

$T\bar{T}$ and $\sqrt{T\bar{T}}$ - deformations

in 4d Chern-Simons Theory

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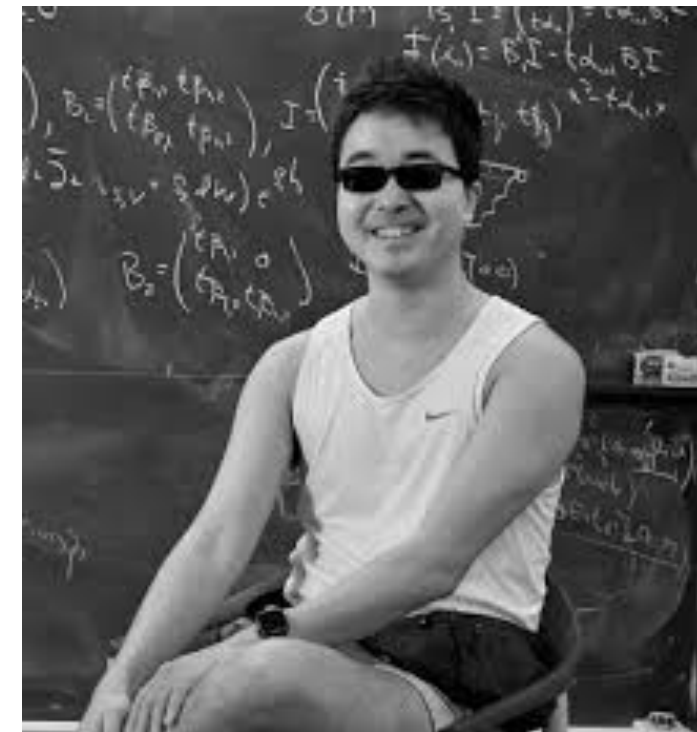
Based on arXiv: 2509.12303 [hep-th]



Roberto
Tateo



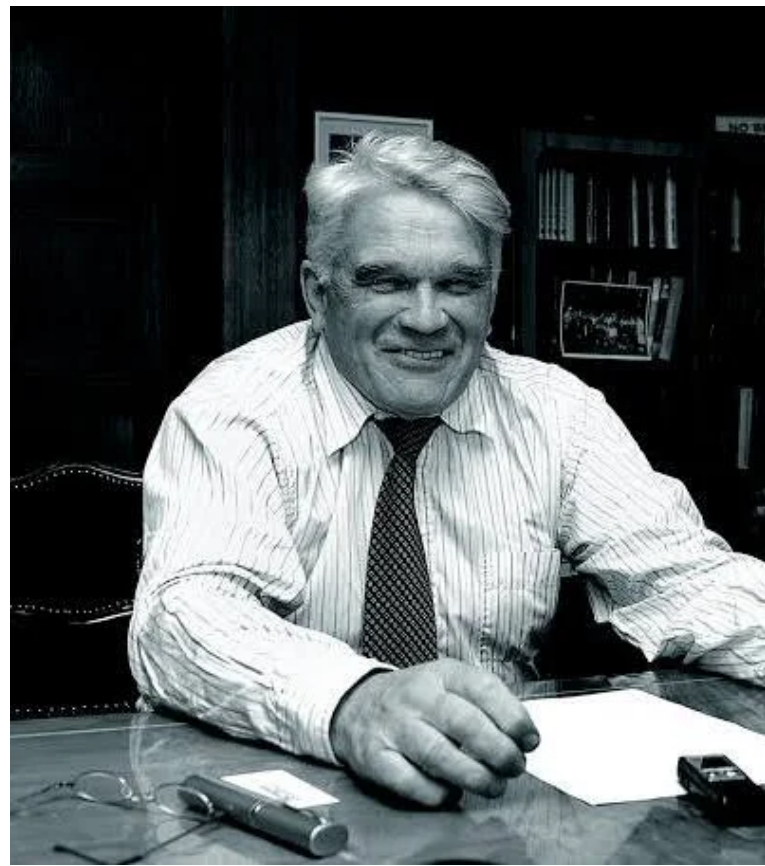
Jun-ichi
Sakamoto



+ M.Y.

Motivations





Faddeev

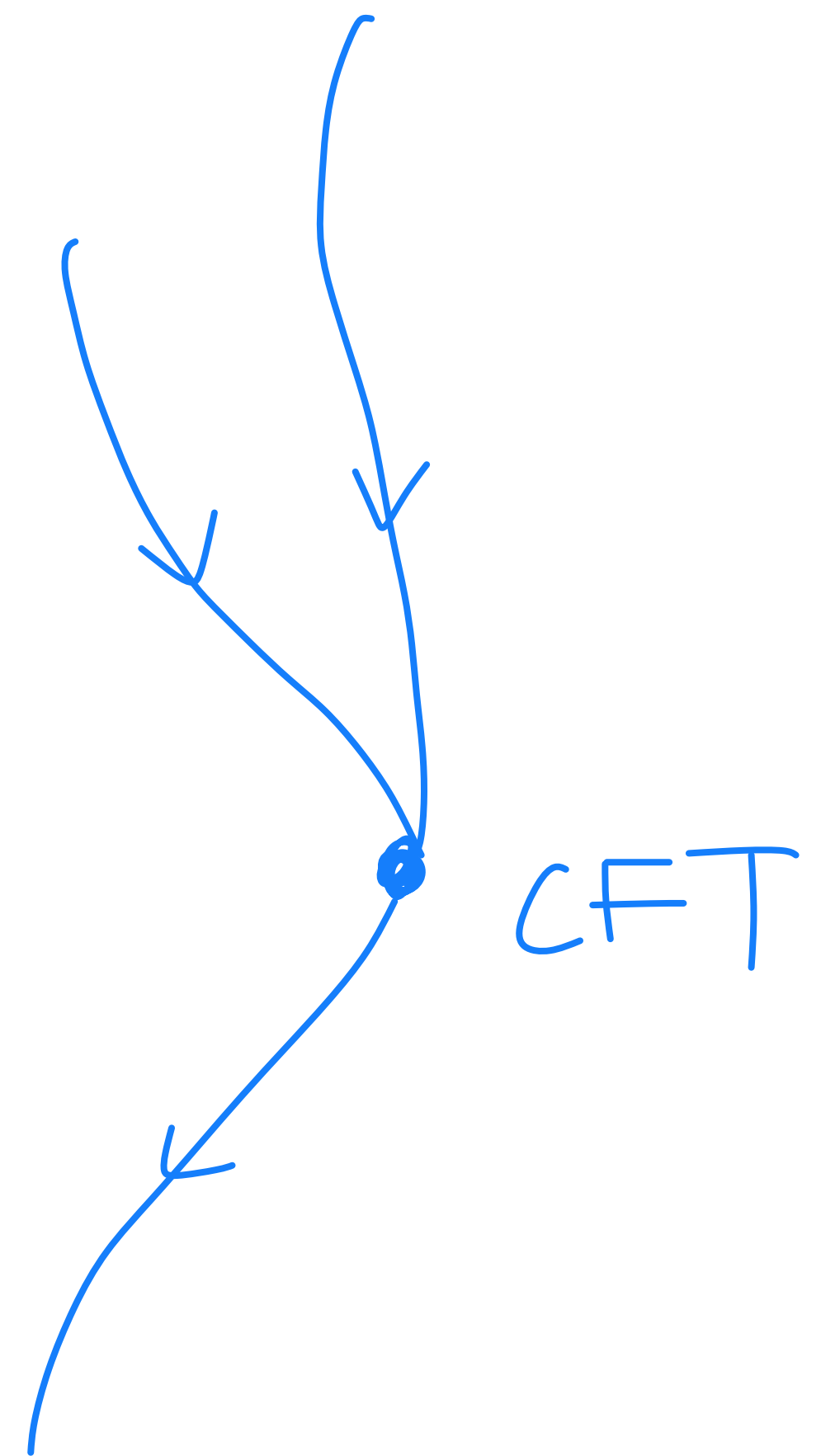
Grand Vision

Integrability $\rightsquigarrow$ QFTs

cf,

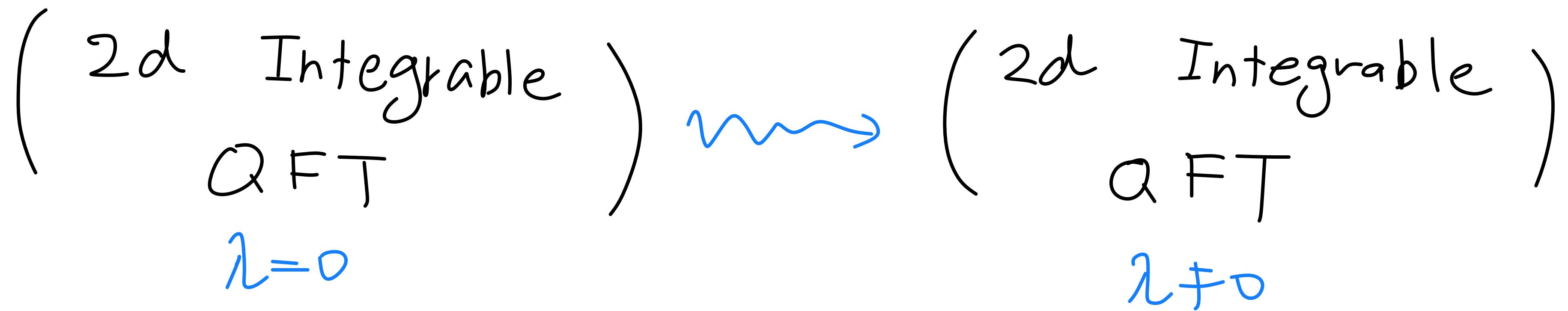
conformal
bootstrap $\rightsquigarrow$ CFTs

RG flow



Question

$T\bar{T}$, $\sqrt{T\bar{T}}$, ... deformations



[Smirnov - Zamolodchikov, Cavaglià - Negro - Szécsényi - Tateo (16)
Conti - Romano - Tateo, Ferko - Sfondrini - Smith - Tortaglino - Mazzucchelli (22)
Babaei - Aghbolagh - Velni - Yekta - Mohammadzadeh (22)]

Question

deformation

(4d CS + defect) $\rightsquigarrow$ (4d CS + defect)
??

[Costello-MY
'19]

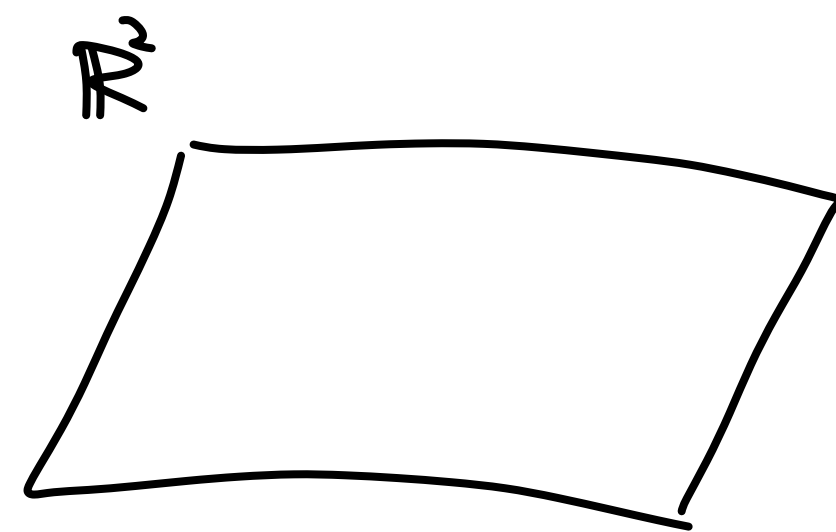
(2d Integrable
QFT
 $\lambda=0$) $\rightsquigarrow$ (2d Integrable
QFT
 $\lambda \neq 0$)
deformation

$T\bar{T}, \sqrt{T\bar{T}}, \dots$

4d CS

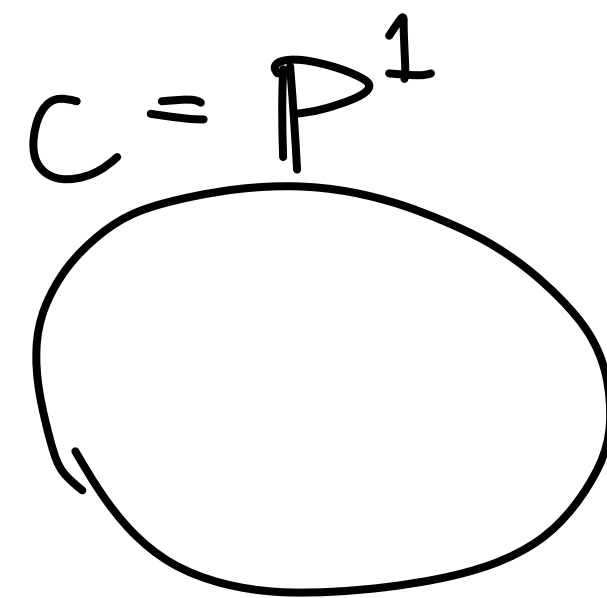
[Costello - Witten - MY ('17, '18)
Costello - MY ('19)]

$$S = \frac{1}{2\pi\hbar} \int_{\mathbb{R}^2 \times C} \omega \wedge \text{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$



topological

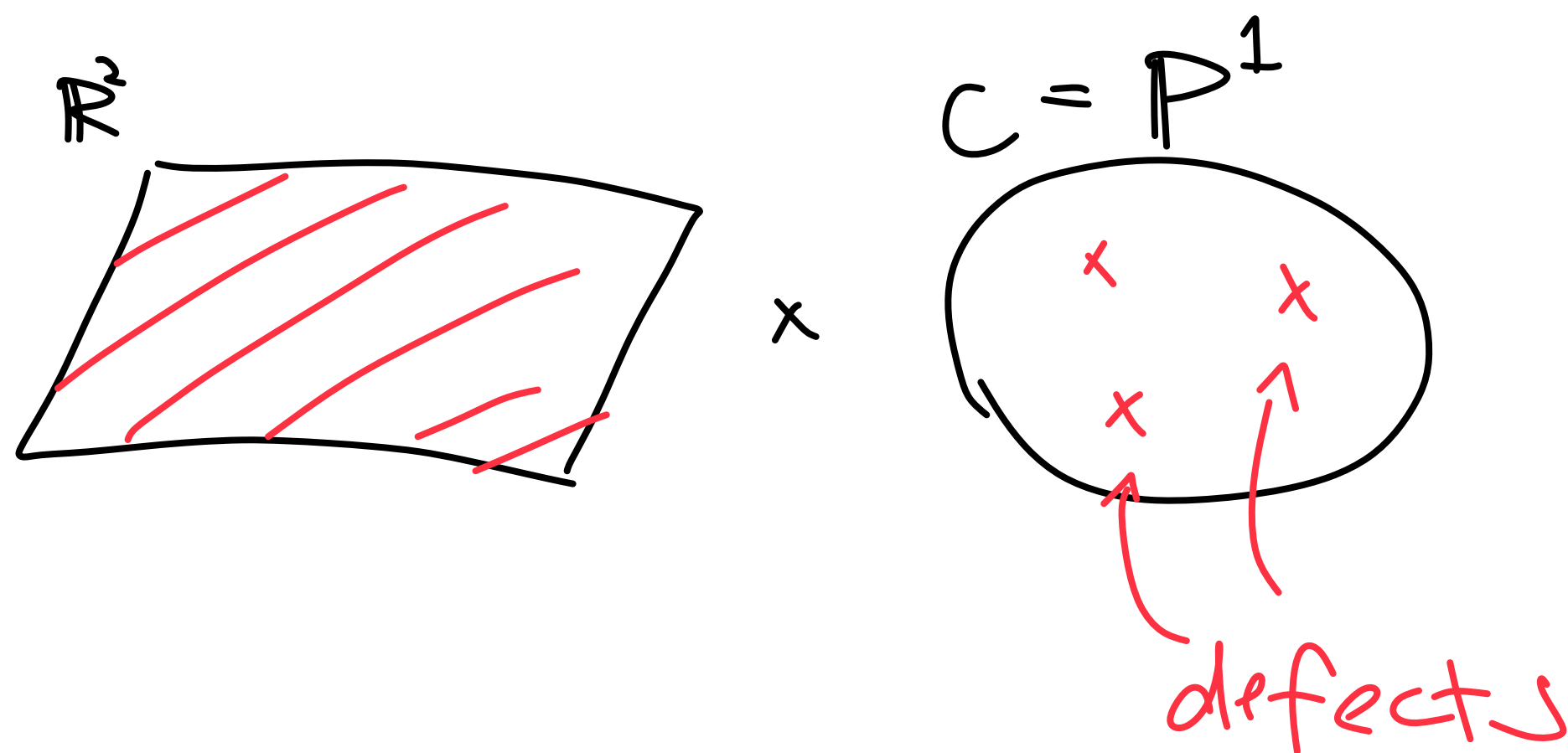
x



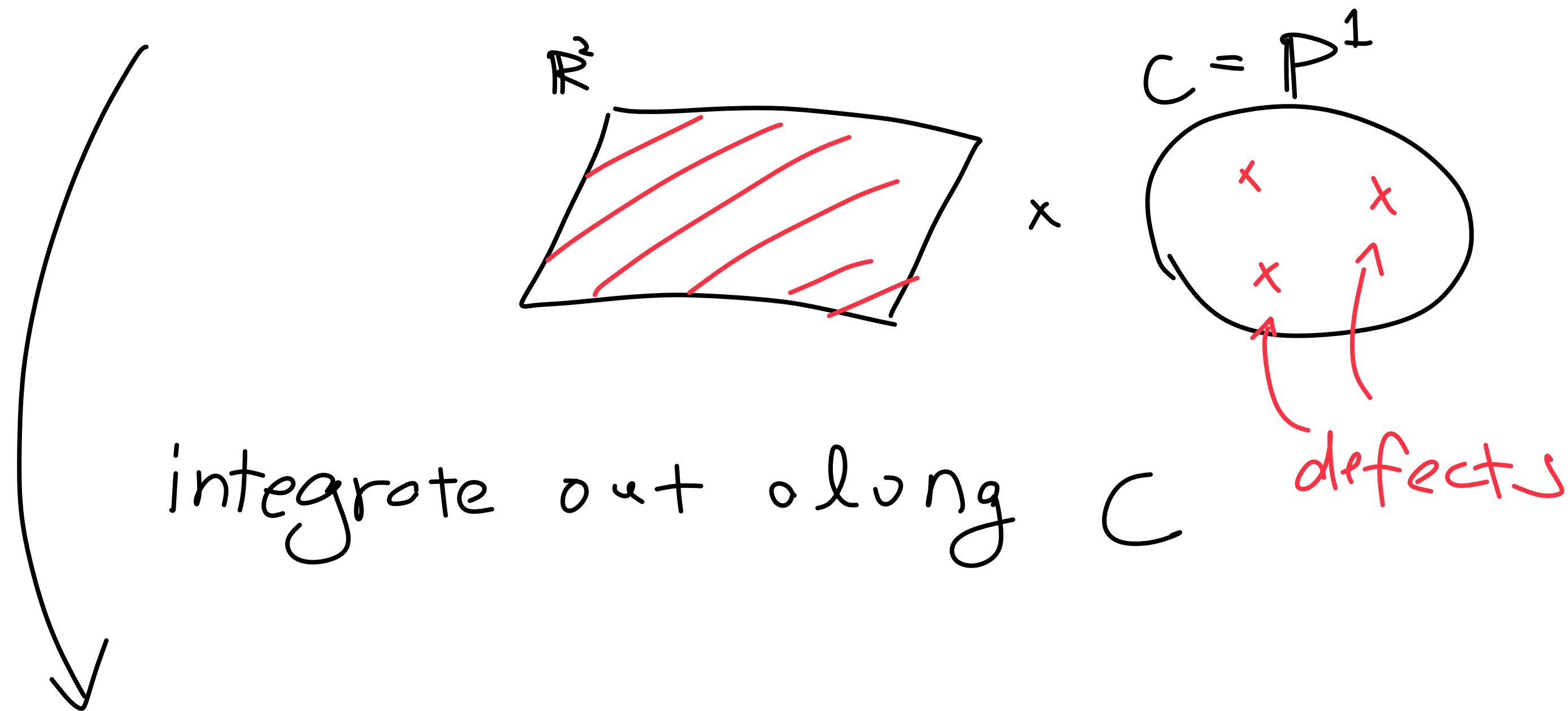
holomorphic

ω : one-form
on C

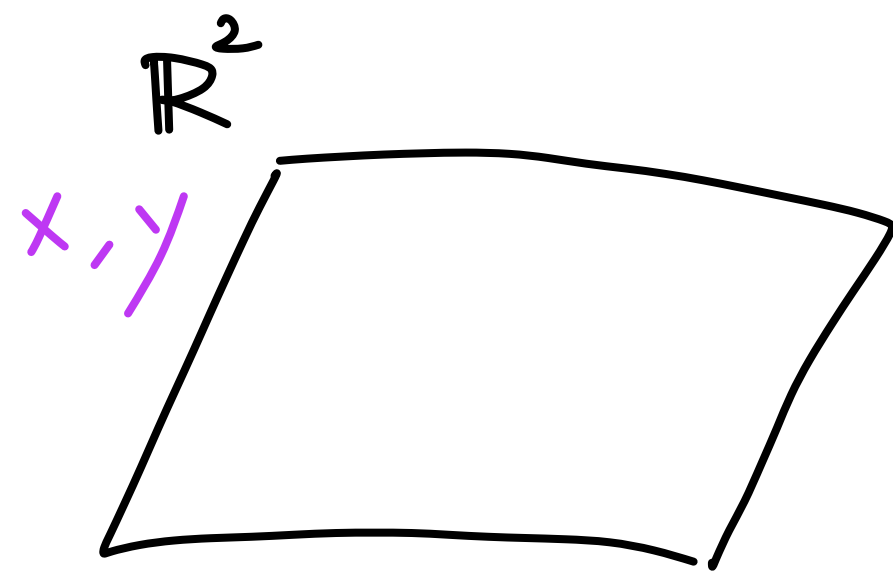
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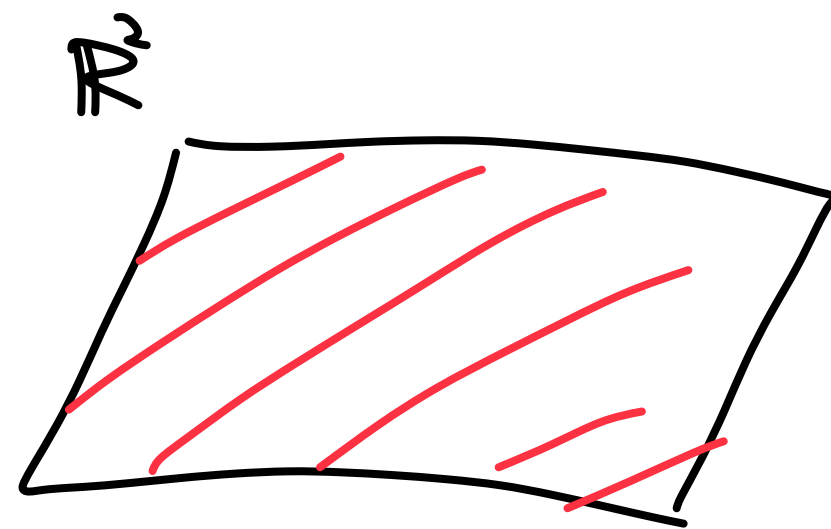
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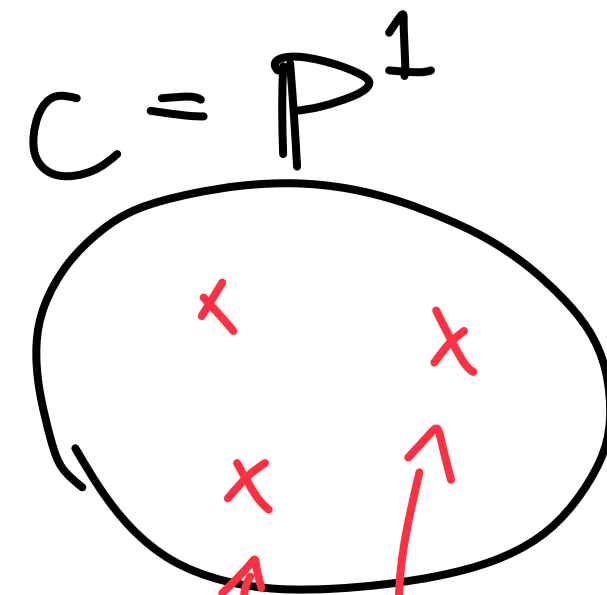
2d IFT



$$S = \frac{1}{2\pi\hbar} \int_{\mathbb{R}^2 \times C} \omega \wedge \text{Tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right)$$



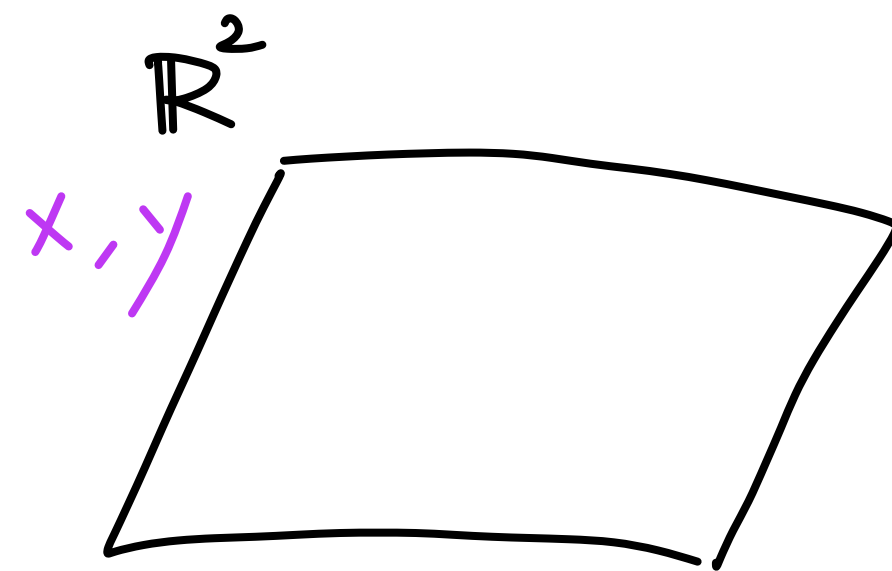
x



$$\left(F = dA + A \wedge A = 0 \right. \\ \left. [e.o.m.] \right)$$

integrate out along C

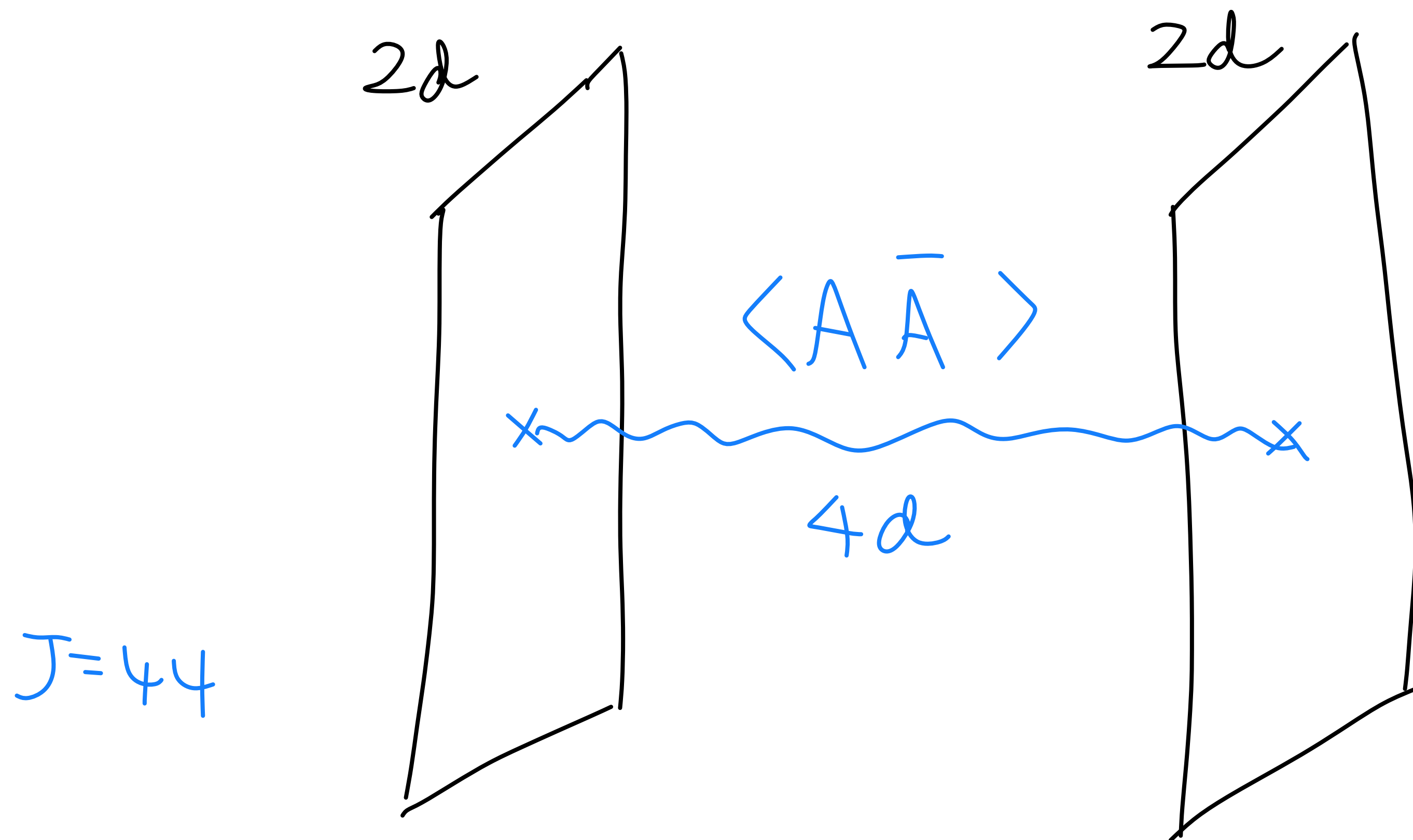
2d IFT



$$\mathcal{L} = A_x(z) dx + A_y(z) dy \quad (A_{\bar{z}} = 0)$$

$$d\mathcal{L} + \mathcal{L} \wedge \mathcal{L} = 0 \quad [Lax \text{ eqn}]$$

e.g. chiral / anti-chiral order defect



$$\psi \bar{\psi} \psi + \psi \bar{A} \psi$$

free chiral

$$\bar{\psi} \partial \bar{\psi} + \bar{\psi} \bar{A} \bar{\psi}$$

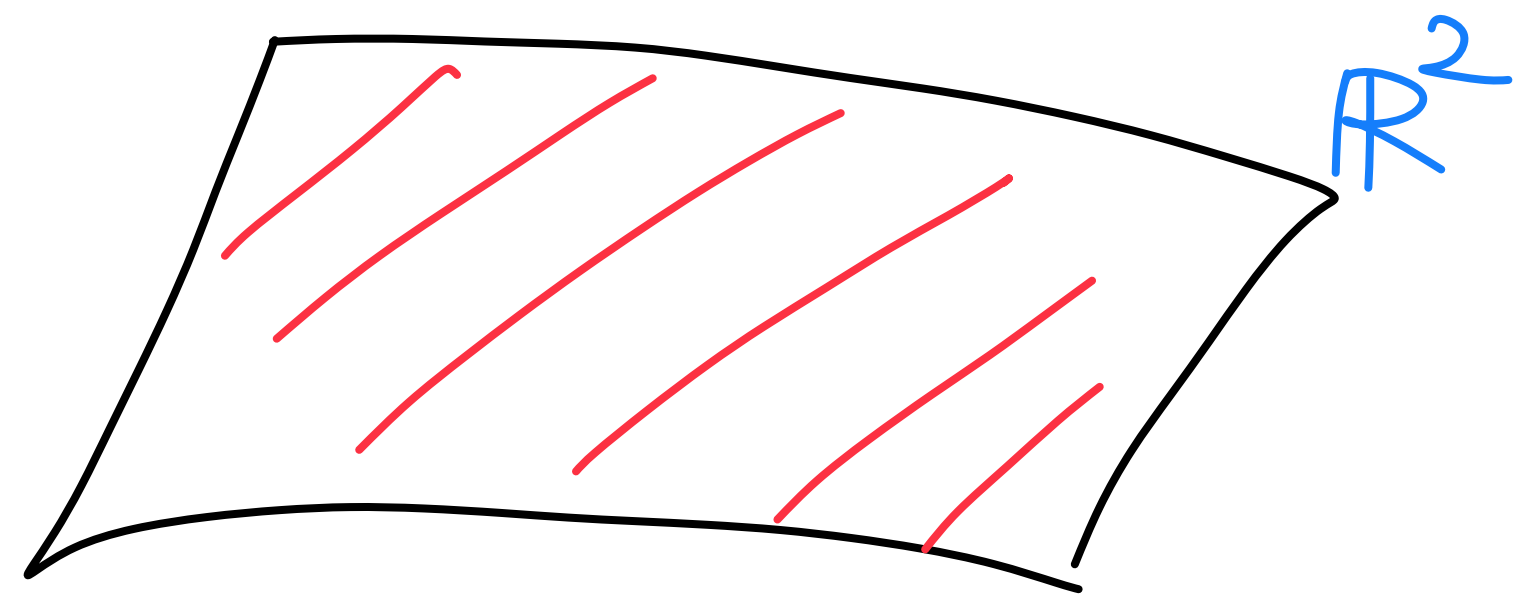
free anti-chiral

4-Fermi interaction

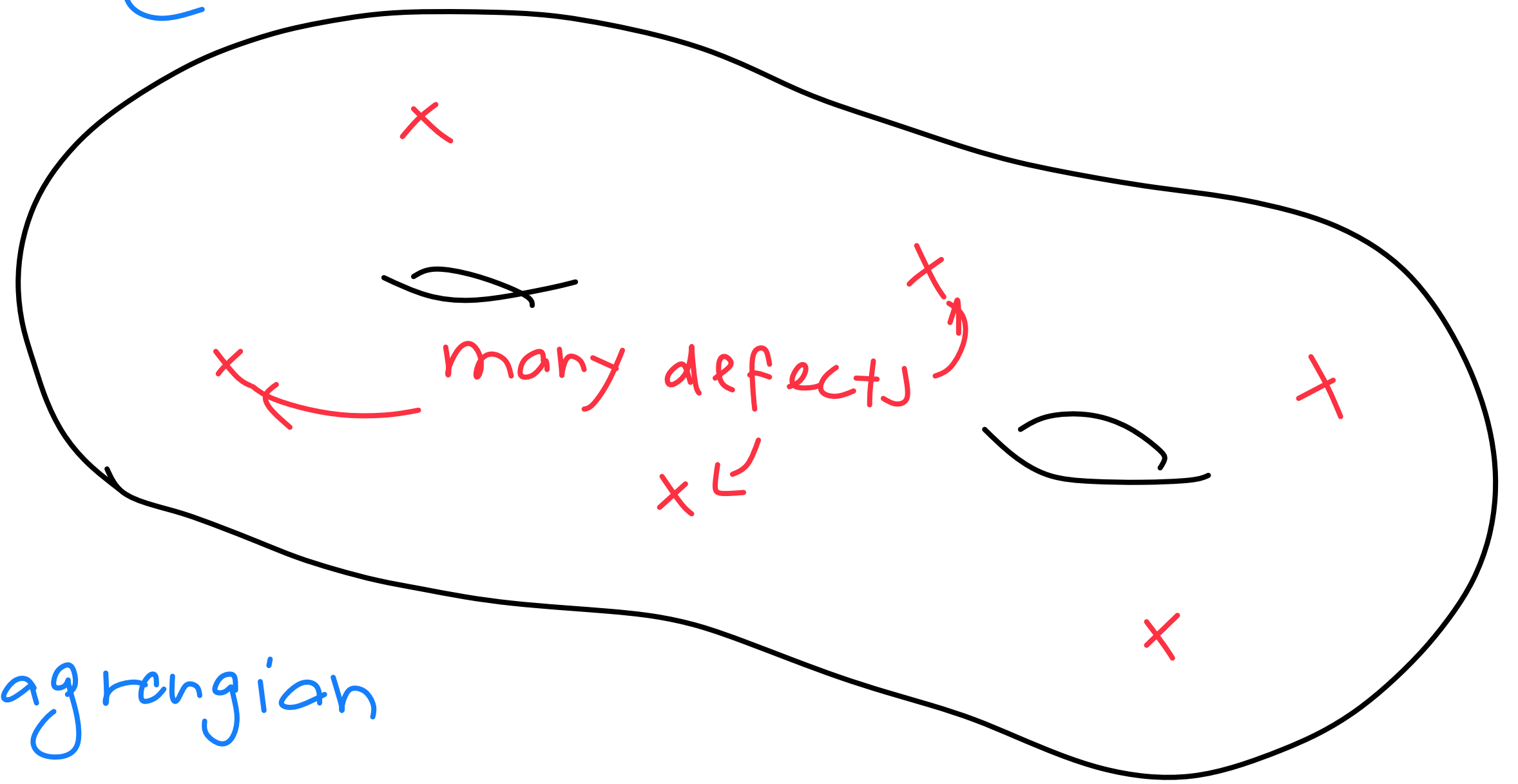
$$J \bar{J} = (\psi \psi) (\bar{\psi} \bar{\psi})$$

$$\bar{J} = \bar{\psi} \bar{\psi}$$

The discussion generalizes



$\times$



$\left\{ \begin{array}{l} \text{Order defect : 2D Lagrangian} \\ \text{disorder defect : singularities of } \omega \end{array} \right.$

- Some deformations, e.g. Yang-Baxter & λ -deformations realized here e.g. [Delduc - Lacroix - Magw - Vicedo (20)]

$T\overline{T}$ in 4d CS?



Questions

- $T\bar{T}$ - deformation: non-linear in λ

$$\frac{d}{d\lambda} L_{\text{PCM}}^{(\lambda)} = \det(T_{\mu\nu}^{(\lambda)})$$

- $T\bar{T}$ - deformation: not the only integrable deformation
e.g. $\sqrt{T\bar{T}}$ - deformation [e.g. Borsato-Ferk-Sfondrini ('22)]

- Extension to general IQFT.

- Gravity should be (at least) 4D, not 3D

We take a different approach
in [Tateo - Sakamoto - MY]

open

$$(b_d \quad c_s)$$

↓ T-duality

(4d CS)
top./hol

$$(2d \text{ IQFT})$$

open

(6d CS)

↓ T-duality

(4d CS)
top./hol

↓

(2d IQFT)

closed

(6d KS gravity)

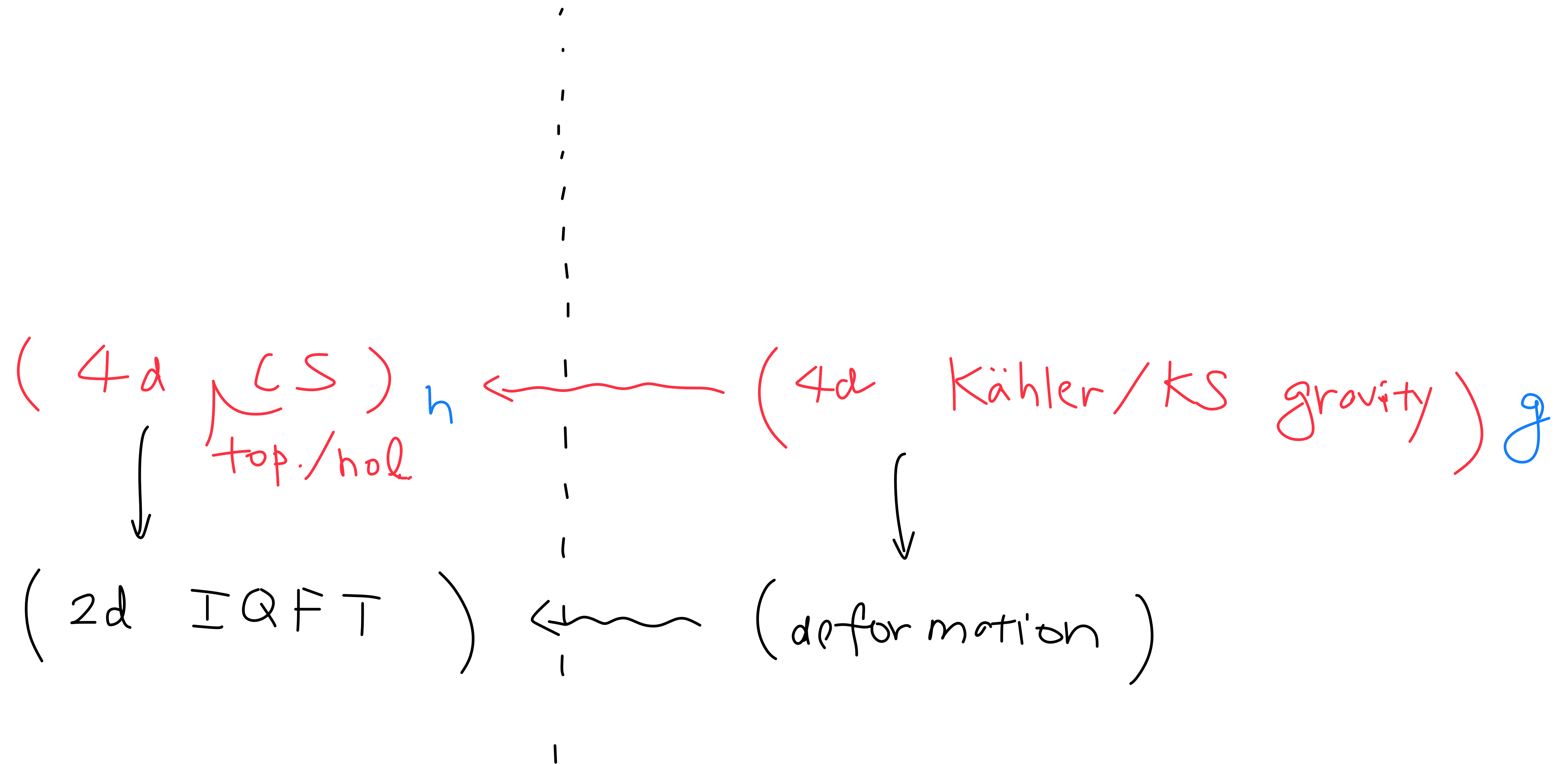
complex str. deformation

↓ T-duality

(4d Kähler/KS gravity)

↓

(deformation)



1.

Solve gravity e.o.m.

$$g_{\mu\nu}[\Phi] = g_{\mu\nu}[T_{\mu\nu}(\Phi)]$$

(4d $\underbrace{CS}_{\text{top./hol}}$)_h



(2d IQFT)

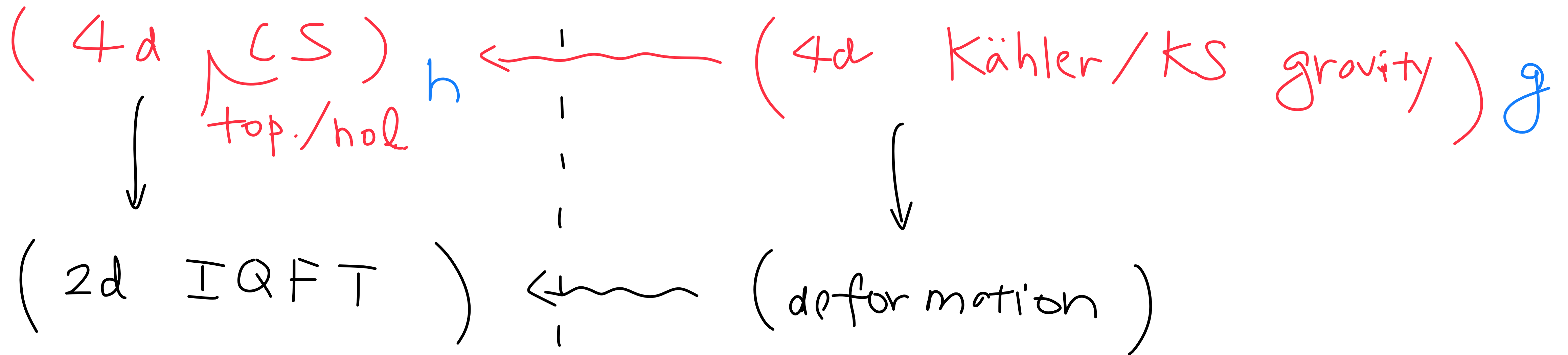


(4d Kähler/KS gravity)_g



(deformation)

2. consider curved bg $g_{\mu\nu}[\Phi]$
1. Solve gravity e.o.m.
 $g_{\mu\nu}[\Phi] = g_{\mu\nu}[T_{\mu\nu}(\Phi)]$
3. coordinate transf.
 Φ -dependent



Indeed, $T\bar{T}$ - deformations in 2D arise from
dynamical / field-dependent transformations

[Dubovsky - Gorbenko - Mirbabayi ('17)]
[Conti - Negro - Tateo ('18)]

We can apply the same idea already in 4D

- 4D / 2D in curved spaces
- $T\bar{T} / \sqrt{T\bar{T}}$ for degenerate $\mathcal{E}$ -model

$T \bar{T}$ from Dynamical

Coordinate Transformation

[Conti - Negro - Tatedo (1/8), Chen - Hou - Tian (1/2)]

$T \bar{T} \leftarrow$ dynamical coordinate transformation

$$ds^2 = - dx^+ dx^-$$

$$\begin{pmatrix} dx^{+'} \\ dx^{-'} \end{pmatrix} = \begin{pmatrix} 1 + \lambda T_{+-}^{(\lambda)} & -\lambda T_{--}^{(\lambda)} \\ -\lambda T_{++}^{(\lambda)} & 1 + \lambda T_{+-}^{(\lambda)} \end{pmatrix} \begin{pmatrix} dx^+ \\ dx^- \end{pmatrix}$$

"Linearizes" the deformation

$$S^{(\lambda)} = \int L^{(\lambda)}(x) dx^+ \wedge dx^-$$

$$= \int \left(L^{(0)}(x) - \lambda \det(T^{(0)}) \right) dx^{+'} \wedge dx^{-'}$$

In general coordinate transf. is harmless in
4D bulk $\Leftarrow$ top. along 2D

$$\mathcal{L} = \mathcal{L}_+ dx^+ + \mathcal{L}_- dx^- = \mathcal{L}'_+ dx^{+'} + \mathcal{L}'_- dx^{-'}$$

In general coordinate transf. is harmless in
4D bulk $\Leftarrow$ top. along 2D

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The situation is non-trivial on the defects.

esp. for field-dependent ones

(a priori no guarantee that integrability be preserved)

The dynamical coordinate transformation was
derived originally for sine-Gordon models

However, the logic should apply much more
generally, e.g. to degenerate Σ -models

[Klimcik - Severa ('95, '96)]

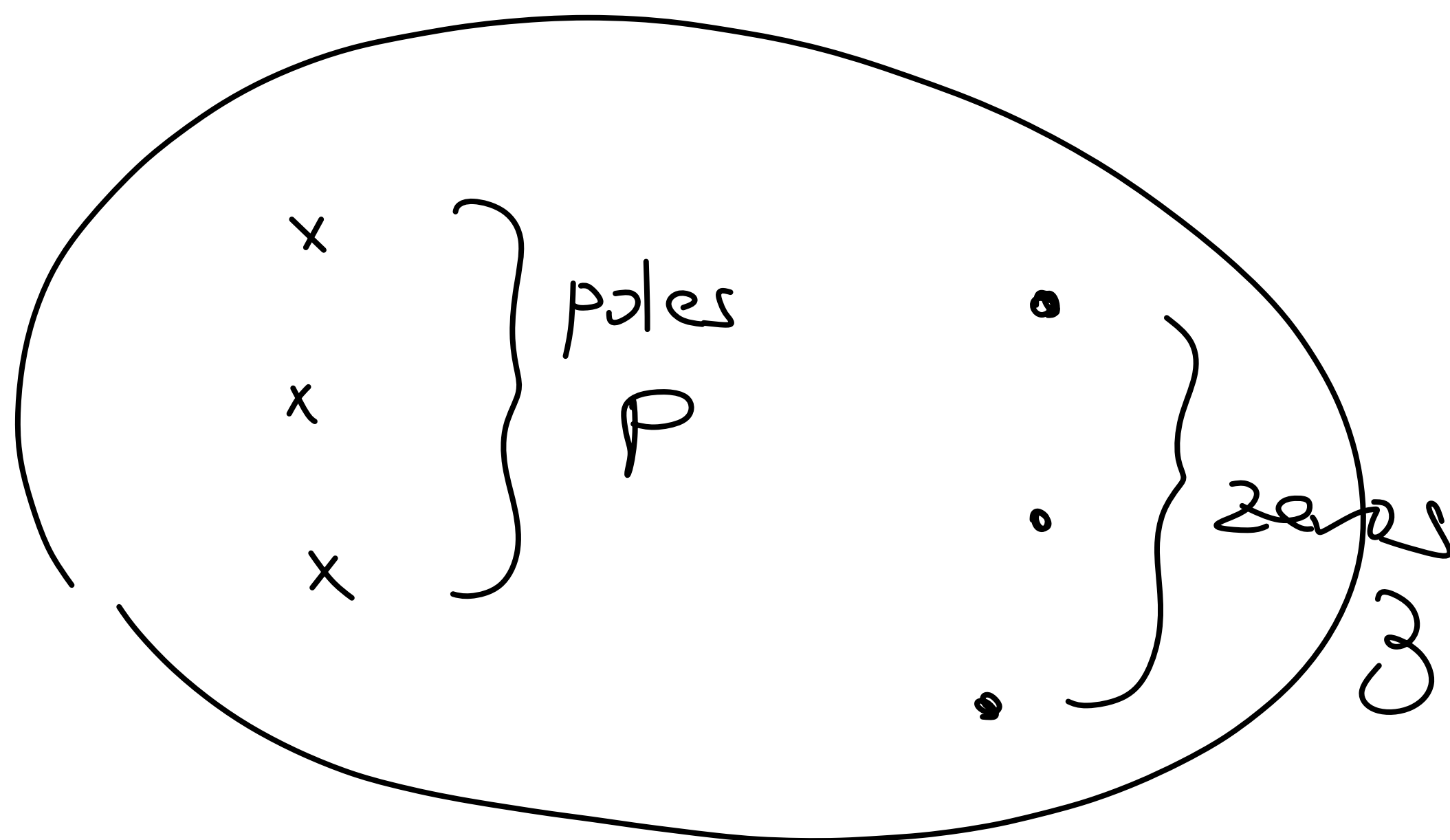
Digression on deg $\mathcal{E}$ -model

[Benini - Schenkel - Vicedo (20)
Lacroix - Vicedo (20), Liníado - Vicedo (23)]

Choose one-form ω to have

set of poles $\mathcal{P}$, zeros $\mathcal{Z}$

$$\omega = \left(\sum_{x \in \mathcal{P}} \sum_{q=0}^{n_x-1} \frac{l_q^x}{(z-x)^{p+1}} - \sum_{q=1}^{n_z-1} l_q^z z^{p-1} \right) dz$$



Need to choose boundary conditions:

poles to w $\rightsquigarrow$ We need expansion of A
up to some order

$$\mathcal{G} = T_x G \rightarrow \underbrace{T_x J^n G}_{\text{jet-bundle}}$$

$$u \mapsto u \otimes e_x^p$$

$$\mathcal{Q} := \prod_{x \in P} \left(\mathcal{G} \otimes \mathbb{R}[\varepsilon_x] / (\varepsilon_x^{n_x}) \right)$$

w/ bilinear pairing $\langle -, - \rangle_{\mathcal{Q}}$

In variation of action

$$\omega \wedge (dA + A \wedge A) = 0$$

We need

$$\int_{\Sigma \times C} \underbrace{d\omega}_{\omega} \wedge \text{Tr}(A \wedge \delta A)$$

only at poles of ω

$$j^*: C^\infty(\Sigma \times C) \rightarrow C^\infty(\Sigma \times P)$$

$$\int_{\Sigma \times P} \langle j^* A, j^* \delta A \rangle_\omega = 0$$



satisfied by

$$A|_P \in \mathcal{K} \subset \mathcal{D}$$

maximally isotropic



When we solve e.o.m. along C

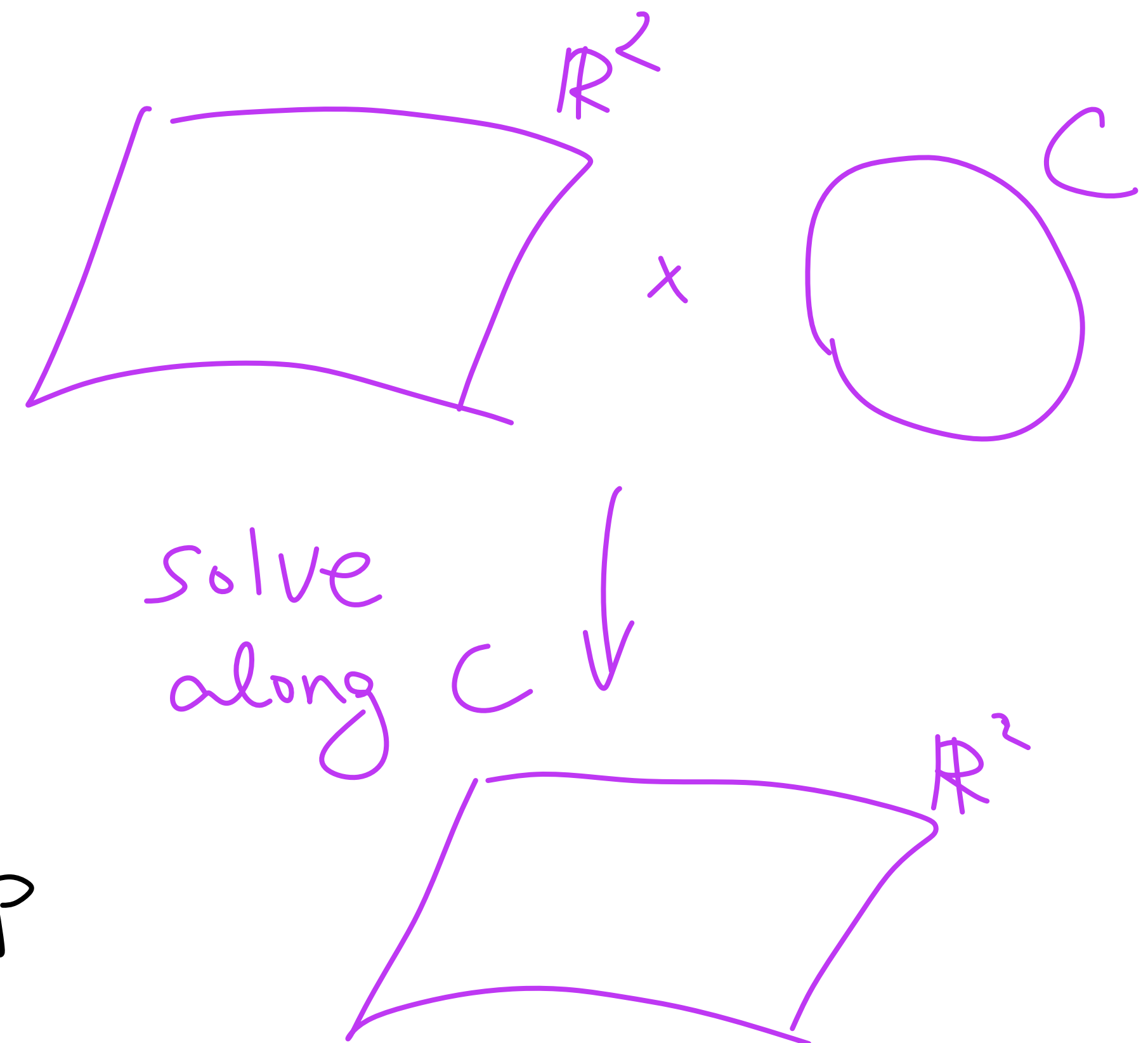
it is useful to make explicit the DOF at poles

h : DOF at the pole of w

$$\uparrow$$
$$C^\infty(\underbrace{\Sigma \times P}_\text{defined @ poles of } w, \underbrace{\mathbb{D}}_\text{takes value in the defect group at poles})$$

defined @
poles of w

takes value
in the defect group
group at poles



h : edge mode defect.

$$S_{\text{defect}}[A, h] = -\frac{1}{2} \int_{\Sigma \times \mathbb{P}} \langle\langle h^{-1} dh, j^* A \rangle\rangle_{\mathcal{L}} \\ + \frac{1}{6} \int_{\Sigma \times \mathbb{P} \times [0, 1]} \langle\langle \hat{h}^{-1} d\hat{h}, [\hat{h}^{-1} d\hat{h}, \hat{h}^{-1} d\hat{h}] \rangle\rangle_{\mathcal{L}}$$

$$[A \mapsto u A u^{-1} - du u^{-1}, \quad h \mapsto h (j^* u)^{-1}]$$

Degenerate $\mathcal{E}$ -model

[Klimcik - Severa ('95, '96)]

$$h \in \mathbb{F} \setminus \mathbb{D} / \mathbb{K}$$

"gauge transf"

diagonal embedding

defect algebra

maximally isotropic
from b.c.

Degenerate $\mathcal{E}$ -model

$$h \in \mathbb{F} \setminus \mathbb{D} / \mathbb{K}$$

we can write down e.g. Lax

$$\mathcal{L}_{\pm}(h) = j_* \left(P_h^{\pm} \underbrace{(h^{-1} \partial_{\pm} h)}_{\substack{\uparrow \\ \mathcal{D}}} \right)$$

$\mathcal{D} \rightarrow$ function
on the whole
 $\mathbb{C}$

$$\left(P_h^{\pm} : \mathcal{A} \rightarrow \mathcal{D} \right)$$

projectors

[Liniado-Vicedo ('23)]

Now ready to apply dynamical
coord. transformation $S \dots$

$$\delta S [\mathcal{L}^{(\lambda)}(h), h]$$

new contribution

$$= \frac{1}{4} \int_{\Sigma \times \mathcal{P}} \delta \gamma^{*(\lambda) \mu \nu} \left\langle h^{-1} \partial_\mu h, (P_h^+ - P_h^-) (h^{-1} \partial_\nu h) \right\rangle_{\mathcal{O}}$$

$$+ \int_{\Sigma \times \mathcal{P}} \left\langle u, d(\mathbb{J}^\dagger \mathcal{L}^{(\lambda)}) + \frac{1}{2} [\mathbb{J}^\dagger \mathcal{L}^{(\lambda)}, \mathbb{J}^\dagger \mathcal{L}^{(\lambda)}] \right\rangle_{\mathcal{O}}$$

flat connection

$$(h \rightarrow e^u h)$$

$$\delta S [\mathcal{L}^{(h)}(h), h]$$

$$= \frac{1}{4} \int_{\Sigma \times \mathcal{P}} \underbrace{\delta \delta^{*(\lambda)} \mu \nu \ll h^{-1} \partial_\mu h, (P_h^+ - P_h^-) (h^{-1} \partial_\nu h) \gg}_\partial$$

$$\parallel \leftarrow ! \quad \left(\text{appendix of our paper} \right)$$

$$4\lambda \delta(\det T^{(x)})$$

This can be cancelled by adding $-\lambda(\det T^{(x)})$
to the action

We thus arrive at

dynamical coord.
transformation

$$\begin{aligned} S^{(\lambda)} &= \int L^{(\lambda)}(x) \, dx^+ \wedge dx^- \\ &= \int \left(L^{(0)}(x) - \pi \det(T^{(0)}) \right) dx^{+'} \wedge dx^{-'} \end{aligned}$$

needed to
preserve integrability

Similar discussion applies to

— $\sqrt{T\bar{T}}$ — deformation

— $T\bar{T}$ combined w/ $\sqrt{T\bar{T}}$

In general involves

both complex / Kähler deformation

Summary

• $T\bar{T}/\sqrt{T\bar{T}}$ deformation : dynamical coord. transf.

in 4d CS + disorder defect,
on $\mathbb{P}^1$



degenerate ε -model

Questions

😊 • $T\bar{T}$ - deformation: non-linear in λ

$$\frac{d}{d\lambda} L_{PCM}^{(\lambda)} = \det(T_{\mu\nu}^{(\lambda)})$$

😊 • $T\bar{T}$ - deformation: not the only integrable deformation

e.g. $\sqrt{T\bar{T}}$ - deformation

😊 • Extension to general IQFT.

(Need to solve gravity e.o.m.)

☹ • Gravity should be (at least) 4D, not 3D

Question:

Can we systematically classify
all integrable deformations?

Questions for closed-string sector

Kähler / BCOV

"deformation as gravity"