow for deep insight into the static and dynamic properties of these many-body systems, and are able to explore not only ground states, but also interactions between complex collective excitations, such as magnons, and the response to

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Algorithm 1 Bethe state preparation

1: Prepare the Dicke state $|D_{L,M}\rangle$ on the system qubits
2: Create permutation labels while applying pieces of A_P
3: Apply $e^{ik_{P_j}x_j}$ using the "faucet" method
4: Reverse permutation label (without phases)
5: Measure permutation label, with success on $|00\cdots0\rangle$

Deterministic method —
without integrability

Deterministic Bethe state preparation

David Raveh and Rafael I. Nepomechie

Department of Physics, PO Box 248046, University of Miami, Coral Gables, FL 33124 USA

We present an explicit quantum circuit that prepares an arbitrary $U(1)$-eigenstate on a quantum computer, including the exact eigenstates of the spin-1/2 XXZ quantum spin chain with either open or closed boundary conditions. The algorithm is deterministic, does not require ancillary qubits, and does not require QR decompositions. The circuit prepares such an L-qubit state with M down-spins using $\binom{L}{M}-1$ multi-controlled rotation gates and $2M(L-M)$ CNOT-gates.

1 Introduction

It is widely believed that the simulation of quantum physics is one of the most promising applications of quantum computers, see e.g. [1, 2]. Among the potential target quantum systems, one-dimensional quantum spin chains are attractive candidates. Indeed, these are many-body quantum systems that appear in a myriad of contexts in fields such as physics (condensed matter [3], statistical mechanics [4, 5], high-energy theory [6]), chemistry [7] and computer science [8]. A subset of those models are quantum integrable, and thus their exact energy eigenstates ("Bethe states") and eigenvalues can be expressed in terms of solutions ("Bethe roots") of so-called Bethe equations. These results can be derived by either the coordinate [9–12] or algebraic [13–15] Bethe ansatz. Given the Bethe roots (say, of the ground state), it would be desirable to prepare the corresponding Bethe state on a quantum computer [16], which would then allow the computation of correlation functions in this state, see e.g. [17, 18].

After [19], an algorithm for preparing Bethe states of the spin-1/2 closed XXZ chain based on the coordinate Bethe ansatz was formulated [20]. This algorithm, which was later generalized to the open chain [21], is restricted to real Bethe roots, requires ancillary qubits, and is probabilistic. Moreover, the success probability was shown to tend super-exponentially to zero with the number of down-spins [22]. A different approach, based instead on the algebraic Bethe ansatz, was subsequently developed for the closed XXZ chain [23]. The latter algorithm is not limited to real Bethe roots, and is deterministic; however, it is not explicit,

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Rafael I. Nepomechie: nepomechie@miami.edu

Deterministic method —
without integrability

Afterwards:

- Yu Li, Guojing Tian, Xiaoyu He, and Xiaoming Sun, arXiv:2508.14470
- Jingquan Luo and Lvzhou Li, arXiv:2508.17197

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David Raveh and Rafael I. Nepomechie

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We present an explicit quantum circuit that prepares an arbitrary $U(1)$-eigenstate on a quantum computer, including the exact eigenstates of the spin-1/2 XXZ quantum spin chain with either open or closed boundary conditions. The algorithm is deterministic, does not require ancillary qubits, and does not require QR decompositions. The circuit prepares such an L-qubit state with M down-spins using $\binom{L}{M}-1$ multi-controlled rotation gates and $2M(L-M)$ CNOT-gates.

1 Introduction

It is widely believed that the simulation of quantum physics is one of the most promising applications of quantum computers, see e.g. [1, 2]. Among the potential target quantum systems, one-dimensional quantum spin chains are attractive candidates. Indeed, these are many-body quantum systems that appear in a myriad of contexts in fields such as physics (condensed matter [3], statistical mechanics [4, 5], high-energy theory [6]), chemistry [7] and computer science [8]. A subset of those models are quantum integrable, and thus their exact energy eigenstates ("Bethe states") and eigenvalues can be expressed in terms of solutions ("Bethe roots") of so-called Bethe equations. These results can be derived by either the coordinate [9–12] or algebraic [13–15] Bethe ansatz. Given the Bethe roots (say, of the ground state), it would be desirable to prepare the corresponding Bethe state on a quantum computer [16], which would then allow the computation of correlation functions in this state, see e.g. [17, 18].

After [19], an algorithm for preparing Bethe states of the spin-1/2 closed XXZ chain based on the coordinate Bethe ansatz was formulated [20]. This algorithm, which was later generalized to the open chain [21], is restricted to real Bethe roots, requires ancillary qubits, and is probabilistic. Moreover, the success probability was shown to tend super-exponentially to zero with the number of down-spins [22]. A different approach, based instead on the algebraic Bethe ansatz, was subsequently developed for the closed XXZ chain [23]. The latter algorithm is not limited to real Bethe roots, and is deterministic; however, it is not explicit,

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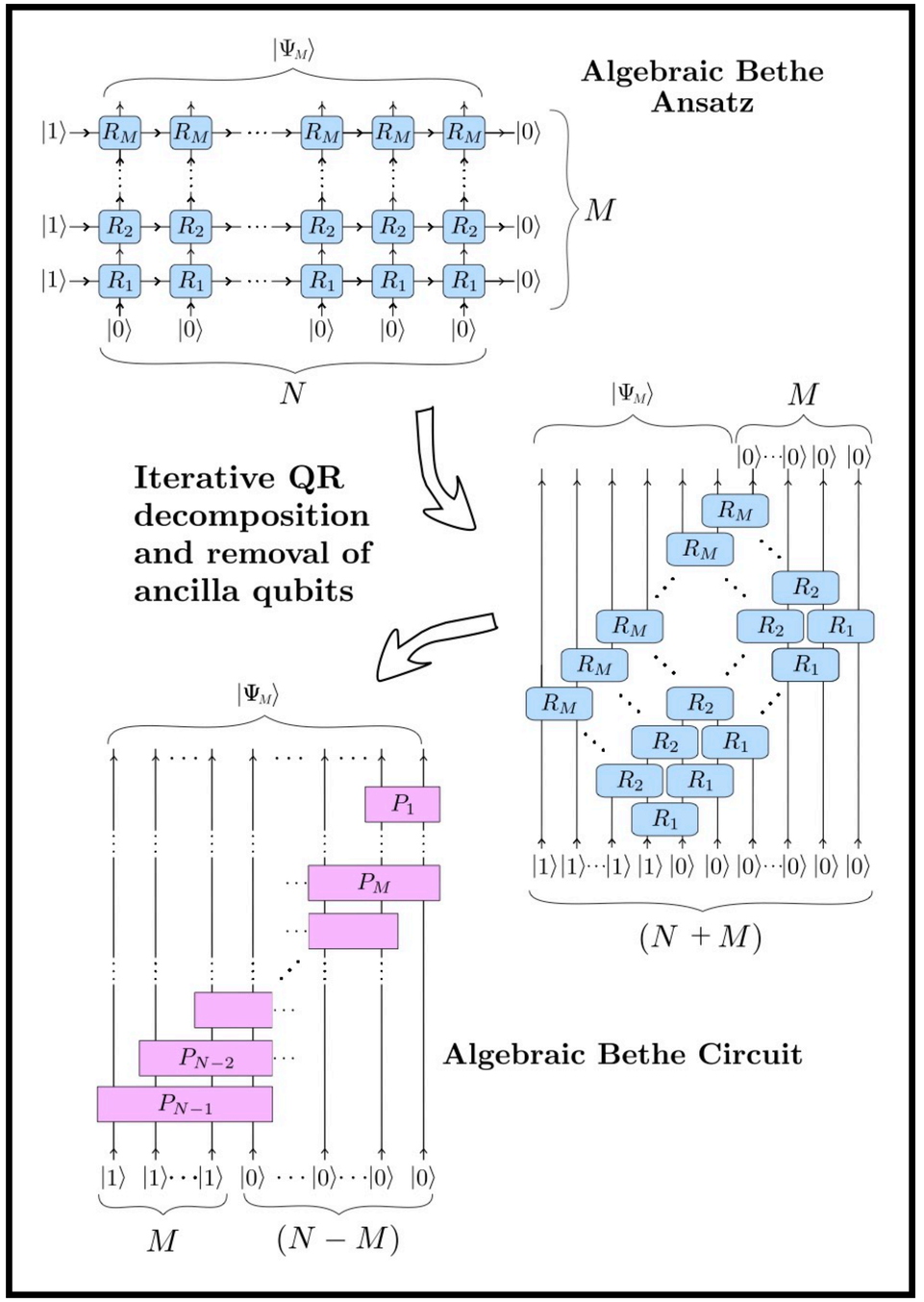
Algebraic Bethe Circuits

- Sopena, M. H. Gordon, D. García-Martín, G. Sierra, E. López, arXiv:2202.04673
- R. Ruiz, A. Sopena, M. H. Gordon, G. Sierra, E. López, arXiv:2309.14430
- R. Ruiz, A. Sopena, E. López, G. Sierra, BP, arXiv:2411.02519



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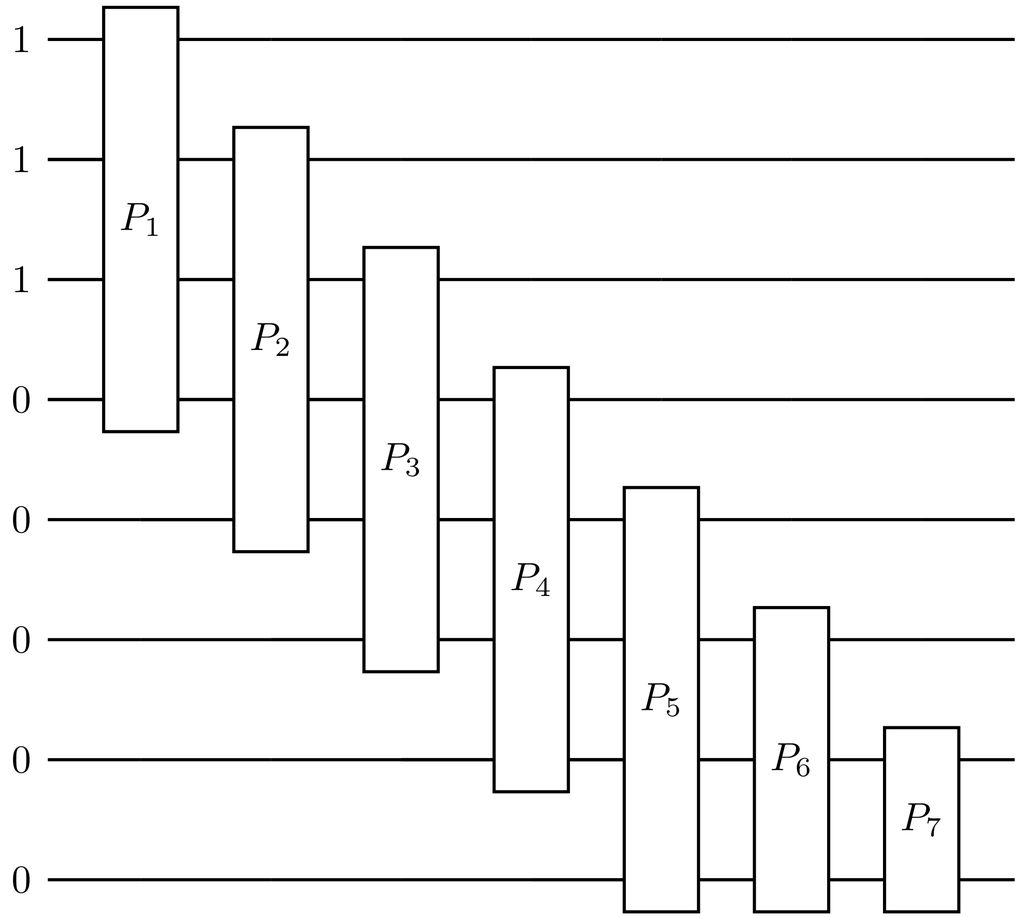

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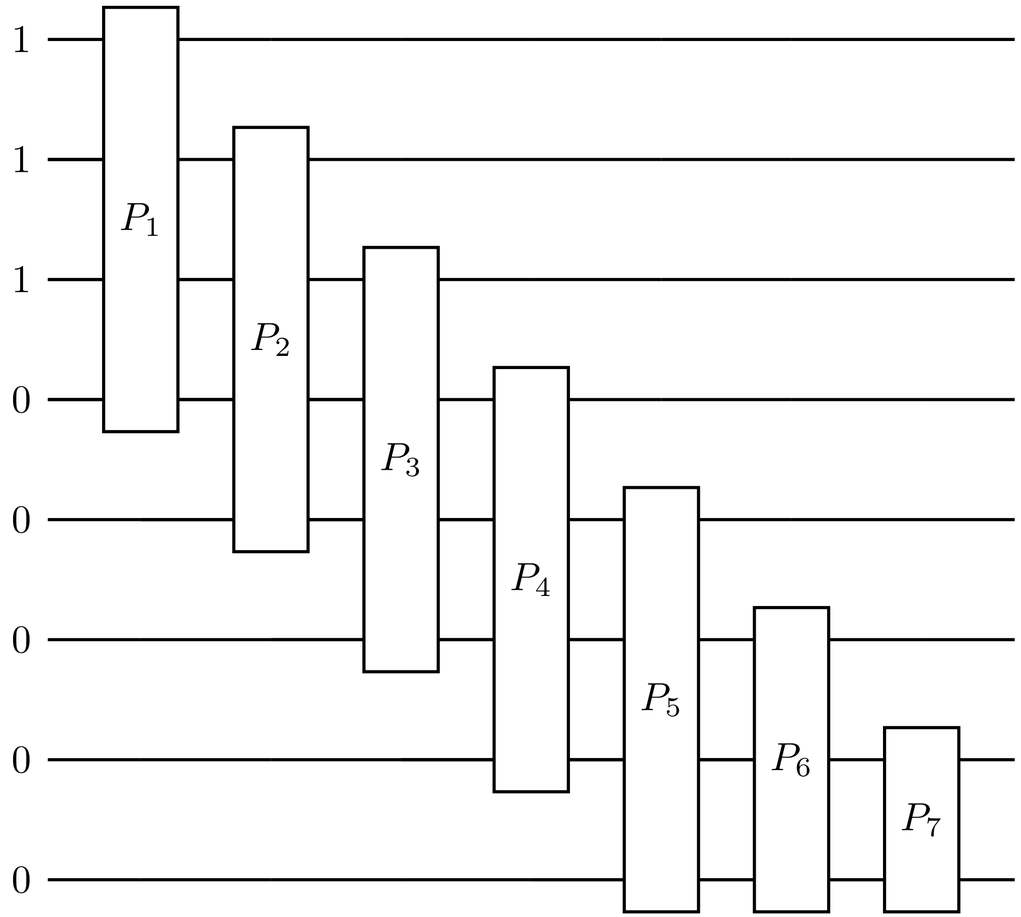

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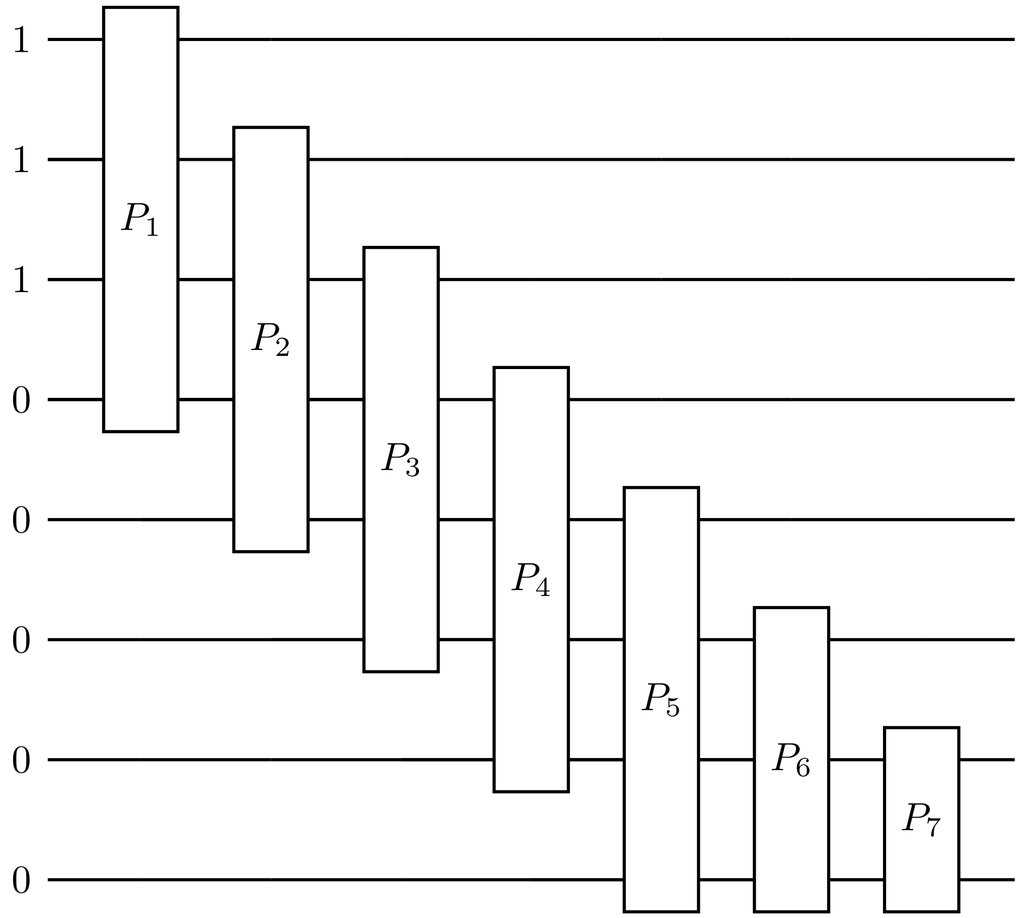

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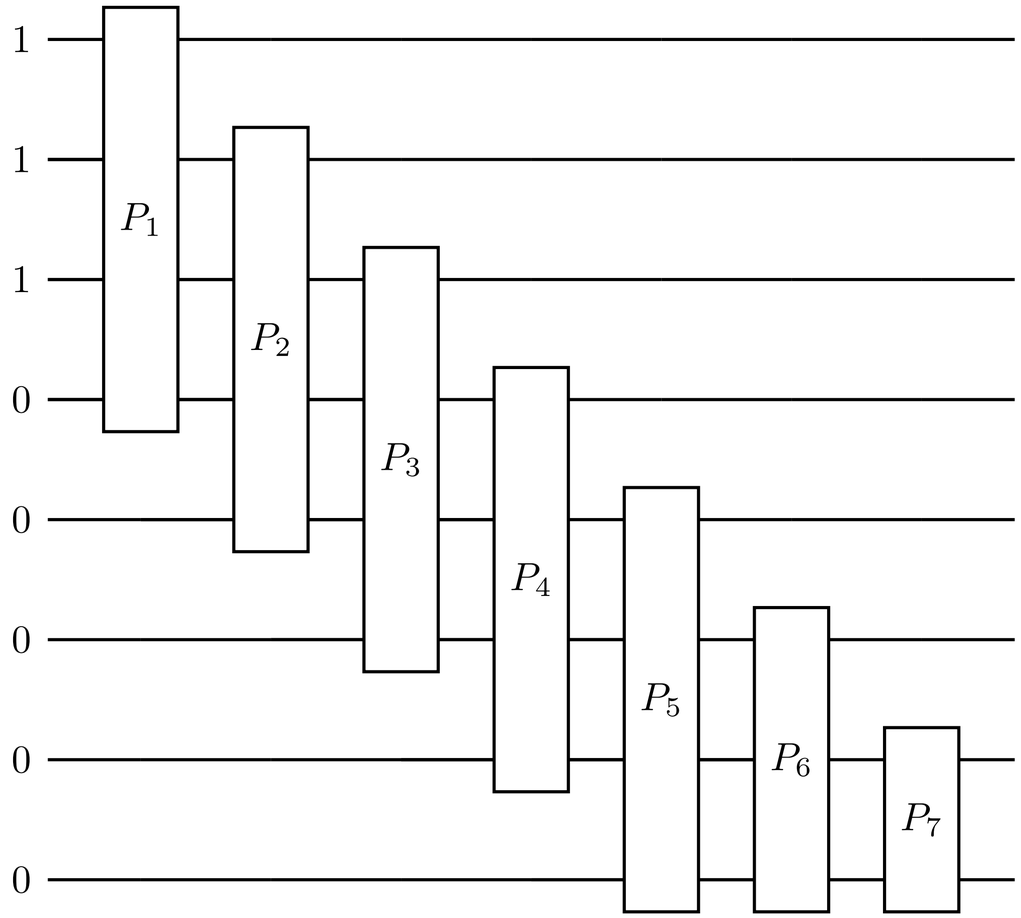

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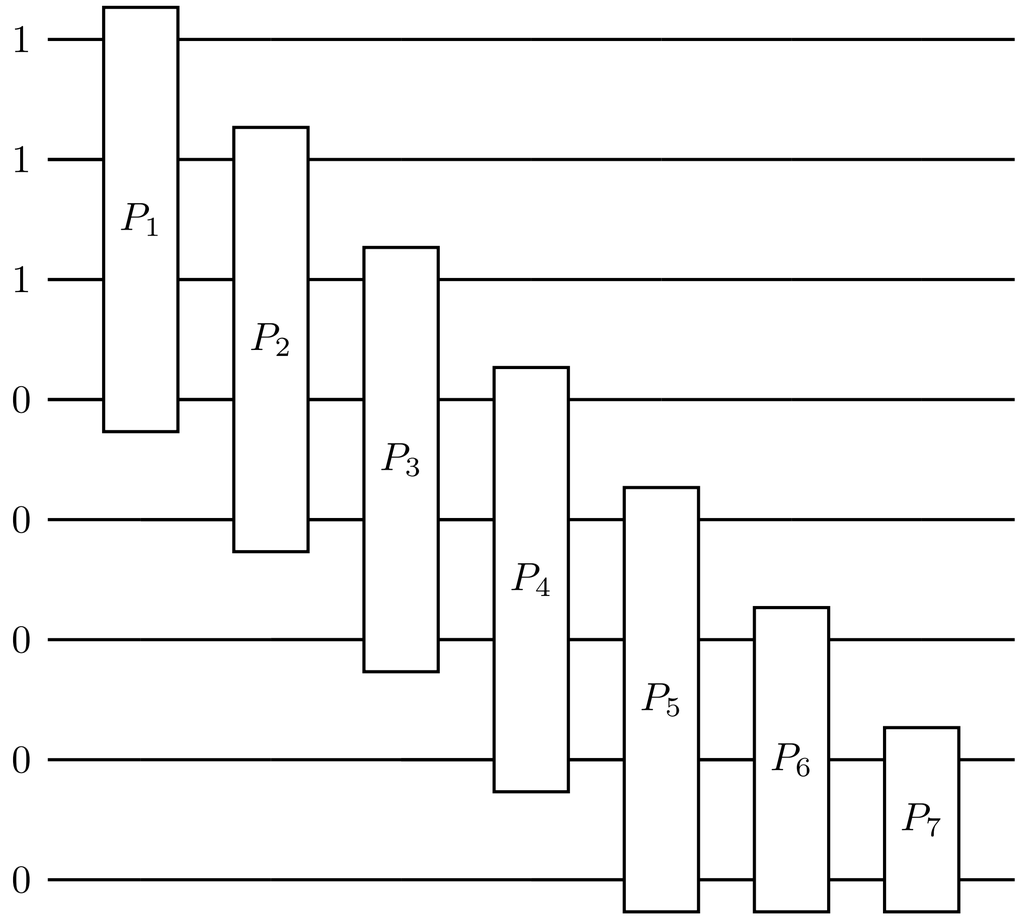

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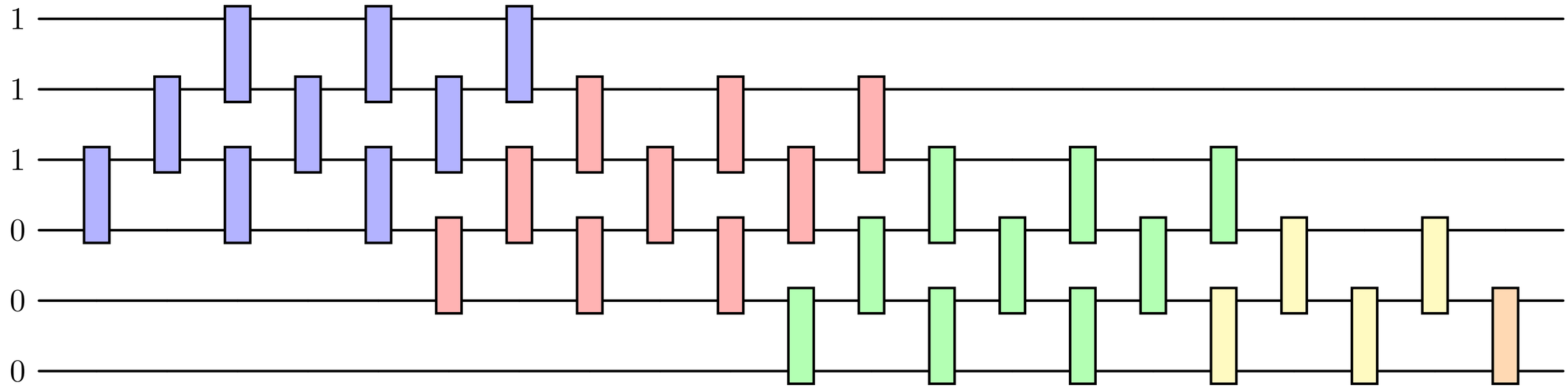
- Factorization unknown!

New idea: Optimization from scratch



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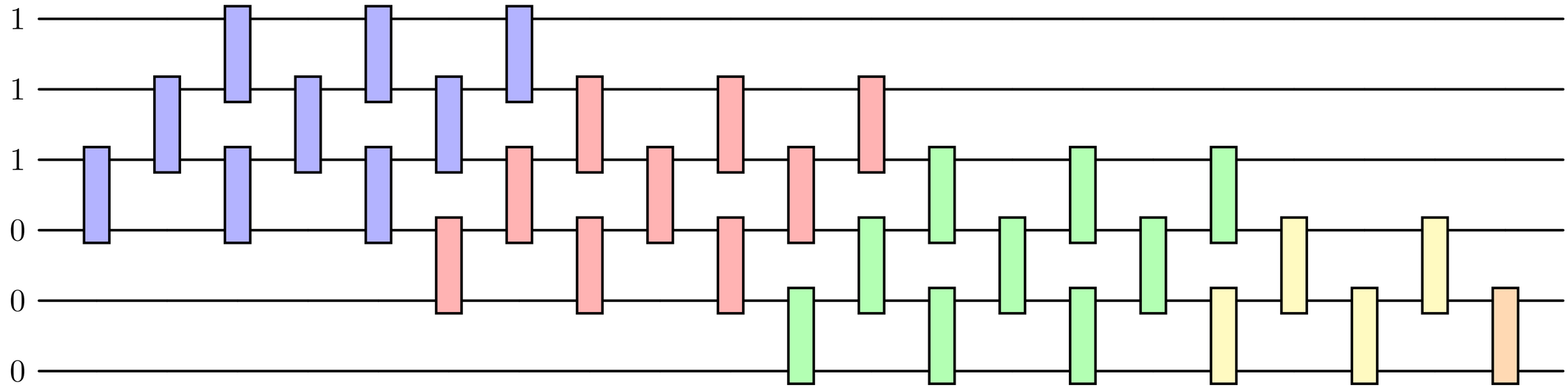

$$F = \begin{pmatrix} 1 & & & \\ & c & d & \\ & -\bar{d} & \bar{c} & \\ & & & e^{i\varphi} \end{pmatrix}, \qquad |c|^2 + |d|^2 = 1$$

Models where polynomial complexity can be proven

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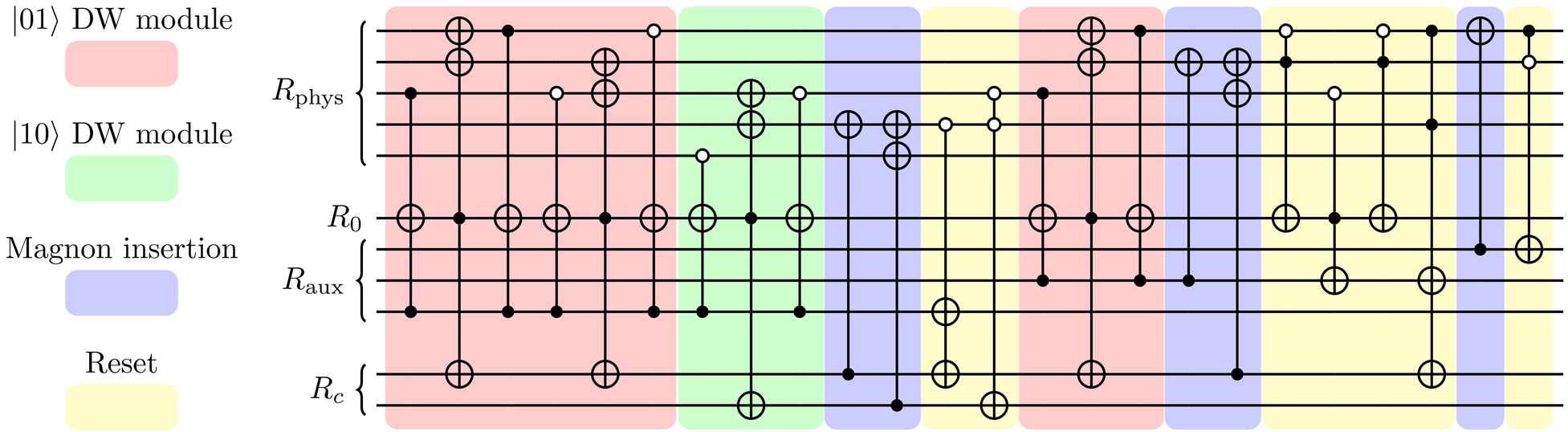
Algorithm:

1. Prepare an eigenstate of the XX model (in smaller volume)
2. Apply the hard rod deformation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Magnon insertion
Reset
R_{phys}
R_0
R_{aux}
R_c

Thank you for the attention!